



OUTLIER DETECTION AND INFLUENTIAL POINT OBSERVATION IN LINEAR REGRESSION USING CLUSTERING TECHNIQUES IN FINANCIAL TIME SERIES DATA

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ABSTRACT

The modern computing technology makes data gathering and storage easier. This creates new range of problems and challenges for data analysis. Detection of outliers in time series data has gained much attention in recent years. We present a new approach based on clustering techniques for outlier. The Expectation Maximization clusters (EM-Cluster) algorithm is used to find the "optimal" parameters of the distributions that maximize the likelihood function. Regression based outlier technique is used to detect influence point. The analysis of outliers and influential points is an important step of the regression diagnostics. Several indicators are used for identifying and analyzing outliers. The proposed approach gave effective results within optimum time and space when applied to synthetic data set. This paper investigates the outliers, volatility clustering and risk-return trade-off in the Indian stock markets NSE Nifty and BSE SENSEX. Engle's ARCH Test and AR (1)-EGARCH (p, q)-in-Mean model were employed to examine the objective of the study. It revealed that volatility is persistent and there is leverage effect in the Indian stock markets.

Keywords: *EM-Cluster, EGARCH-M, ADF, Centriod, DFBETAS, Cut Value*

1. INTRODUCTION

Data analysis is an area of high relevance in the fields of engineering and technology, bioinformatics, medical science application, financial engineering, natural language processing and customer relationship management. Computationally intelligent method is very much needed to predict and explore optimum decision. Clustering and outlier technique is playing a significant role in computer science research and other fields. The clustering algorithm was driven by biologist Sneath and Sokal in 1963 in numerical taxonomy before being taken by the statisticians. In many clustering methods, clusters are often determined by estimating the location and dispersion of different sample groups within a given dataset [47]. In the literature outliers are found as a by-product of clustering algorithms that are neither a part of a cluster nor a part of the background noise; rather they are specifically points that behave very differently from

the norm. [49][57][34][27][68]. According to [32] Outlier is defined as a data point that is very different from the rest of the data. Such a data point often contains useful information on abnormal behavior in the system that is characterized by the data. [36] Classified outlier detecting approach into two categories: classic outlier approach and spatial outlier approach. The classic outlier approach analyzes outlier based on transaction dataset, which can be grouped into statistical-based approach, distance-based approach, deviation-based approach and density-based approach. The spatial outlier approaches analyze outlier based on spatial dataset, which can be grouped into space-based approach and graph-based approach. [67] discusses a cluster-outlier iterative detection algorithm, tending to detect the clusters and outliers in another perspective for noisy data sets. In this algorithm, clusters are detected and adjusted according to the intra-relationship within clusters and the inter-relationship between clusters and outliers, and vice



versa. In this study we have taken time series data analysis in finance domain. A financial time series is an ordered sequence of observations made through time. Financial time series are mainly dealing with financial market variables such as stock prices, bond values, and foreign exchange rates. There are two main objectives of investigating financial time series. First, it is important to understand how prices behave. The variance of the time series is particularly relevant. Tomorrow's price is uncertain and it must therefore be described by a probability distribution. This means that statistical methods are the natural way to investigate prices. Usually one builds a model, which is a detailed description of how successive prices are determined. The second objective is to use the relevant information of price behaviour to reduce risk or take better decisions. In modelling the financial time series, one key problem is its high noise or the effect of unknown external events which can have a large impact on individual observations, called outliers, which differ greatly from others. Learning observations with outliers without awareness may lead to fitting those unwanted data and may corrupt the approximation function. Returns from financial market variables measured over the period of time (i.e. intra-daily, daily, or weekly) are uncorrelated, but not independent. In particular, it has been observed that although the signs of successive price movements seem to be independent, their magnitude, as represented by the absolute value or square of the price increments, is correlated in time. This phenomena is denoted as volatility clustering, and indicates that the volatility of the series is time varying. Small changes in the price tend to be followed by small changes and large changes by large ones. Since volatility clustering implies a strong autocorrelation in the absolute values of returns, a statistical method for detecting volatility clustering is calculating the autocorrelation function of the absolute values of returns. If the volatility is clustered, the autocorrelation function will have positive values for a relatively large number of lags. Besides, the issue of modelling returns accounting for time-varying volatility has been widely analysed in financial econometrics literature by employing the Generalised Autoregressive Conditional Heteroscedasticity (GARCH)-models [8] techniques.

2. RELATED WORK

In many engineering applications, noisy and missing samples often exist, causing negative

effects on performance of data analysis. The review of the related study has been discussed. [52] proposed PAM clustering algorithm to identify outliers. [1] has proposed context-sensitive clustering technique based on the Bayes decision theory to estimate an unsupervised way the statistical parameters of classes to be used in the Bayesian decision rule. An investigation done by [2] reveals all data clustering algorithms have some ambiguity in some data when clustered. In web access navigation behavior EM-Clustering algorithm shows improved result when compared with k-mean algorithm [58]. The EM algorithm is introduced in order to estimate and improve the parameters of the mixture of densities recursively in color image segmentations [69]. The EM algorithm is presented [34] to estimate the parameters corresponding to a probability density function when we have missing data. In this case the class labels are the missing data. [11] when items are missing the EM algorithm is a convenient way to estimate the covariance matrix at each iteration step of the BACON algorithm. A version of the EM algorithm for survey data following a multivariate normal model, the EEM algorithm (Estimated Expectation Maximization), is proposed. The combination of the two algorithms, the BACON-EEM algorithm, is applied to two datasets and compared with alternative methods. In [3] a search for outliers in two real data sets is shown. It is stressed that identifying outliers should not be done on the basis of asymptotical cutoffs derived under assumption of normality of the analyzed data.[6] both dynamic programming approach (DPA) and grid-based pruning approach (GPA) are used for detecting outliers on uncertain data based on the definition of distance-based method. Despite of outliers and volatility clustering in financial time-series, understanding the risk-return trade-off in existence of it fundamental to equilibrium asset pricing and has been an important topic in research. Many theoretical asset pricing models [61][45][53][50][51] postulates the return of an asset to its own return variance. However, whether such a relationship is positive or negative has been controversial. Many traditional asset-pricing models [61][51] postulate a positive relationship between a stock portfolio's expected return and the conditional variance as a proxy for risk. On the other hand, theoretical works by [7][26][5][65][67] consistently asserts that stock market volatility should be negatively correlated with stock returns. Empirical studies pertaining to the relationship between expected returns and conditional volatility also provides mixed finding. Earlier studies by



[25][16][59][28][41][29][35][37] establishes a positive and significant relationship between expected returns and conditional variance. Besides, the earlier works by [4][63][17][22][42][56][44][15] report a positive but insignificant relationship stock market returns and conditional variance. Furthermore, consistent with the asymmetric volatility argument, several researchers [10][64][54][43][21][9] report a negative and often significant relationship. Given the conflicting results cited above, it is primarily an empirical question whether the conditional first and second moments of equity returns are positively related. Besides, the several emerging markets like India are not weak-form efficient and subject to have asymmetric properties in risk-return characteristics. Hence, the usage of asymmetric econometric models in examining risk-return trade-off could provide more precise results, as Exponential GARCH-in-Mean (EGARCH-M) accommodates an asymmetric relationship between stock price returns and volatility changes under the assumption that both the magnitude and sign of volatility was important in determining the risk-return correlation. Thus, the negative and positive sign of the conditional variance allowed the stock price returns to respond asymmetrically (bad and good news) to rises and falls in stock prices. The purpose of this paper is to investigate the outliers, volatility clustering and risk-return trade-off in the Indian stock markets. The rest of this paper is organized as follows. Section-2 discusses the empirical methodology, Section-3 reports the experimentation, and finally, Section-4 concludes the paper.

3. METHODOLOGY

Jarque-Bera Test: The Jarque-Bera test estimates whether the skewness (S) and kurtosis (K) of the sample matches the normal distribution. The test statistic JB is defined as

$$JB = \frac{n}{6} \left(S^2 + \frac{1}{4} (K - 3)^2 \right) \quad (1)$$

where n is the number of sample, S is the skewness and K is the kurtosis. If the sample comes from a normal distribution, the JB will be asymptotically 2 degrees of freedom chi-square distribution. It should be noted that the Jarque-Bera test is not suitable for small sample because the chi-squared approximation is overly sensitive and often wrongly reject the null hypothesis that the sample is from a normal distribution.

Augmented Dickey-Fuller Test: The testing procedure for the ADF test is the same as for the Dickey-Fuller test but it is applied to the model

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \delta_1 \Delta y_{t-1} + \dots + \delta_{p-1} \Delta y_{t-p+1} + \varepsilon \quad (2)$$

where α is a constant, β the coefficient on a time trend and p the lag order of the autoregressive process. Imposing the constraints $\alpha = 0$ and $\beta = 0$ corresponds to modelling a random walk and using the constraint $\beta = 0$ corresponds to modelling a random walk with a drift. By including lags of the order p the ADF formulation allows for higher-order autoregressive processes. This means that the lag length p has to be determined when applying the test. One possible approach is to test down from high orders and examine the t-values on coefficients. An alternative approach is to examine information criteria such as the Akaike information criterion, Bayesian information criterion or the Hannan-Quinn information criterion. The unit root test is then carried out under the null hypothesis $\gamma = 0$ against the alternative hypothesis of $\gamma < 0$ Once a value for the test statistic

$$DF_\tau = \frac{\hat{\gamma}}{SE(\hat{\gamma})} \quad (3)$$

is computed it can be compared to the relevant critical value for the Dickey-Fuller Test. If the test statistic is less (this test is non symmetrical so we do not consider an absolute value) than (a larger negative) the critical value, then the null hypothesis of $\gamma = 0$ is rejected and no unit root is present.

Ljung-Box Q-Test: The sample autocorrelation function (ACF) and partial autocorrelation function (PACF) are useful qualitative tools to assess the presence of autocorrelation at individual lags. The Ljung-Box Q-test is a more quantitative way to test for autocorrelation at multiple lags *jointly*. The null hypothesis for this test is that the first m autocorrelations are jointly zero,

$$H_0 : \rho_1 = \rho_2 = \dots = \rho_m = 0. \quad (4)$$

The choice of m affects test performance. If N is the length of your observed time series, choosing $m = \ln(N)$ is recommended for power. We can test at multiple values of m . If seasonal autocorrelation is possible, you might consider testing at larger values of m , such as 10 or 15.

The Ljung-Box test statistic is given by

$$Q(m) = N(N+2) \sum_{h=1}^m \frac{\rho_h^2}{N-h}. \quad (5)$$



This is a modification of the Box-Pierce Portmanteau "Q" statistic. Under the null hypothesis, $Q(m)$ follows a χ_m^2 distribution. We can use the Ljung-Box Q-test to assess autocorrelation in any series with a constant mean. This includes residual series, which can be tested for autocorrelation during model diagnostic checks. If the residuals result from fitting a model with g parameters, we should compare the test statistic to a χ^2 distribution with $m - g$ degrees of freedom. It can also be possible to test for conditional heteroscedasticity by conducting a Ljung-Box Q-test on a squared residual series. An alternative test for conditional heteroscedasticity is Engle's ARCH test

Engle's ARCH Test: An uncorrelated time series can still be serially dependent due to a dynamic conditional variance process. A time series exhibiting conditional heteroscedasticity—or autocorrelation in the squared series—is said to have *autoregressive conditional heteroscedastic* (ARCH) effects. Engle's ARCH test is a Lagrange multiplier test to assess the significance of ARCH effects.

Consider a time series

$$y_t = \mu_t + \varepsilon_t, \tag{6}$$

where μ_t is the conditional mean of the process, and ε_t is an innovation process with mean zero. Suppose the innovations are generated as $\varepsilon_t = \sigma_t z_t$, where z_t is an independent and identically distributed process with mean 0 and variance 1. Thus, $E(\varepsilon_t \varepsilon_{t+h}) = 0$ for all lags $h \neq 0$ and the innovations are uncorrelated. Let H_t denote the history of the process available at time t . The conditional variance of y_t is $Var(y_t | H_{t-1}) = Var(\varepsilon_t | H_{t-1}) = E(\varepsilon_t^2 | H_{t-1}) = \sigma^2$ (7) Thus, conditional heteroscedasticity in the variance process is equivalent to autocorrelation in the squared innovation process.

Define the residual series

$$e_t = y_t - \hat{\mu}_t. \tag{8}$$

If all autocorrelation in the original series, y_t , is accounted for in the conditional mean model, then the residuals are uncorrelated with mean zero. However, the residuals can still be serially dependent. The alternative hypothesis for Engle's ARCH test is autocorrelation in the squared residuals, given by the regression

$$H_a : e_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \dots + \alpha_m e_{t-m}^2 + u_t, \tag{9}$$

where u_t is a white noise error process. The null hypothesis is

$$H_0 : \alpha_0 = \alpha_1 = \dots = \alpha_m = 0. \tag{10}$$

To conduct Engle's ARCH test, we need to specify the lag m in the alternative hypothesis. One way to choose m is to compare loglikelihood values for different choices of m . We can use the likelihood ratio test or information criteria to compare loglikelihood values. To generalize to a GARCH alternative, note that a GARCH(P, Q) model is locally equivalent to an ARCH($P + Q$) model. This suggests also considering values $m = P + Q$ for reasonable choices of P and Q . The test statistic for Engle's ARCH test is the usual F statistic for the regression on the squared residuals. Under the null

hypothesis, the F statistic follows a χ^2 distribution with m degrees of freedom. A large critical value indicates rejection of the null hypothesis in favor of the alternative. As an alternative to Engle's ARCH test, you can check for serial dependence (ARCH effects) in a residual series by conducting a Ljung-Box Q-test on the first m lags of the squared residual series with `lbqtest`. Similarly, you can explore the sample autocorrelation and partial autocorrelation functions of the squared residual series for evidence of significant autocorrelation.

Exponential Generalized Auto Regressive Conditional Heteroscedasticity Model:

In order to capture the asymmetric response of volatility to news, [54] proposed EGARCH-M model which allows the conditional volatility to have asymmetric relation with past data. Statistically, this effect occurs when an unexpected drop in stock price due to bad news increases volatility more than an unexpected increase in price due to good news of similar magnitude. Two explanations for asymmetric responses have been put forward. The traditional explanation for this phenomenon was the so-called 'leverage effect' whereby a fall in price results in greater financial leverage, leading to an increase in risk premiums [7][18] Moreover [18] acknowledged that financial leverage alone was not a sufficient explanation to account for the actual size of the observed asymmetries, and an alternative explanation based on market dynamics and the role of noise traders have been expounded [40] and [60]. The EGARCH-M model expresses the conditional variance of a given variable as a non-linear function of its own past values of standardised innovations that can react asymmetrically to good and bad news. The AR(1)-



EGARCH(p, q)-in-Mean model can be specified as follows:

$$R_t = \beta_0 + \beta_1 R_{t-1} + \xi \sigma_t^2 + \varepsilon_t \quad (11)$$

$$\ln(\sigma_t^2) = \alpha_0 + \alpha_1 \ln(\sigma_{t-1}^2) + \delta_1 \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| + \gamma_1 \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \quad (12)$$

where, R_t is the stock market returns of the S&P CNX Nifty and BSE SENSEX Indices at time 't'. R_{t-1} is a proxy for the mean of R_t conditional on past information.

β_0 is comparable to the risk-free rate in the Capital Asset Pricing Model. $\xi \sigma_t^2$ is the market risk premium for expected volatility. This is the most relevant parameter for this study, because the sign and significance of the parameter ξ directly shed light on the nature of the relationship between stock market returns and its volatility. The expected volatility is approximated by σ_t^2 , the conditional variance of R_t such that:

$$\sigma_t^2 = \text{var} (R_t / \psi_{t-1}) \quad (13)$$

where ψ_{t-1} is the information set up to time $t-1$ and, $\text{var}(\cdot)$ is the variance operator. In terms of conditional variance equation (12), $\ln(\sigma_t^2)$ is the one-period ahead volatility forecast. This implies that the leverage effect is exponential rather than quadratic and forecast of conditional variance are guaranteed to be nonnegative. σ_{t-1}^2 denotes the estimation of the variance of the previous time period that stands for the linkage between current and past volatility. In other words, it measures the degree of volatility persistence of conditional

variance in the previous period. $\left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right|$: represents

information concerning the volatility of the previous time period. It signifies the magnitude impact (size effect) coming from the unexpected

shocks. $\frac{\varepsilon_{t-1}}{\sigma_{t-1}}$: indicates information concerning

the asymmetry effects. Unlike the GARCH model, the EGARCH model allows for leverage effect. If γ_1 is negative, leverage effect exists. That is an unexpected drop in price (bad news) increases predictable volatility more than an unexpected increase in price (good news) of similar magnitude. If δ_1 is positive, then the conditional volatility tends to rise (fall) when the absolute value of the standardized residuals is larger (smaller). α 's, β 's,

ξ , δ and γ_1 are the constant parameters to be estimated. ε_t represents the innovations distributed as a Generalized error distribution (GED), a special case of which is the normal distribution.

4. EXPERIMENTATION AND INTERPRETATION

The daily closing prices of two major indexes of Indian stock exchanges, viz., S&P CNX NIFTY and the SENSEX indexes of National Stock Exchange (NSE) and Bombay Stock Exchange (BSE), respectively were used for the study. The database was considered from July 1, 1997 to March 31, 2012, comprising a total of 3,808 observations. The necessary information regarding the daily closing values of the NSE S&P CNX NIFTY and the BSE SENSEX indexes were collected from the official website of National Stock Exchange, Mumbai and Bombay Stock Exchange, Mumbai, respectively. Throughout this paper, stock market returns are defined as continuously compounded or log returns (hereafter returns) at time t , R_t , calculated as follows:

$$R_t = \log (P_t / P_{t-1}) = \log P_t - \log P_{t-1} \quad (14)$$

where P_t and P_{t-1} are the daily closing values of the NSE S&P CNX Nifty and the BSE SENSEX indexes at days t and $t-1$, respectively. To assess the distributional properties of stock market return series of NSE Nifty and BSE SENSEX, descriptive statistics are reported in Table-1. The mean and the standard deviation of NSE Nifty and BSE SENSEX market returns indicates, on average, the positive association between risk and returns in Indian stock markets. Besides, the skewness values of both market return series are negative, indicating that the asymmetric tail extends more towards negative values than positive ones. This reflects that both the market return series are non-symmetric. The kurtosis values of market return series was much higher than three, indicating that the return distribution is fat-tailed or leptokurtic. The market return series of NSE Nifty and BSE SENSEX are non-normal according to the Jarque-Bera test, which rejects normality at one per cent level. Further, the study employed Quantile- Quantile (QQ) plots to assess whether the data in a single series follow a specified theoretical distribution; e.g. whether the data are normally distributed [12]. If the two distributions are the same, the QQ plot should lie on a straight line. If the QQ-plot does not lie on a straight line, the two distributions differ along some dimension. The pattern of deviation

from linearity provides an indication of the nature of the mismatch. Figures -1 and 2 clearly validate that the distribution of the stock market returns series show a strong departure from normality. As can be seen from the figure-3 & 4 of return series for ‘in sample period’, large changes are followed by large changes and small changes are followed by small ones [48]. Therefore it is possible to suspect that there is an outliers and ARCH effect. Table-2 & 3 shows the autocorrelation and partial autocorrelation

functions up to the specified order of lags for stock market return series. These functions characterize the pattern of temporal dependence in the series and typically make sense only for time series data. Both the figures provide the evidence of significant autocorrelation at lag one. As evident from Table-5, the Ljung-Box test statistics $Q(12)$ and $Q^2(12)$ for the return and squared returns series of NSE Nifty and BSE SENSEX confirms the presence of autocorrelation. We can also observe that the both stock market return shows evidence of ARCH effects judging from the significant ARCH-LM test statistics, proposed by [23]. The outlier and heteroscedasticity effects were further confirmed by the visual representation of residual series of S&P Nifty and BSE Sensex returns of figure-5 & 6, respectively. The Augmented Dickey-Fuller (ADF) test was employed to test the stationarity of both market return series and the results are presented in Table-4. Both unit root tests strongly reject the hypothesis of non-stationarity in the case of two market return series. However, despite the unit root test results that the market return series should be considered stationary, returns display a degree of time dependence.

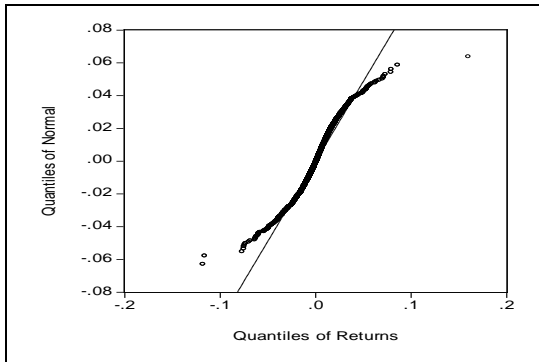


Figure 2 (QQ) Plot Of BSE Sensex Return Series

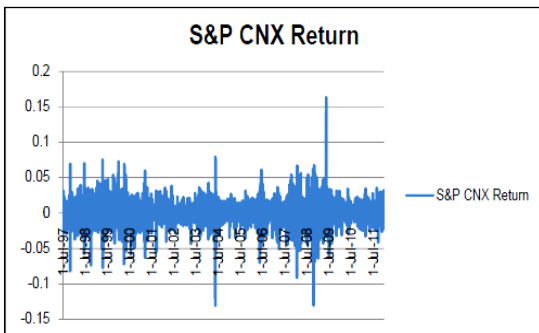


Figure 3 S&P Nifty Return Series

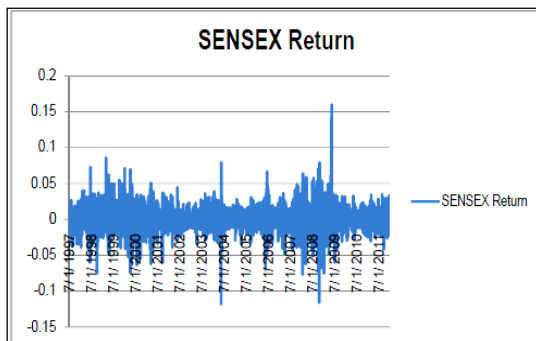


Figure 4 BSE Sensex Return Series

Table .2 Autocorrelation Functions Of S&P CNX Nifty Returns

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob
1	-0.450	-0.450	771.06	0.000	
2	-0.068	-0.340	788.82	0.000	
3	0.012	-0.257	789.40	0.000	
4	0.020	-0.184	790.92	0.000	
5	0.006	-0.127	791.08	0.000	
6	-0.051	-0.166	801.07	0.000	
7	0.015	-0.160	801.90	0.000	
8	0.023	-0.129	803.85	0.000	
9	-0.014	-0.131	804.64	0.000	
10	0.038	-0.060	810.05	0.000	
11	-0.029	-0.062	813.30	0.000	
12	-0.022	-0.097	815.19	0.000	
13	0.013	-0.102	815.80	0.000	
14	0.032	-0.054	819.64	0.000	
15	-0.021	-0.054	821.39	0.000	
16	-0.024	-0.082	823.66	0.000	
17	0.063	-0.000	838.80	0.000	
18	-0.051	-0.043	848.93	0.000	
19	0.029	-0.003	852.05	0.000	
20	-0.048	-0.062	860.83	0.000	
21	0.039	-0.034	866.70	0.000	
22	-0.008	-0.040	866.97	0.000	
23	-0.017	-0.061	868.04	0.000	
24	0.021	-0.052	869.72	0.000	
25	0.009	-0.028	870.03	0.000	
26	-0.019	-0.040	871.38	0.000	
27	0.013	-0.028	872.01	0.000	
28	0.005	-0.004	872.10	0.000	
29	-0.017	-0.018	873.18	0.000	
30	-0.000	-0.018	873.18	0.000	
31	-0.002	-0.034	873.19	0.000	
32	0.014	-0.019	873.93	0.000	
33	-0.014	-0.030	874.67	0.000	
34	0.012	-0.017	875.24	0.000	
35	-0.006	-0.021	875.37	0.000	
36	0.001	-0.022	875.37	0.000	

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob
1	-0.681	-0.681	368.43	0.000	
2	0.182	-0.526	394.74	0.000	
3	0.037	-0.343	395.80	0.000	
4	-0.082	-0.324	401.21	0.000	
5	0.069	-0.274	405.06	0.000	
6	-0.044	-0.286	406.61	0.000	
7	0.035	-0.254	407.60	0.000	
8	-0.020	-0.237	407.91	0.000	
9	0.003	-0.225	407.92	0.000	
10	0.002	-0.228	407.92	0.000	
11	0.020	-0.125	408.25	0.000	
12	-0.065	-0.176	411.67	0.000	
13	0.081	-0.145	417.00	0.000	
14	-0.066	-0.194	420.47	0.000	
15	0.060	-0.125	423.38	0.000	
16	-0.054	-0.119	425.75	0.000	
17	0.025	-0.120	426.25	0.000	
18	0.005	-0.118	426.27	0.000	
19	-0.005	-0.048	426.30	0.000	
20	-0.030	-0.125	427.03	0.000	
21	0.075	-0.041	431.60	0.000	
22	-0.082	-0.016	437.06	0.000	
23	0.043	0.006	438.55	0.000	
24	-0.010	-0.029	438.63	0.000	
25	0.017	0.045	438.85	0.000	
26	-0.028	0.073	439.50	0.000	
27	0.010	0.089	439.58	0.000	
28	-0.002	0.003	439.59	0.000	
29	0.024	0.006	440.07	0.000	
30	-0.034	-0.036	441.00	0.000	
31	0.028	-0.022	441.66	0.000	
32	-0.017	-0.016	441.90	0.000	
33	-0.011	-0.028	442.01	0.000	
34	0.034	-0.015	442.97	0.000	
35	-0.035	-0.002	444.00	0.000	
36	0.019	-0.034	444.30	0.000	

By and large, the return series of NSE Nifty and BSE SENSEX seem to be best described by an unconditional leptokurtic distribution and volatility clustering, and possesses significant ARCH effects.

Table.3 Autocorrelation Function of BSE Sensex

Table-4 Unit Root Test Results Of S&P CNX Nifty & BSE-30 Sensex Returns

Augmented Dickey-Fuller Test			
Variables	Intercept	With Intercept & trend	Without Intercept & trend
NIFTY	-25.448*	-25.462*	-25.401*
SENSEX	-27.931*	-27.949*	-27.894*

Notes: * – indicates significance at one per cent level. Optimal lag length is determined by the Schwarz Information Criterion (SIC).

Thus, the EGARCH-M model is capable with generalised error distribution (GED) is deemed fit for modeling the conditional variance. Further, the EGARCH-M model is capable of capturing, at least partially, the leptokurtosis of a non-conditional return distribution of an economic element as well as the valuable information about the dependence in the squared values of return [24]. Table-6 reports the results of AR(1)-EGARCH(1, 1)-in-Mean estimates for NSE Nifty and BSE SENSEX stock markets. In the mean equation (6), the coefficient ζ turns out to be positive but statistically insignificant. This implies that stock returns are not affected by volatility trends. In other words, conditional variance lacks predictive power for stock returns.

Table-5 Result Of Portmanteau Ljung-Box Test Statistics And Langrange Multiplier Test

Parameters	S&P CNX Nifty Return	BSE-30 SENSEX Return
LB- Q[12]	30.024* (0.003)	35.760* (0.000)
LB ² -Q[12]	643.25* (0.000)	721.96* (0.000)
ARCH-LM[12]	28.405* (0.000)	28.578* (0.000)

Notes: Figures in the parenthesis () indicates p-value. *- denote the significance at one level. Q[12] and Q²[12] represents Portmanteau Ljung-Box (1978) Q-statistics for the return and squared return series respectively. They test for existence of autocorrelation in return and squared return series for 12 lags respectively. Ljung-Box Q test statistic tests the null hypothesis of absence of autocorrelation. ARCH-LM[12] is a Lagrange multiplier test for ARCH effects up to order 12 in the residuals (Engle, 1982).

This result is consistent with the findings of [25][4][14].

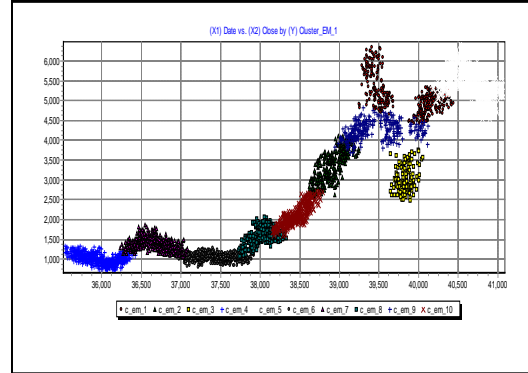


Figure 5 Cluster Wise Representation For BSE Sensex

The present study suggests that investors are not rewarded for the risk they had taken on the Indian stock exchanges. In terms of the conditional variance equation (7), the persistence parameter α_1 was 0.9458 and 0.9563 for the NSE and BSE stock markets, respectively. This suggests that the degree

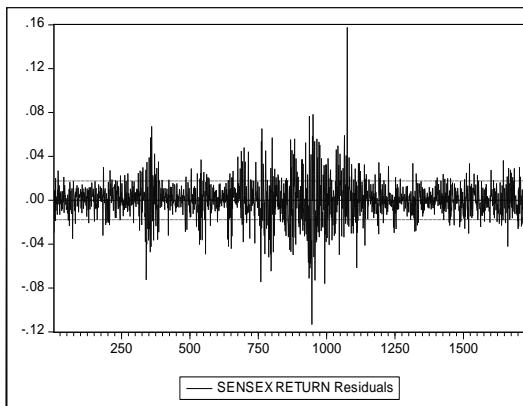
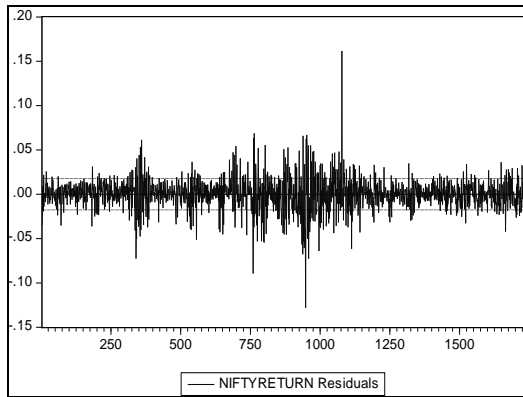


Figure 4 Residual Series Of BSE Sensex Returns

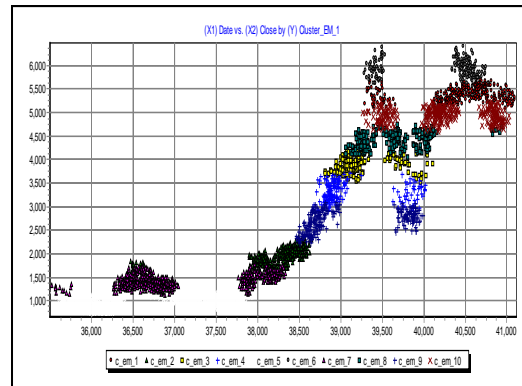


Figure 6 Cluster Wise Representation For NSE Nifty

of persistence is high and very close to one. In other words, once volatility increases, it is likely to remain high and takes longer time to dissipate. The positive and statistically significant coefficient α_1 in the case of both stock markets confirms that the ARCH effects are very pronounced implying the presence of volatility clustering. Conditional volatility tends to rise (fall) when the absolute value of the standardized residuals is larger (smaller) [41]. Besides, the asymmetric coefficient γ_1 in the case of both Indian stock markets was found to be



negative and statistically significant at one per cent level, implying the presence of asymmetric effects. This suggest that there is a larger impact on volatility due to the noise traders in the Indian stock markets during market downward movement than market upward movement under the same magnitude of innovation, i.e. the volatility of negative innovations is larger than that of positive innovations. In addition, Table-6 shows the results of the diagnostic checks on the estimated AR(1)-EGARCH(1, 1)-in-Mean estimates for NSE Nifty and BSE SENSEX stock markets. The Ljung-Box $Q^2(12)$ statistics of the squared standardized residuals are found to be insignificant, confirming the absence of ARCH in the variance equations. The ARCH-LM test statistics further showed that the standardized residuals did not exhibit additional ARCH effect. This shows that the variance equations are well specified in the case of both estimates. In other words, the AR (1)-EGARCH (1, 1)-M process generally provides a good approximation of the data generating process for stock returns under consideration.

5. CLUSTERING

There are a large number of clustering techniques to detect outliers. EM clustering algorithm is chosen to cluster data for the following reasons among others. It has strong statistical basis and linear to data base size. EM is robust to noisy data and can accept the desired number of clusters as input. The EM algorithm proceeds by estimating the missing data (the E-Step) and then estimating the parameters of the model, via maximum likelihood (the M-Step). The number of clusters is a parameter of the algorithm. But we can also detect the “optimal” number of clusters by evaluating several values, i.e. testing 1 cluster, 2 clusters, etc. and choosing the best one (which maximizes the likelihood or another criterion such as AIC or BIC). Detecting automatically the number of clusters and choosing the right number of clusters is a crucial question in a clustering problem. In order to obtain an unbiased result, we must add two enhancements in this basic outline: we use a resampling method (cross-validation) for obtaining an honest likelihood estimate; the likelihood must be counterbalanced with the model complexity (number of clusters) for avoiding the solutions with a high number of clusters (i.e. AIC Akaike or BIC Schwartz criteria). The EM-selection component has been added with the EM-clustering component. It tests various numbers of clusters and select the best one according to the chosen criterion. In our

dataset, we use a resubstitution AIC estimate. We evaluate a number of clusters from 1 to 10. The associated EM-clustering component is automatically updated. We obtain the result as shown in the Table-7. Cluster centriod and cluster size for identified cluster is shown in the Table-8 for the NSE and BSE market data. The visual representation is depicted in the figure 7& 8 for both the return series respectively.

6. OUTLIERS AND INFLUENTIAL POINTS DETECTION FOR REGRESSION

The analysis of outliers and influential points is an important step of the regression diagnostics. The goal is to detect the points which are very different to the others. We implement several indicators for the analysis of outliers and influential points cut of values as shown in Table-11.

Table-7 Em- Clustering Quality Criterion

Parameter	BSE	NSE
Clusters	10	10
# Cluster-Start value	1	1
# Cluster-End value	10	10
Evaluation type	Cross Validation	Cross Validation
Folds For CV	5	5
Random generator	Standard	Standard
Computation Time	172ms	141ms
Criterion Value	710818	631202
AIC	70024.3314	61977.4495
BIC	70204.1636	62157.5983
Log-Likelihood	-34983.1657	-30959.7247
Maximum iteration	10	10
Seed random generator	Standard	Standard
Ridge	0.00100	0.00100
Allocated Memory	34kbs	34kbs

The choice of the cut values is very important. It enables to highlight the suspicious individuals. The threshold value is $2/\sqrt{n}$. Outlier detection evaluates the overall influence of each observation the results have been described in the Table-10 with detail statistics. DFBETAS evaluates the influence of each observation on each estimated coefficient the results are shown in Table-9.

Table-8 Cluster Centriod And Cluster Size

Cluster	EM-Cluster			
	Cluster	Centriod	Cluster Size	
	BSE	NSE	BSE	NSE
C1	3287.8526	5385.4994	919	316



7. CONCLUSION

This paper investigates the outliers, volatility clustering and risk-return trade-off in the Indian stock markets. Jarque-Bera Test, Augmented Dickey-Fuller Test, Ljung-Box Q-Test, Engle's ARCH Test and AR(1)-EGARCH(p, q)-in-Mean model were employed to examine the objective of the study. According to Jarque-Bera test, the stock market return series are not normally distributed. The Ljung-Box test statistics $Q(12)$ and $Q^2(12)$ for the return and squared returns series of NSE Nifty and BSE SENSEX confirms the presence of autocorrelation. Further, we can also observe that the both stock market return shows evidence of volatility clustering/ARCH effects judging from the significant ARCH-LM test statistics, proposed by [23]. The Augmented Dickey Fuller test provided the evidence of stationarity in the case of two market return series. However, despite the unit root test result, the market returns display a degree of time dependence. The empirical results of AR (1)-EGARCH (p, q)-in-Mean model revealed that volatility is persistent and there is leverage effect supporting the work of [54] in the Indian stock markets. Besides, the study revealed positive but insignificant relationship between stock return and risk for NSE Nifty and BSE SENSEX stock markets. This is in accordance with the findings of [17][14][62][38] for the emerging stock markets. The study results also show that market returns are contributed to the high volatility persistence, implying that Indian stock markets are not weak form efficient signifying that there is systematic way to exploit trading opportunities and acquire excess profits. Gaussian mixture model with EM algorithm is a powerful approach for clustering. It is surprisingly rarely implemented in the data mining and statistical software. We use a resampling method (cross-validation) for obtaining an honest likelihood estimate. The choice of the cut values is very important. It enables to highlight the suspicious individuals. The analysis of the residuals is a key step in the linear regression diagnostic process. There are outliers and influential points that are detected and described in this experiment. We use the threshold values in order to highlight the suspicious individuals according to the indicators. But these thresholds are not irrevocable. In most of the situations, it is more convenient to sorting the data according to the indicators in order to highlight the individuals which differ strongly from the others.

Table-9 DFBETAS For Regression Diagnostics

Parameter	# detected	
	BSE	NSE
Intercept	256	266
Close	83	88
Cut Value	0.0331	0.0329
Computation time	390 ms.	374 ms.
Significance level	0.0500	0.0500

Note: #detected according to coefficients

Table-11 Indicators Used To Find Influential Points

Indicator	Cut value
Leverage	$2 * p/n$
RStandard	-
RStudent	2
DFFITs	$2 * \text{SQRT}(p/n)$
Cook's D	$4/(n-p)$
COVRATIO-1	$3 * p/n$

Note: n is the dataset size, p is the number of estimated parameters (number of descriptors + 1 for a regression with intercept).



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Table- 10 Outliers And Influential Point's Detection For Regression

Statistic	Lower bound		Upper bound		# detected	
	BSE	NSE	BSE	NSE	BSE	NSE
Leverage	-	-	0.0011	0.0011	149	161
RStandard	-	-	-	-	0	0
RStudent	-2.0000	-2.0000	2.0000	2.0000	171	124
DFFITs	-0.0468	-0.0466	0.0468	0.0466	102	104
Cook's D	-	-	0.0011	0.0011	102	104
COVRATIO-1	0.9984	0.9984	1.0016	1.0016	87	88

Table-6 Results Of Estimated AR (1)-EGARCH (1, 1)-Mean Model

$$R_t = \beta_0 + \beta_1 R_{t-1} + \xi \sigma_t^2 + \epsilon_t \quad \dots\dots\dots (4)$$

$$\ln(\sigma_t^2) = \alpha_0 + \alpha_1 \ln(\sigma_{t-1}^2) + \delta_1 \left| \frac{\epsilon_{t-1}}{\sigma_{t-1}} \right| + \gamma_1 \frac{\epsilon_{t-1}}{\sigma_{t-1}} \quad \dots\dots\dots (5)$$

S&P CNX Nifty Return								
β_0	β_1	ξ	α_0	α_1	δ_1	γ_1	$Q^2[12]$	ARCH-LM[12]
0.0006 (1.863)***	0.0988 (5.524)*	0.1183 (0.082)	-0.6655 (-13.03)*	0.9458 (176.70)*	0.2739 (17.07)*	-0.1148 (-11.47)*	6.4934	0.5261
SENSEX (BSE-30) Return								
0.0007 (2.193)**	0.0994 (5.261)*	-0.7996 (-0.554)	-0.5539 (-12.55)*	0.9563 (209.80)*	0.2461 (16.49)*	-0.1056 (-11.25)*	9.0710	0.7198

Notes: Figures in parenthesis are z-statistics, *, ** and ***- denotes the significance at one, five and ten percent level, respectively. $Q^2(12)$ and $LM^2(12)$ represents the Liung Box Q statistics for the model squared standardized