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THE ANALYZATION OF ADAPTIVE ALGORITHMS' PERFORMANCES OF AIRSPACE ANTI-JAMMING TECHNOLOGY

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ABSTRACT

This paper briefly discussed the principle of the linear constraints minimum variance (LCMV) algorithm and the least mean square (LMS) algorithm on the basis of the construction of an uniform circular antenna array. Then it compared the parameters of the two, such as signal-to-noise ratio, bit error rate, the average error, convergence rate, and so on, and then it was simulated by MATLAB. The simulation results showed that performances of LCMV algorithm were a little bit worse than LMS algorithm when interference was relatively strong, but much better than LMS when SNR was above 0dB. At last, we found out the appropriate algorithm for each environment, and proposed the improvement measures.

Keywords: *Adaptive algorithm, antenna array, LCMV algorithm, LMS algorithm*

1. INTRODUCTION

Airspace anti-jamming technology is an effective way to solve the problem which GNSS system is easily to be interfered. Its essence is to control the antenna array by adaptive algorithm, so as to achieve the goals that could enhance the useful signals and suppress the coherent interference and broadband interference effectively. Antenna array is composed by the multi-antenna array and real-time adaptive processors. The processors adjust a group of weights according to the certain laws and algorithms while system is working, so as to adjust and optimize the antenna direction pattern, besides, to search and track the useful signals, and suppress or eliminate the interferences. The standards to measure the performances of the adaptive algorithms are in the following aspects: (1) the computational complexity of the algorithms, which means the required numbers of multiplication and plus operation. Its performances determine the hardware performances requirements and cost [1]. (2) The convergence rate of the algorithms, which is the iteration numbers of the optimal solution of the converged algorithms in a static environment. (3) The robustness of the algorithms, which means the algorithms can work normally while input abnormally, or the algorithms can converge on what conditions. Nowadays, LCMV algorithm is widely used in airspace anti-jamming technology, because the interference energy can be greatly weakened while the useful signals are basically unaffected, by using the characteristic that the power of GPS signals is much lower than the power of noise. While LMS algorithm is also widely used because of its simplicity, unrequirement of data storage, small amount of calculation, etc [2].

This paper will be structured as follows: section 2 includes two part, the first one is the construction of the signal model of multi-antenna array, the second part is the analysis and description of the two algorithms' principles; section 3 are the results which we simulated these algorithms' parameters, such as convergence rate, average value of error, relationship between SNR and BER, and so on; section 4 is the conclusion to the whole paper, and proposes several research directions to improve the performances of the algorithms.

2. ALGORITHM DESCRIPTION

2.1 Signal Model of Multi-Antenna Array [3]

In this paper, we first constructed an uniform circular multi-antenna array (M elements), and q signals arrived at the antenna (the angles of incidence direction were $\theta_1, \theta_2, \cdots, \theta_q$). Then sampled the received data from antenna array, and the k times' sampled data was represented as follows.

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$$x_k = \sum_{i=1}^{q} s_i(k)a(\theta_i) + N(k), k = 1, 2, \cdots \quad (1)$$

The expression of the correlation matrix of the received data is as follows, when the desired signals, interferences and noise were unrelated.

$$\begin{aligned} \vec{R}_x &= E\{x(k)x(k)^H\} = \sigma_1^2 a(\theta_1)a^H(\theta_1) \\ &+ \sum_{i=2}^{q} \sigma_i^2 a(\theta_i)a^H(\theta_i) + \vec{R}_N \\ &= \vec{R}_s + \vec{R}_i + \vec{R}_N \end{aligned} \quad (2)$$

The output is as follows when the received data from antenna array were beamformed.

$$y(k) = \vec{W}^H \vec{x}(k) \quad (3)$$

2.2 Principle of LCMV Algorithm

The main idea of LCMV algorithm is to minimize the output signals' power under the certain linear constraints. The established equation which based on LCMV algorithm criteria is as follows, when the desired signals' steering vectors are assumed [4-8].

$$\begin{cases} MinP_{out} = E\left\{|y[n]|^2\right\} P_{out} = E\left\{|y[n]|^2\right\} P_{out} = E\left\{|y[n]|^2\right\} \\ \vec{W}^H a(\theta_i) = 1 \end{cases} \quad (4)$$

The optimal solution of weight vector by using Lagrange algorithm is as follows.

$$\begin{cases} \vec{w}_{opt} = P_{\min out} \vec{R}_{xx}^{-1} a \\ P_{\min out} = (a(\theta_i)\vec{R}_{xx}^{-1} a(\theta_i))^{-1}(\theta_i) \end{cases} \quad (5)$$

In order to make sure the desired signals pass through without power loss, and inhibit the interference and noise effectively, use the adaptive beam-forming technology which based on LCMV criterion, while the direction of arrival (DOA) of the desired signals is known. Its mathematical expressions are as follows.

$$w = \arg\min \vec{w}^H \vec{R} \vec{w} \quad s.t. \vec{w}^H \tilde{a} = 1 \quad (6)$$

$\tilde{a}$ is the desired signal steering vector which is assumed. The optimal weight vector is as follows which is calculated by using the Lagrange operator.

$$\vec{W}_{opt} = \vec{R}^{-1}\tilde{a}(\tilde{a}^H \vec{R}^{-1}\tilde{a})^{-1} \quad (7)$$

The minimum output power of the system when the interference and noise can be suppressed effectively under the circumstance, that the obtained optimal-weight-vector could ensure the certain direction gain of the desired signals. Thus, we get the antenna arrays' output power.

$$P = \frac{1}{\tilde{a}^H \vec{R}^{-1} \tilde{a}} \quad (8)$$

2.3 Principle of LMS Algorithm [9-13]

LMS algorithm is the gradient steepest descent method, which means it could solute the optimum weights iteratively under the minimum mean square error (MMSE) criterion.

$$E\{|\varepsilon(n)|^2\} = E\{|d(n)|^2\} - 2\vec{w}^H \vec{R}_{xd} \vec{w} + \vec{w}^H \vec{R}_{xx} \vec{w} \quad (9)$$

$d(n)$ is the useful signal, $\vec{w}^H \vec{x}(n)$ is the output signal, $E\{|\varepsilon(n)|^2\}$ is the mathematical expectation of mean square error between the output signals and statistical signals.

$$\begin{cases} \vec{R}_{xd} = E\{d^*(n)\vec{x}(n)\} \\ \vec{R}_{xx} = E\{\vec{x}(n)\vec{x}^H(n)\} \end{cases} \quad (10)$$

The optimal value is converged to the minimum mean square error finally, by searching the mean square error at the negative gradient direction. To iterate the initial weight value by using Wiener optimal solution as the best value, and the $n+1$ times iterated weight value is as follows.

$$\vec{w}_{n+1} = \vec{w}_n + \mu[\vec{R}_{xd} - \vec{R}_{xx}\vec{w}_n] \quad (11)$$

$\vec{R}_{xd}, \vec{R}_{xx}$ are replaced by estimated value in the practice. Taken the mathematical expectation on both sides of the equation, and the simplified equation is as follows.

$$\begin{aligned} E\{\vec{w}_{n+1}\} &= E\{\vec{w}_n\} + \mu[\vec{R}_{xd} - \vec{R}_{xx}E\{\vec{w}_n\}] \\ &= (1 - \mu\vec{R}_{xx})E\{\vec{w}_n\} + \mu\vec{R}_{xx}\vec{w}_{opt} \end{aligned} \quad (12)$$

It is assumed that $\vec{w}_d$ is the weight deviation vector, which is the remainder of $\vec{w}_n$ and $\vec{w}_{opt}$, which the former one is the n times weight, and the latter one is the optimal weight. $\vec{Q}$ is the orthonormalized matrix of $\vec{R}_{xx}$. The equation is as follows.

$$E\{\vec{w}'_{d(n+1)}\} = (\vec{I} - \mu\vec{\Lambda})^n E\{\vec{w}'_{d(0)}\} \quad (13)$$

3. MATLAB SIMULATION

3.1 SNR and BER

Antenna array output:

$$SNR = \frac{\vec{w}_{opt}^H \vec{R}_\tau \vec{w}_{opt}}{\vec{w}_{opt}^H (\vec{R}_i + \vec{R}_N)\vec{w}_{opt}} = \frac{\sigma^2 |\vec{w}_{opt}^H a(\Delta\phi)|^2}{\vec{w}_{opt}^H (\vec{R}_i + \vec{R}_N)\vec{w}_{opt}} \quad (14)$$

So $\tilde{a} \approx a(\Delta\phi) \quad \vec{w}^H a(\Delta\phi) = 1$ (15)

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$$SNR = \frac{\sigma^2}{\vec{w}_{opt}^H (\vec{R}_i + \vec{R}_N)\vec{w}_{opt}} \qquad (16)$$

In the simulation, we chose a 12-element uniform circular antenna array as the research subject, and input eight signals from different incident directions, the signal wavelength(λ) is 0.1903m (corresponding f is the GPS f_1 carrier frequency). The experimental results were shown in Figure 1. The BER (bit error rate) of them were both about 10^{-1} when SNR was -10dB. But the BER of the two were rapidly reduced to 10^{-5} when SNR increased to 0dB, thus it could be ignored. And the BER of them were rapidly declining continuously while SNR was increasing, and they were almost the same.

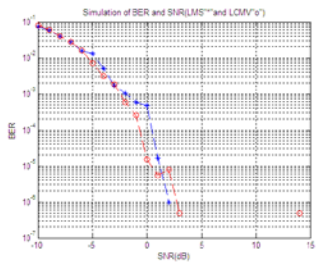

Fig1 Comparison of BER between LCMV and LMS

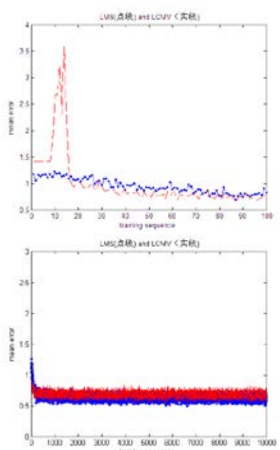

Fig2 Comparison of convergence rate and average error between LCMV (red) and LMS (blue) when SNR=-10dB

3.2 Convergence Rate and Average Error

In regard to LMS algorithm, if the i th element of $w_d^{'}$ is $w_{di(n)}^{'}$, its expression is as follows.

$$w_{di(n)}^{'} = (1-\mu\lambda_i)^n w_{di(0)}^{'} \qquad (17)$$

According to the principle and the expression of LMS algorithm: (1) the convergence rate of the corresponding algorithm became faster while the value of μ became larger, when it satisfied the convergence condition; (2) the convergence rate changed relatively slow, when the distribution range of the characteristic value was relatively large.

The two algorithms were compared in different conditions that the SNR (signal-to-noise ratio) were at -10, 0, and 20dB, and experiment conditions and parameters were still the same. The results were shown in Fig 2 to Fig4.

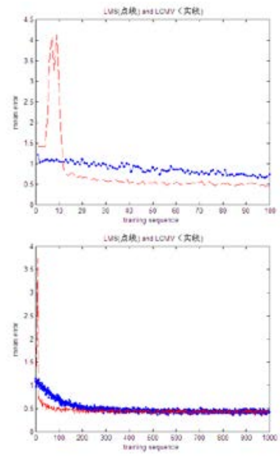

Fig3 Comparison of convergence rate and average error between LCMV(red) and LMS (blue)when SNR=0dB

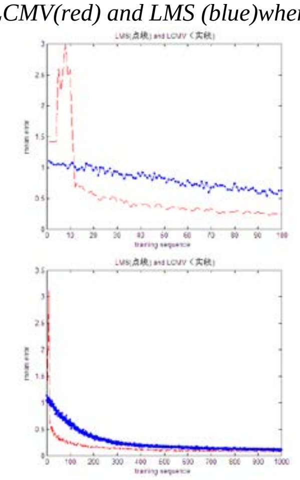

Fig3 Comparison of convergence rate and average error between LCMV (red) and LMS (blue) when SNR=20dB

Comparing the two algorithms by analyzing the results of the three kinds of situation of the simulation, we could find out: (1)the average error at the beginning of LCMV was larger than LMS（μ =0.0005）, and the oscillation amplitude was also more severe than LMS; (2) the average error and the oscillation amplitude were both reduced at the steady time, while the SNR was increasing, but LCMV was much better than LMS; (3) the convergence rate of the two algorithms were both decreased, while SNR was increasing, but LCMV

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was from 100 times to about 200 times which was much faster than LMS, which was from 100 times to about 700 times .

3.3 Comprehensive Analysis

Comparing the results of 3.1 and 3.2, the conclusion is as follows. (1) When interference was greater, such as SNR = -10dB, the BER of them were both larger and similar, which from 10^{-1} to 10^{-3}. Also, the average error of them was similar. The convergence rate and the initial error of LMS were better than LCMV algorithm. (2) Interference became smaller which SNR was from 0dB to 20dB. The BER of them were all smaller and closer to each other which were only 10^{-5}, which means it could be ignored. The initial error of LMS was smaller than LCMV, but the convergence rate and average error at the steady time were much worse than LCMV. (3) The program running time of LCMV was longer than the LMS algorithm, because LCMV algorithm is much complex than LMS in the same conditions.

4. SUMMARY

This paper illustrated the principle and related performances of LCMV and LMS, which on the basis of using antenna array to control the airspace anti-jamming technology's performances by adaptive algorithms, and simulated them by MATLAB programs. Generally speaking, the average error and the oscillation amplitude at the steady time of LCMV were worse than LMS when interferences were relatively strong which had verified by experiment already, but the performances of convergence rate and average error were much better than LMS while SNR increased above 0dB. So, in my opinion, there are many ways to improve the performances according to the characteristics of the two in future studies, such as changing the step size to adjust the convergence rate of LMS algorithm under strong interference [14], or using power inversion and other method to improve the real-time performance of LCMV algorithm [15], or adjusting the array's structure and number, and both of them will play a more important role in the future.

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