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# IRREVERSIBLE EMERGENCY MATERIAL ALLOCATION MODEL WITH DISASTER INFORMATION UPDATE

[1]GE HONGLEI [2]LIU NAN

[1]Ningbo Institute of Technology, Zhejiang University, Ningbo, China
[2]School of management, Zhejiang University, Hangzhou, China
E-mail: [1]gehonglei33@126.com , [2] nliu@zju.edu.cn

**ABSTRACT**

As the allocation of emergency materials in emergency response stage is an irreversible decision and the disaster information is continuously observed and updated, this paper brings the determination of decision-making time and the formulation of decision scheme into a systematic framework, builds the Bayes sequential decision model for the multiple rescue points selection problem, thus making the total loss of the affected point be the minimum and the response time be the shortest. Through simulation, this paper analyzes the relation among relevant parameters including prior mean value of disaster information, prior standard deviation of disaster information, observation standard deviation of disaster information, maximum observation frequency and total expected decision loss.
**Keywords:** *Allocation Of Emergency Material; Rescue Point Selection; Disaster Information Uptade; Irreversible Decision; Bayes Sequential Decision*

## 1. INTRODUCTION

In emergency response stage, the uncertainty of emergency evolution makes emergency logistics decision full of challenges. So the emergency logistics plans must have elasticity so they can be adjusted [1]-[2]. In current research, there are two dynamic methods for emergency logistics modeling. One is Scheduling and Re-scheduling method and the other is Disruption Management method. The Scheduling and Re-scheduling method means to firstly build a mathematics model and realize global optimization according to the original disaster information. When getting new information, people rebuild the model and realize new global optimization with new disaster information [3]-[4]. The Disruption Management method means that in the beginning, people use a optimized model to get a good operation plan. During the implement of the plan, the interference accidents make the original plan infeasible and we need a new plan at the real time. The new plan, while considering the original optimization object, needs to minimize the negative effect brought by the interference [5-7]. For both two methods, the precondition for the dynamic adjustment of emergency response scheme is that the scheme is easy to adjust, or the decision is totally or partly reversible [8]-[9].

The Scheduling and Re-scheduling method doesn't consider any adjustment cost or deviation cost. In this case, the emergency response scheme is totally reversible and can be totally reset under the condition of obtaining new information. The Disruption Management method considers limited adjustment cost. So the emergency response scheme is partly reversible and focuses on local adjustment in order to minimize the adjustment cost. However, many emergency response decisions are usually irreversible, which makes them, once carried out, unable to adjust [10]. For example, the decision of using helicopter to air-drop materials to affected points is usually irreversible. As many affected points are not suitable for helicopters to land, the air-dropped materials to these points will be impossible to transfer to other points. Even we use cars to transport emergency materials, the adjustment cost for emergency materials allocation scheme may be very high. As in many cases roads are in ruins in disaster and cars can only move in one direction. When emergency material allocation decision is

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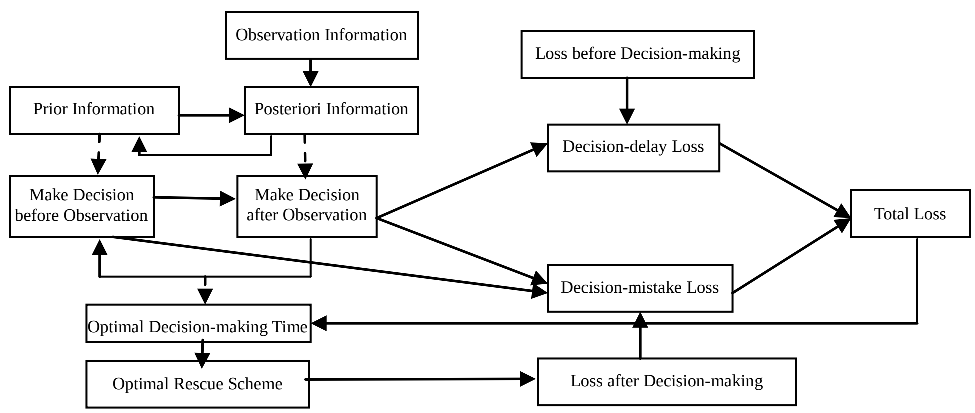
irreversible, the allocation scheme is hard to adjust and it is the only way for us to choose the optimal decision-making time [2].

Based on the irreversibility of emergency material allocation decision in emergency response stage, under the condition that the disaster information is continuously observed and updated, this paper uses the Bayes decision theory to establish a mathematic model for the rescue points selection problem, which brings the solution of emergency material allocation scheme and decision-making time into a systematic framework. The rescue points selection problem is a kind of the emergency material allocation problem. In rescue points selection problem, there are one affected point and many rescue points, which are selected to provide emergency material to the affected point. In each decision period, we only make and carry out one emergency material allocation scheme and do no adjustment later. In each decision period, the higher observation frequency of disaster information is, the more accurate the material demand information is, and the less loss improper material allocation decision will cause. However, along with the increase of the observation frequency, the loss caused by material allocation delay will be larger [11]. So in order to reduce the total loss, we need to determine the optimal decision-making time. The basic framework of the Bayes sequential decision model for multiple rescue points selection is shown in Picture 1.




Picture 1 Basic Framework Of The Bayes Sequential Decision Model

As shown in Picture1, in the first time after disaster, we can only get the prior disaster information calculated by the historical data. At this time, the decision-maker needs to choose whether to make emergency material allocation decision right now or to observe disaster information for another period of time. If decision-maker decides to make emergency material allocation decision right now, the loss caused by decision delay will be 0 and the loss caused by uncertain disaster information will be serious. If decision-maker decides to continue to observe disaster information, he can get posteriori disaster information through observation information and prior information. So the posteriori disaster information will be more accurate and the loss caused by improper decision (decision-mistake loss) will be reduced. But the emergency material allocation decision will be delayed and there will be decision-delay loss. When people keep continuous observation on disaster information, the posteriori information obtained in this observation becomes the prior information for the next observation. So, the optimal decision-making time depends on the trade off between the decision-mistake loss and the decision-delay loss to make the total loss minimal. After obtaining the optimal decision-making time, we can work out the optimal emergency material allocation scheme with selected rescue points and their material supply amount. When there is a serious disaster, the observation cost can be ignored compared with the loss of affected point, so this paper doesn't consider any observation cost.

A mathematics model is established in the second part. The third part gives the process of solving the model. The results are discussed in the fourth part. The fifth part gives an application example. The sixth part performs numerical simulation of the relation among different variables, and the seventh part is the conclusion.

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## 2 MODEL CONSTRUCTION

### 2.1 Definition of Symbols

$i$ : code for alternative rescue point , $i = 1, 2, \cdots, p$

$b_i$ : amount of emergency material reserves in rescue point $i$

$t_i$ : transportation time from rescue point $i$ to affected point

$B$ : total population in affected point

$\theta$ : affected proportion of the population in affected point

$\mu$ : prior mean of affected proportion

$\tau$ : prior standard deviation of affected proportion

$\sigma$ : standard deviation of observation example of affected proportion

$d$ : quantity-demanded for emergency material of each affected people

$n$ : observation frequency of disaster information

$N$ : maximum observation frequency of disaster information, $N \geq 1$

$n^*$ : optimal emergency material allocation decision-making time expressed by observation frequency

$n^*_{\text{int}}$ : integral value of optimal decision-making time expressed by observation frequency

$\overline{T}^*$ : actual optimal decision-making time

$S$ : amount of emergency materials allocation for affected point

$S^*$ : optimal amount of emergency material allocation

$D$ : quantity-demanded for emergency material of affected point, $D = d\theta B$

$L_f$ : decision-mistake loss

$L_d$ : decision-delay loss

$L$ : total decision loss

$r$ : Bayes risk

$\rho$ : posteriori expected loss

### 2.2 Suppose

(1) Suppose that the affected proportion of the population in affected point $\theta \in \Theta$ has the prior distribution $\pi(\theta)$ and obeys the normal distribution $N(\mu, \tau^2)$, then the prior distribution $\pi(D)$ obeys $N(dB\mu, d^2B^2\tau^2)$.

(2) $\mathbf{X}^n = (X_1, \cdots, X_n)$ is the observed sequential example of affected proportion in affected point, and the observation example's conditional distribution $f(x|\theta)$ obeys the normal distribution $N(\theta, \sigma^2)$, $x \in \Omega$. Suppose that the posteriori distribution of affected proportion $\theta$ after observing the example value $x$ is $\pi(\theta|x)$.

(3) Loss associated with emergency material allocation decision only includes the decision-delay loss and the decision-mistake loss. Suppose that the decision-delay loss $L_d$ is the square form of unsatisfied demand amount of affected point and calculated according to the prior mean of emergency material quantity-demanded, then $L_d = (dB\mu - 0)^2 = (dB\mu)^2$ . Suppose that decision-mistake loss $L_f$ is calculated according to the posteriori mean of emergency material quantity-demanded, then $L_f(\theta, S) = (d\theta B - S)^2$.

Suppose that the interval between each observation is equal to $1/N$ . After observing disaster information for $n$ times and distributing the material, the total decision loss is as follows.

$$L(\theta, S, n) = (N - n)\frac{1}{N}L_f(\theta, S) + n\frac{1}{N}L_d$$
$$= \frac{1}{N}\left[(N - n)L_f(\theta, S) + nL_d\right]$$

### 2.3 Bayes Risk Function

The expected total decision loss of this problem can be expressed by the Bayes Risk Function:

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$$r^n(\pi) = E^\pi E_\theta^{X^n}\left[L\left(\theta, \delta_n^\pi\left(\mathbf{X}^n\right), n\right)\right]$$
$$= E^\pi E_\theta^{X^n}\left[\frac{N-n}{N} L_f\left(\theta, \delta_n^\pi\left(\mathbf{X}^n\right)\right)\right] + \frac{n}{N} L_d \quad (1)$$

Here, $\delta_n^\pi\left(\mathbf{X}^n\right)$ is the Bayes decision rule of the decision loss $L(\theta, S, n)$.

Suppose $r\left(\pi, \delta_n^\pi\right) = E^\pi E_\theta^{X^n}\left[\frac{N-n}{N} L_f\left(\theta, \delta_n^\pi\left(X^n\right)\right)\right]$, which means the expected decision-mistake loss. Suppose $DL = \frac{n}{N} L_d$, which means the expected decision-delay loss. Then formula (1) can be expressed as:

$$r^n(\pi) = r\left(\pi, \delta_n^\pi\right) + DL$$

Define the posteriori expected loss of the problem as follows.

$$\rho\left(\pi(\theta|x), S, n\right) = \int_\Theta L(\theta, S, n)\pi(\theta|x)\,d\theta \quad (2)$$

### 2.4 Model

For this problem, we can build a two-stage model. The first stage ensures optimal decision-making time and optimal total material allocation quantity to minimize the expected total decision loss. The second stage formulates optimal rescue scheme to minimize response time.

The model for the first stage is:

$$\min_{n,S} r^n(\pi) \quad (3)$$

Suppose $\phi$ is one feasible scheme for rescue points selection, and expressed as $\phi=\left\{\left(i_1, b_{i_1}'\right), \left(i_2, b_{i_2}'\right), \ldots, \left(i_m, b_{i_m}'\right)\right\}$, where $0 \le b_{i_l}' \le b_{i_l}$, $\sum_{l=1}^{m} b_{i_l}' = S^*$. $i_1, i_2, \ldots, i_m$ are a permutation of $1, 2, \cdots, p$ 's subsequence. $\Xi$ represents the set of feasible schemes, and $\phi^*$ represents the optimal rescue scheme.

The response time is the time for the last emergency material to arrive at the affected point, and then the response time can be expressed as:

$$T(\phi) = \max_{l=1,2,\ldots,m} t_{i_l}$$

Model for the second stage is:

$$\min_{\phi\in\Xi} T(\phi) \quad (4)$$

## 3 SOLUTION

The Bayes decision model in the first stage can be solved by Bayesian analysis. The multiple rescue points selection model in the second stage can be solved by combinatorial optimization algorithm. The specific steps are as follows.

**Step 1.** Using Bayes formula to get the posteriori distribution of affected proportion $\pi(\theta|x)$ and the posteriori distribution of material demand $\pi(D|x)$.

**Theorem 1.** Suppose the prior distribution $\pi(\theta)$ of random variable $\theta$ follows $N(\mu, \tau^2)$, $\mu$ and $\tau^2$ are known, the conditional distribution $f(x|\theta)$ of the observation example $X$ is $N(\theta, \sigma^2)$, here, $\theta$ is unknown, $\sigma^2$ is known. According to Bayes formula, set the sequential example $X^n$ down, the posteriori distribution $\pi(\theta|x)$ of $\theta$ obeys $N\left(\mu_n\left(\bar{x}_n\right), \phi_n\right)$,

where $\mu_n\left(\bar{x}_n\right) = \frac{\sigma^2}{\sigma^2 + n\tau^2}\mu + \frac{n\tau^2}{\sigma^2 + n\tau^2}\bar{x}_n$,

$\phi_n = \frac{\sigma^2\tau^2}{\sigma^2 + n\tau^2}$, and $\bar{x}_n$ is the mean of the observed value of sequential example [12].

According to theorem 1, the posteriori variance obeys the rules $\phi_n < \phi_{n-1}$, which means that using observed value to update the original disaster information can reduce the uncertainty of disaster information. At the same time, we can get that the posteriori distribution of the emergency material demand $\pi(D|x)$ obeys $N\left(dB\mu_n\left(\bar{x}_n\right), d^2B^2\phi_n\right)$.

**Step 2.** Minimizing the Bayes risk to get the Bayes decision rule.

By minimizing the Bayes risk or the posteriori expected loss, we can get the Bayes decision rule

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[12]. In this problem, it is easier to use the posteriori expected loss to solve the Bayes rule. Put $L(\theta, S, n)$ into formula (2), then we get:

$$\rho(\pi(\theta|x), S, n) = \int_{\Theta}\left[\frac{N-n}{N}(d\theta B - S)^2 + \frac{n}{N}(dB\mu)^2\right]\pi(\theta|x)\,\mathrm{d}\theta$$

set $\frac{\mathrm{d}\rho}{\mathrm{d}S} = 0$, then we get:

$$-2dB\int_{\Theta}\theta\,\pi(\theta|x)\,\mathrm{d}\theta + 2S = 0$$

The Bayes decision rule of this problem can be solved out.

$$\delta_n^{\pi}(X^n) = dBE^{\pi(\theta|x)}(\theta) = dB\mu_n(\overline{x}_n) \quad (5)$$

So the Bayes rule is equal to the posteriori mean of material demand.

**Step3.** Put Bayes decision rule into the Bayes risk function, then minimize the Bayes risk, and we can get the optimal decision-making time and the optimal material allocation quantity.

Put $\delta_n^{\pi}(X^n) = dB\mu_n(\overline{x}_n)$ into formula (1) and get the Bayes risk of material distribution decision.

$$r^n(\pi) = \frac{N-n}{N}d^2B^2\frac{\sigma^2\tau^2}{\sigma^2 + n\tau^2} + \frac{n}{N}(dB\mu)^2 \quad (6)$$

At the same time, we can get the expression of expected decision-mistake loss.

$$r(\pi, \delta_n^{\pi}) = \frac{N-n}{N}d^2B^2\frac{\sigma^2\tau^2}{\sigma^2 + n\tau^2} \quad (7)$$

Suppose $n$ is a continuous variable. Set $\frac{\mathrm{d}r^n(\pi)}{\mathrm{d}n} = 0$ to get the optimal decision time.

$$n^* = \mu^{-1}\tau^{-2}\left[\sigma\tau\left(\sigma^2 + N\tau^2\right)^{1/2} - \mu\sigma^2\right] \quad (8)$$

Calculate the second derivative of $r^n(\pi)$ to $n$ and get:

$$\frac{\mathrm{d}^2 r^n(\pi)}{\mathrm{d}n^2} = 2d^2B^2\sigma^2\tau^2\frac{\sigma^2 + N\tau^2}{N\left(\sigma^2 + n\tau^2\right)^3}$$

$\frac{\mathrm{d}^2 r^n(\pi)}{\mathrm{d}n^2} > 0$, so $n^*$ is the observation frequency when $r^n(\pi)$ is minimum, i.e. the optimal decision-making time. When $n^* \le 0$, set $n^* = 0$; when $n^* > N$, set $n^* = N$. When $N > n^* > 0$ and $n^*$ is a decimal, set the former integer of $n^*$ as $[n^*]$. The integer value of optimal observed frequency $n_{\text{int}}^*$ is $[n^*]$ or $[n^*]+1$ which makes $r^n(\pi)$ smaller.

It is necessary to emphasize that $n^*$ or $n_{\text{int}}^*$ is the optimal decision time expressed by the observation frequency. The actual optimal decision time $\overline{T}^*$ is:

$$\overline{T}^* = n_{\text{int}}^* \times \frac{1}{N} = \frac{\mu^{-1}\tau^{-2}\left[\sigma\tau\left(\sigma^2 + N\tau^2\right)^{1/2} - \mu\sigma^2\right]}{N} \quad (9)$$

In formula (9), the unit of $\overline{T}^*$ is one decision period.

Put $n^*$ or $n_{\text{int}}^*$ into the Bayes decision rule in formula (5), and we can get the optimal material distribution quantity:

$$S^*(n^*, X^n) = dB\left[\overline{x}_{n^*} + \sigma\tau^{-1}\mu\left(\sigma^2 + N\tau^2\right)^{-1/2}\left(\mu - \overline{x}_{n^*}\right)\right] \quad (10)$$

$$S^*(n_{\text{int}}^*, X^n) = dB\frac{\sigma^2\mu + n_{\text{int}}^*\tau^2\overline{x}_{n_{\text{int}}^*}}{\sigma^2 + n_{\text{int}}^*\tau^2} \quad (11)$$

Formula（10） can be seen as the approximate value of formula（11）.

**Step4.** According to the optimal material distribution quantity $S^*$ calculated out in the first stage model, we can make a decision on rescue points selection and their material supply to get the optimal rescue scheme $\phi^*$.

Set $t_1 \le t_2 \le \cdots \le t_p$

**Definition 1**. For sequence $b_{i_1}, b_{i_2}, \cdots, b_{i_m}$, if $q$ exists, which satisfies the condition that $1 \le q \le m \le p$ and $\sum_{l=1}^{q} b_{i_l} \ge S^* > \sum_{l=1}^{q-1} b_{i_l}$, we call $q$ as the critical subscript of this sequence,

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relative to $S^*$.

**Theorem 2**. The necessary and sufficient condition for feasibility of scheme $\phi$ is that sequence $b_{i_1}, b_{i_2}, \cdots, b_{i_m}$ has the critical subscript $q$ [13].

Proof:

If $q$ is the critical subscript of sequence $b_1, b_2, \cdots, b_p$ in relative to $S^*$, it will optimize the emergency material allocation scheme by taking $1, 2, \cdots, q$ as the rescue points, which makes $\mathrm{T}\left(\phi^*\right) = \max_{l=1,2,\ldots,q} t_l = t_q$. The characteristics of this scheme are as follows. Firstly choose rescue point 1 for rescue, which is the nearest to the affected point. If the entire emergency material supply $b_1$ in this point is less than the optimal material distribution quantity $S^*$, then choose rescue point 2 for rescue, which is the second nearest to the affected point. If the sum of emergency material supply in point 1 and 2 $b_1 + b_2$ is still less than $S^*$, then choose point 3,4,5……till the total material supply of selected rescue points is not less than $S^*$. This is all.

**Theorem 3.** The scheme which choose $1, 2, \cdots, q$ as rescue points will make the response time shortest. $\mathrm{T}\left(\phi^*\right) = \max_{l=1,2,\ldots,q} t_l = t_q$.

When $l < q$, the emergency materials supply in the rescue point $l$ $s_l^* = b_l$ and When $l = q$, $s_q^* = S^* - \sum_{l=1}^{q-1} b_l$.

## 4 RESULTS ANALYSIS

### 4.1 Economical meaning of observation frequency and decision-making time

As $\frac{\mathrm{d}\, r\left(\pi, \delta_n^\pi\right)}{\mathrm{d}\, n} = -\frac{d^2 B^2 \sigma^2 \tau^2}{N} \frac{\left(\sigma^2 + N\tau^2\right)}{\left(\sigma^2 + n\tau^2\right)^2}$, we can see that the expected decision-mistake loss is a decreasing function of the observation frequency of disaster information. Set $MR = -\frac{\mathrm{d}\, r\left(\pi, \delta_n^\pi\right)}{\mathrm{d}\, n}$, which means the marginal benefit of the observation decreasing along with increasing of observation frequency.

As $\frac{\mathrm{d}\, DL}{\mathrm{d}\, n} = \frac{d^2 B^2 \mu^2}{N}$, we can see that the decision-delay loss is an increasing function of the observation frequency of disaster information. Set the marginal cost of observation $MC = \frac{\mathrm{d}\, DL}{\mathrm{d}\, n}$, which doesn't change with observation frequency.

When the marginal benefit of observation is equal to the marginal cost, i.e. $MR = MC$, we can get the optimal decision-making time $n^*$. The result is the same as formula (6).

### 4.2 Optimal decision-making time

（1）relation between optimal decision-making time and maximum observation frequency of disaster information

1）relation between optimal decision-making time expressed by observation frequency $n^*$ and maximum observation frequency $N$

$\frac{\mathrm{d}\, n^*}{\mathrm{d}\, N} = \frac{1}{2} \mu^{-1} \sigma\tau \left(\sigma^2 + N\tau^2\right)^{-1/2}$. $\frac{\mathrm{d}\, n^*}{\mathrm{d}\, N} > 0$, which shows that optimal decision-making time $n^*$ is an increasing function of maximum observation frequency $N$. When $0 < N \le \tau^{-2}\sigma^2\left(\mu^2\tau^{-2} - 1\right)$, $n^* = 0$; when $N > \tau^{-2}\sigma^2\left(\mu^2\tau^{-2} - 1\right)$, $n^* > 0$. Specially, when $\mu < \tau$, $N > 0 > \tau^{-2}\sigma^2\left(\mu^2\tau^{-2} - 1\right)$, and $n^* > 0$. That is to say, when the prior mean of affected proportion is smaller than its prior standard deviation, we need to observe disaster information before we make emergency material distribution decision.

2) relation between actual optimal decision-making time $\overline{T}^*$ and maximum observation frequency $N$

$$\frac{\mathrm{d}\, \overline{T}^*}{\mathrm{d}\, N} = -\frac{\frac{1}{2}\mu^{-1}\sigma\tau\left(\sigma^2 + N\tau^2\right)^{-1/2}\left[2\tau^{-2}\sigma^2 + N\right] + \mu^{-1}\tau^{-2}\mu\sigma^2}{N^2}$$

. $\frac{\mathrm{d}\, \overline{T}^*}{\mathrm{d}\, N} < 0$, which shows that the actual optimal

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decision-making time $\overline{T}^*$ is a decreasing function of $N$. Specially, when $N \to \infty$, $\lim_{N\to\infty} \overline{T}^* = 0$. That is to say, if we do uninterrupted observation of disaster information, the actual optimal decision-making time is almost equal to 0. In this situation, disaster information is updated so quickly that the uncertainty of the information decreases in short time. So we can make decision right now.

Along with the increase of maximum observation frequency of disaster information, though the optimal decision-making time expressed by observation frequency will increase, as the observation interval decreases, the actual optimal decision-making time will still decrease. So, after disaster, we can use the uninterrupted observation skills including remote sensing to improve decision-making speed and efficiency.

(2) relation between optimal decision-making time and the prior mean value of affected proportion

$$\frac{\mathrm{d}n^*}{\mathrm{d}\mu} = -\mu^{-2}\tau^{-1}\sigma\left(\sigma^2 + N\tau^2\right)^{1/2} . \quad \frac{\mathrm{d}n^*}{\mathrm{d}\mu} < 0 ,$$

which shows that the optimal decision-making time is a decreasing function of the prior mean value of affected proportion. The bigger the prior mean value of affected proportion is, the more serious the disaster situation known according to the prior information is, the higher the marginal cost of delaying decision is, and the shorter time for distributing emergency material should be. When $0 < \mu < \sigma^{-1}\tau\left(\sigma^2 + N\tau^2\right)^{1/2}, n^* > 0$. When

$\mu \ge \sigma^{-1}\tau\left(\sigma^2 + N\tau^2\right)^{1/2}$, $n^* = 0$。

(3) relation between optimal decision-making time and the prior standard deviation of affected proportion

$\frac{\mathrm{d}n^*}{\mathrm{d}\tau} = \sigma^2\tau^{-2}\left[2\tau^{-1} - \mu^{-1}\sigma\left(\sigma^2 + N\tau^2\right)^{-1/2}\right]$ .when $4N - \mu^{-2}\sigma^2 \ge 0$, $\frac{\mathrm{d}n^*}{\mathrm{d}\tau} > 0$.

While $4N - \mu^{-2}\sigma^2 < 0$, the relation as follows exits.

$$\begin{cases} 0 < \tau^2 < 4\sigma^2\left(\mu^{-2}\sigma - 4N\right)^{-1} : \frac{\mathrm{d}n^*}{\mathrm{d}\tau} > 0 \\ \tau^2 = 4\sigma^2\left(\mu^{-2}\sigma - 4N\right)^{-1} : \frac{\mathrm{d}n^*}{\mathrm{d}\tau} = 0 \\ \tau^2 > 4\sigma^2\left(\mu^{-2}\sigma - 4N\right)^{-1} : \frac{\mathrm{d}n^*}{\mathrm{d}\tau} < 0 \end{cases}$$

It can be seen that relation between optimal decision-making time and the prior standard deviation of affected proportion is varied.

(4) relation between optimal decision-making time and the observation standard deviation of affected proportion

$$\frac{\mathrm{d}n^*}{\mathrm{d}\sigma} = \mu^{-1}\tau^{-1}\left(\sigma^2 + N\tau^2\right)^{-1/2}\left(2\sigma^2 + N\tau^2\right) - 2\tau^{-2}\sigma$$

. When $\mu < \tau$, $\frac{\mathrm{d}n^*}{\mathrm{d}\sigma} > 0$. Suppose $A = N\sqrt{\mu^2 - \tau^2}\left(\mu - \sqrt{\mu^2 - \tau^2}\right)\left(2\mu^2\tau^{-2} - 2\right)^{-1}$, when $\mu > \tau$:

$$\begin{cases} 0 < \sigma^2 < A : \frac{\mathrm{d}n^*}{\mathrm{d}\sigma} > 0 \\ \sigma^2 = A : \frac{\mathrm{d}n^*}{\mathrm{d}\sigma} = 0 \\ \sigma^2 > A : \frac{\mathrm{d}n^*}{\mathrm{d}\sigma} < 0 \end{cases}$$

It can be seen that the relation between optimal decision-making time and the observation standard deviation of affected proportion is varied.

### 4.3 Optimal materials distribution quantity

The optimal material distribution quantity $S^*\left(n_{\text{int}}^*, X^n\right)$ in formula (11) is obtained according to the posteriori information of affected proportion. In addition, we can directly use the prior information of affected proportion to distribute emergency material, signed as $S\left(\mu\right)$,

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and $S(\mu) = dB\mu$. We can also directly solve out material distribution quantity according to the mean of observed sequential example of affected proportion $\overline{x}_{n_{\text{int}}^*}$, signed as $S\left(\overline{x}_{n_{\text{int}}^*}\right)$, and $S\left(\overline{x}_{n_{\text{int}}^*}\right) = dB\overline{x}_{n_{\text{int}}^*}$. Formula（11） can be written as:

$$S^*\left(n_{\text{int}}^*, X^n\right) = S(\mu) + \frac{n_{\text{int}}^*\tau^2}{\sigma^2 + n_{\text{int}}^*\tau^2}\left(S\left(\overline{x}_{n_{\text{int}}^*}\right) - S(\mu)\right) \tag{12}$$

$$S^*\left(n_{\text{int}}^*, X^n\right) = S\left(\overline{x}_{n_{\text{int}}^*}\right) + \frac{\sigma^2}{\sigma^2 + n_{\text{int}}^*\tau^2}\left(S(\mu) - S\left(\overline{x}_{n_{\text{int}}^*}\right)\right) \tag{13}$$

Combine formula(12) and (13)，we can get the relation among $S^*\left(n_{\text{int}}^*, X^n\right)$, $S(\mu)$ and $S\left(\overline{x}_{n_{\text{int}}^*}\right)$,

$$\begin{cases} \mu > \overline{x}_{n_{\text{int}}^*} : S\left(\overline{x}_{n_{\text{int}}^*}\right) < S^*\left(n_{\text{int}}^*, X^n\right) < S(\mu) \\ \mu = \overline{x}_{n_{\text{int}}^*} : S^*\left(n_{\text{int}}^*, X^n\right) = S(\mu) = S\left(\overline{x}_{n_{\text{int}}^*}\right) \\ \mu < \overline{x}_{n_{\text{int}}^*} : S(\mu) < S^*\left(n_{\text{int}}^*, X^n\right) < S\left(\overline{x}_{n_{\text{int}}^*}\right) \end{cases}$$

It can be seen that $S^*\left(n_{\text{int}}^*, X^n\right)$ is always between $S(\mu)$ and $S\left(\overline{x}_{n_{\text{int}}^*}\right)$. The posteriori information, combining the prior information and the posteriori information, makes the emergency material allocation decision more reasonable.

## 5 NUMERICAL EXAMPLE

### 5.1 Emergency scenario

Suppose that in a certain area P1, there has an earthquake. The prior distribution of affected proportion of population $\theta$ obeys $N(0.27, 0.32^2)$. Suppose the observation example obeys $N(\theta, \sigma^2)$, and $\sigma = 0.4$. The disaster information is observed and updated every two hours and will be told to the emergency headquarters, who is responsible for emergency material distribution decision. The decision must be made in each day, so $N = 12$. The object of the rescue program is to minimize the total expected decision loss of emergency material distribution.

Suppose the total population in the affected point P1 is 795 thousands. Each person needs 6 L of drinking water every day, i.e. $d = 6$. There are 7 rescue points: P1-P7. The arriving time $t_i$ from each rescue point to P1 and the reserves of drinking water $b_i$ in each rescue point are shown in Tab. 1.

Tab. 1 Arriving Time $t_i$ (Hour) From Each Rescue Point To P1 And Reserves Of Drinking Water $b_i$ (Ten Thousand L)

| | P1 | P2 | P3 | P4 | P5 | P6 | P7 |
|---|---|---|---|---|---|---|---|
| $t_i$ | 0 | 2.17 | 1.92 | 1.82 | 2.47 | 3.3 | 3.52 |
| $b_i$ | 80 | 50 | 30 | 60 | 25 | 10 | 10 |

### 5.2 Solution of the example

（1）optimal decision-making time

According to formula (8), we can get the optimal decision-making time $n^*$=3.8934，and the integer value of optimal decision-making time $n_{\text{int}}^* = 4$. According to formula (9), we can get the actual optimal decision-making time $\overline{T}^* = 8$（hour）. When the earthquake has happened for 8 hours, the emergency headquarters make the optimal emergency allocation distribution decision. At this time, the expected total decision loss is minimal.

（2）optimal emergency material distribution quantity

In MATLABR2007b, use random() function to produce quantities who obey $N(0.27, 0.4^2)$. Choose the first 12 quantities which are bigger than 0 and smaller than 1 as the observed sequential example of affected proportion in P1, and we can get $\mathbf{X}^{12} = (0.2463$, 0.5158, 0.4731, 0.9470, 0.5065, 0.0126, 0.4221, 0.2622, 0.2507, 0.2700, 0.1429, 0.7080). When the optimal decision time is 4, mean of the sequential example $\overline{x}_4 = 0.5455$，and the posteriori mean of affected proportion $\mu_4\left(\overline{x}_4\right) = 0.4681$. According to formula（11）, we can get the optimal emergency material distribution quantity $S^*\left(4, X^4\right) = 223.3049$. And we can also get $S(\mu) = 128.7900$，$S\left(\overline{x}_4\right) = 260.2248$. As

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$\bar{x}_4 > \mu$， $S(\mu) < S^*(4, X^4) < S(\bar{x}_4)$.

（3）optimal rescue scheme

The steps for calculating out the critical subscript of rescue points sequence in relative to the optimal emergency material distribution quantity $S^*(4, X^4)$ are shown in Tab.2. As 220 ＜223.3049＜245, we can know that the critical subscript is 5，and the rescue points are P1、P4、P3、P2、P5， with the following emergency material supply respectively ： 80, 60, 30, 50 and 21.6951。

Tab.2 Steps For Calculating Critical Subscript

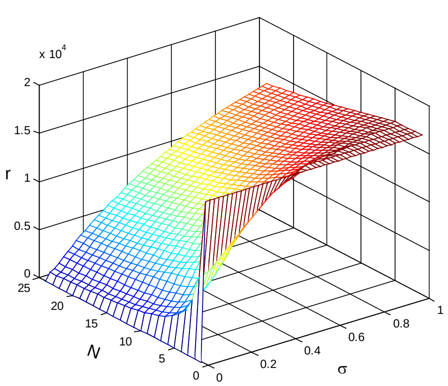
| Step | Rescue sequence | Total material supply |
|---|---|---|
| 1 | P1 | 80 |
| 2 | P1、P4 | 140 |
| 3 | P1、P4、P3 | 170 |
| 4 | P1、P4、P3、P2 | 220 |
| 5 | P1、P4、P3、P2、P5 | 245 |

## 6. NUMERICAL SIMULATION

This part does data simulation of the relation between some variables and the expected total decision loss $r$ in MATLABR2007b.

（1）simulation of the relation among $\sigma$， $N$ and $r$

Suppose $\theta \sim N(0.27, 0.32^2)$, $\sigma$ belongs to[0, 1] and $N$ belongs to[1, 24]. The relation among $\sigma$, $N$ and $r$ is shown in picture 2.




Picture 2 Simulation Of The Relation Among $\sigma$， $N$ And $r$

From picture 2 we can see that when $\sigma$ is invariable, basically, the expected total decision loss $r$ is a decreasing function of the maximum observation frequency of disaster information $N$. The bigger the maximum observation frequency is, the smaller the expected total decision loss is. To reduce the loss in the affected point, we need increase the maximum observation frequency and decrease the observation intervals. When $N$ is invariable, the expected total decision loss $r$ is basically an increasing function of the observation standard deviation of affected proportion $\sigma$（when $N = 1$，the expected total decision loss is invariable）.The bigger $\sigma$ is, the larger the expected total decision loss is. To reduce the loss in the affected point, we should use more precise observation method.

（2）simulation of the relation among $\mu$， $\tau$ and $r$

Suppose $\sigma = 0.4$ ， $N = 12$ ， $\theta \sim N(\mu, \tau^2)$ ， $\mu$ belongs to [0, 1]，and $\tau$ belongs to [0, 1]. The relation among $\mu$， $\tau$ and $r$ is shown in picture 3.

From picture 3 we can see that when $\mu$ is invariable, basically, the expected total decision loss $r$ is an increasing function of the prior standard deviation of affected proportion $\tau$. The bigger the prior standard deviation is, the smaller the expected total decision loss is. That is to say, the more uncertain the prior disaster information is, the more loss the affected point will suffer. At the same time. when $\tau$ is invariable, the expected total decision loss $r$ is basically an increasing function of the prior mean of affected proportion $\mu$. The bigger the prior mean of affected proportion is, the larger the expected total decision loss is. That is to say, the more serious the prior emergency situation is, the more loss the affected point will suffer. In order to reduce the loss, we should establish emergency database, keep updating the data, and divide emergency situation into different scenarios, so that we can get more precise prior disaster information.

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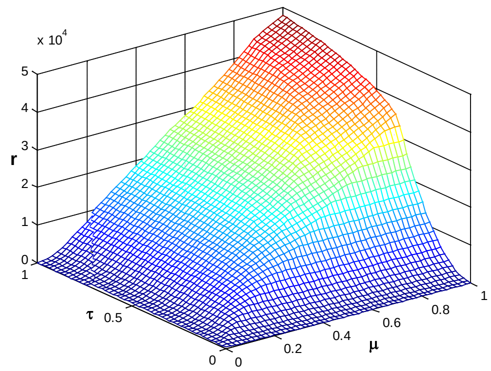
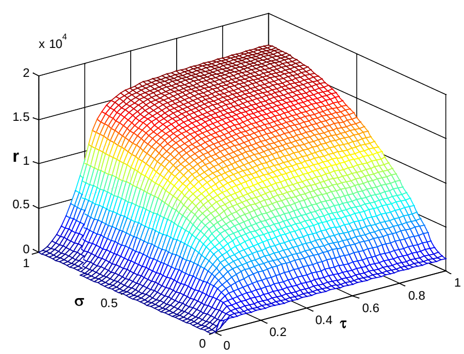



Picture 3 Simulation Of The Relation Among $\mu$, $\tau$ And $r$

（3）relation simulation among $\tau$, $\sigma$ and $r$

Suppose $N=12$, $\theta \sim N(\mu,\tau^2)$, $\mu=0.27$, $\tau$ belongs to [0, 1] and $\sigma$ belongs to[0, 1]. The relation among $\tau$, $\sigma$ and $r$ is shown in picture 4.




Picture 4 Simulation Of The Relation Among $\tau$, $\sigma$ And $r$

From picture 4 we can see that when $\sigma$ is invariable, basically, the expected total decision loss $r$ is an increasing function of the prior standard deviation of affected proportion $\tau$. At the same time, when $\tau$ is invariable, the expected total decision loss $r$ is basically an increasing function of the observation standard deviation of affected proportion $\sigma$. This simulation results are the same as the former ones.

## 7. CONCLUSIONS

The model built in this paper can be applied into emergency rescue program in natural disasters. It can help decision-makers to determine optimal decision-making time and optimal rescue plans. The research results can offer guidance to the enrichment of emergency preparation system and emergency plans. Of course, this paper only does research into the rescue points selection problem with one decision period and one affected point. The future study may consider the emergency material distribution problem for multiple decision periods and multiple affected points.

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