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THE FUNDAMENTAL OPERATION ON CONNECTION NUMBER AND ITS APPLICATIONS

[1]CHUNFENG LIU, [2]LING ZHANG,[3,4]AIMIN YANG
[1, 2, 3] College of Science, Hebei United University, Tangshan 063009, Hebei, China
[4] Colleges of Mechanical Engineering, Yanshan University, Qinhuangdao 066004, China
E-mail: [1]liucf403@126.com , [2]abczhangling@126.com , [3]mailto:4aimin_heut@163.com

ABSTRACT

In this paper, we introduced the basic knowledge of set pair analysis at the first place. Then we defined the general form of basic operations and the properties of connection number, in the end we give the corresponding application examples. The concepts and methods of the operations are the basis points of this paper. The ccomposition operation that put forward in this article makes the results that we need more realistic and it more detailed and more reasonable when describe a problem.The operations also broaden the application scope of connection number and enrich the connection number theory of set pair analysis. The mathematical meaning is clear and the method is simple of this operation, it is easy to learn and use .This research also provides the computational basis for the next studies, it has a certain theoretical and practical significance.

Keywords: *Connection Number, Set Pair Analysis, Fundamental Operations, Compositional Operations*

1. INTRODUCTION

The world is a contradictory unity of certainty and uncertainty. A variety of systems and all kinds of things reflect certainty under certain conditions, whereas they reflect the uncertainty in another condition. We can found that certainty and uncertainty happen to be a system from the system's point. Certainty and uncertainty has been fully developed in this system. The uncertainty has been plaguing people for a long time in the field of information, systems and control. So the scientists had to create numerous theories and methods to study the uncertainty. So far, people understanding the random uncertainty more clearly. But random uncertainty theory emphasizes the relative independence of the observed object. In fact, in many cases, it is difficult to test this independence so as to bring a lot of difficulties in the practical application.

The research of fuzzy uncertainty began in the 1960s, the work of Zadeh [1] (LAZadeh) who is an automatically cybernetics expert and an applied mathematician. In recent decades, the fuzzy mathematics [2] has been more widely used in information, systems and control field. But the uncertain fuzzy theory proposed by Zadeh essentially still used traditional mathematical theories and methods , to deal with fuzzy object by transforming fuzzy set into the classical set through the "cut set" . The membership degree is an important concept in fuzzy set, however, the determination often with subjective arbitrariness of the membership degree, On the other hand, membership degree describes uncertainty with a given value of [0,1] ,it ignores the uncertainty within a certain range of fuzzy uncertainty. Intermediary uncertainty which different from fuzzy and random uncertainty [3-5] between opposites objects. For example, the abstention of the decision-making process and the uncertainty caused by incomplete of information. We often need quantitative analysis and research of these uncertainties, But exist theories is difficult to provide a satisfactory method, in order to go further study of above problem, set pair analysis came into being.

The set pair analysis that proposed by Zhao Keqin in 1989 consider the uncertainty from the perspective of similarity, difference and contrary of two sets. There is a fundamental difference with the above two uncertainty theory in the basic concepts, research ideas and approach. It views the certainty and uncertainty as a system and seeking the rule of selecting value of uncertainty according to the transformation process of certainty and uncertainty [6-14].Taking the attitude of objectively admit, the depict for systems and specific analysis for the

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uncertainty so that the result is more close to reality. Recent years, set pair theory has been involved in many areas such as mathematics, physics, systems science, computer science, management science, prediction theory, agricultural engineering, hydraulic engineering, military science and artificial intelligence. So it has the theoretical significance and application prospects. In view of the so much applications of set pair analysis, it is necessary to further study its theoretical. In the aspect of mathematical properties of set pair theory, Huang decai mainly study the operations and properties [7] of $\mu=a+bi+cj$,but the general form was not given. This paper begins with a brief introduction to the basic concepts and the theoretical basis of set pair analysis and followed by the general form of connection number's operations and properties, then gives the compositional operations and the application examples.

2. SET PAIR ANALYSIS AND ITS UNCERTAINTY THEORY

Set pair [8-9] is constituted by two sets that have some connection. The basic idea of set pair analysis is analyze the characteristics of the two sets under a certain background and established connection degree expression $\mu=a+bi+cj$ .And then extended to the more complex systems that consisted by the number of sets is $m>2$, then go further research.

The uncertainty results from the incomplete and not comprehensive when people understand of things. While set pair analysis is an feasible and effective method to resolve this uncertainty. The connection degree is an extremely important concept of set pair analysis, to some extent, connection degree is the cornerstone of set pair analysis. A lot of theories and methods in set pair analysis exported by connection degree directly. The following highlights the connection degree.

2.1. The Connection Degree μ

The connection degree[10] is an important part of set pair analysis, expressed by μ , in the general case it represented as $\mu=\frac{S}{N}+\frac{F}{N}i+\frac{P}{N}j$, where N is the total number of characteristics, S is the number of common characteristics of the two sets, P is the number of opposite characteristics of the two sets, $F=N-S-P$ is the two sets neither common nor opposite characteristics. The S/N , F/N , P/N respectively called the similarity degree, difference degree and contrary degree. j is the coefficient of contrary degree ,its value is -1; i is the coefficient of difference degree, its values depend on the circumstances and restrain in $[-1,1]$. For simplicity, enable the $S/N=a, F/N=b, P/N=c$, So $\mu=\frac{S}{N}+\frac{F}{N}i+\frac{P}{N}j$ can be written as $\mu=a+bi+cj$.

The function of connection degree μ : connection degree describes uncertainty issues with clear ideas and easy to operate, it is also objective reasonable and convenient for further study. In addition connection degree has some other functions, for example, it has the function of objective reaction some dialectical thinking in the human brain and it can convert into connection angle, etc. The information of connection degree including transform the certainty information into uncertainty and transform contrary information into similarity. This information can establish the transformation system based on set pair analysis. From vector view, a,b,c in connection degree μ can regarded as a particular vector, thereby it can be dialed by the rules of relevant vectors.

In addition connection degree μ contains the new theory of uncertainty which different from probability theory and Zadeh fuzzy set theory. In short, the connection degree is very important from the contact structure or from the information it contained. The connection number is exported by the connection degree; it has more important significance in the further study of the set pair analysis.

2.2. Connection Number

When we analyze connection degree, it can be seen as a number and called the connection number [11]. For clarity, the connection number is defined as follows:

Definition: like $a+bi+cj, a+bi$, $a+cj$, $bi+cj$ called the connection number, where a,b,c are arbitrary numbers, $j=-1$, $i\in[-1,1]$ the value is uncertainty.

Seen from the definition of connection number, connection number is implied by the concept of the connection degree, but it different from connection degree, that is the a,b,c in connection number is no longer confined to $[0,1]$, while it can be any positive number. Specifically, if the connection number $a+bi+cj$, and $a+b+c=k, k>1$, then $a+bi+cj$ can be equivalently expressed as $a+bi$, $a+cj$ or

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$bi+cj$ .when $a+b+c=1$ and $a>0,b>0,c>0$,it is the connection degree.

The original purpose of connection number is convenient to use, but its theoretical significance extent the number concept. Connection number's significance not only linked in a specific number and its range but also because it is linked certainty and uncertainty within the range, making certainty and uncertainty reflect in a number with linkages, penetration, checks and transforming objectively, it is a new mathematical tool for the study of the uncertainty of many complex systems. In order to make the connection number can widely used in the system control, error handling and other areas that exist uncertainty, predecessors provided the subtraction, multiplication and division definition[12] of connection number and study the basic law of the $a+bi+cj$ .While $a+bi+cj$ is only a special example of connection number. So, we give the general form of the operations and the properties in the following.

3. THE OPERATIONS OF CONNECTION NUMBER

3.1. The General Form Of Operations And Properties Of Connection Number

Definition 1 : $u_m = a_m + \sum_{k=1}^{n-2} b_{mk} i_k + c_m j$, we called u_m is connection number of n . where a_m and c_m are arbitrary real numbers and have the same symbol, $b_{mk}(k=1,2,\cdots,n-2)$ is non-negative real number. b_{mk} and c_m respectively called uncertainty number and contrary number. $i_k(k=1,2,\cdots,n-2)$ is an uncertain amount, $i_k \in [-1,1]$ and its value depend on the specific situation. Sometimes i_k also only used as a symbol of an uncertain amount. j is the contrary symbol, it can be taken 1 or -1 according to the background of practical application in the quantitative calculation.

Definition 2: There are two connection number $u_p = a_p + \sum_{k=1}^{n-2} b_{pk} i_k + c_p j$, $u_q = a_q + \sum_{k=1}^{n-2} b_{qk} i_k + c_q j$ the sum of them is also a connection number

$u_m = a_m + \sum_{k=1}^{n-2} b_{mk} i_k + c_m j$ .Denoted $u_m = u_p + u_q$,

where $a_m = a_p + a_q$, $b_{mk} = b_{pk} + b_{qk}$, $c_m = c_p + c_q$ .It is easy to see that the addition meet commutative and associative law by definition 2. Next we give the definition of subtraction, multiplication and division of the connection number.

Definition 3: There are two connection number

$$u_p = a_p + \sum_{k=1}^{n-2} b_{pk} i_k + c_p j \ , u_q = a_q + \sum_{k=1}^{n-2} b_{qk} i_k + c_q j$$

the difference of them is $u_m = a_m + \sum_{k=1}^{n-2} b_{mk} i_k + c_m j$, denoted: $u_m = u_p - u_q$,where $a_m = a_p - a_q$, $b_{mk} = b_{pk} + b_{qk}$, $c_m = c_p - c_q$ .The actual meaning of the above definition is the difference between two indeterminate amount is still an uncertain amount.

Theorem 1:

1) If $u_m = a_m + \sum_{k=1}^{n-2} b_{mk} i_k + c_m j$,then $-u_m$ is also a connection number :

$$-u_m = -a_m + \sum_{k=1}^{n-2} b_{mk} i_k - c_m j$$

2) If $u_m = a_m + \sum_{k=1}^{n-2} b_{mk} i_k + c_m j$,then

$$u_m - u_m = 2\sum_{k=1}^{n-2} b_{mk} i_k$$

3) If $u_m = \sum_{k=1}^{n-2} b_{mk} i_k$,then $-u_m = \sum_{k=1}^{n-2} b_{mk} i_k = u_m$

4) If connection number:

$$u_p = a_p + \sum_{k=1}^{n-2} b_{pk} i_k + c_p j,$$

$$u_q = a_q + \sum_{k=1}^{n-2} b_{qk} i_k + c_q j,$$

$$u_r = a_r + \sum_{k=1}^{n-2} b_{rk} i_k + c_r j,$$

then

$$u_p - (u_q + u_r) = u_p - u_q - u_r = u_p - u_r - u_q .$$

Prove:

$$u_p - (u_q + u_r) = u_p - [(a_q + a_r) + \sum_{k=1}^{n-2} (b_{qk} + b_{rk}) i_k + (c_q + c_r) j]$$

$$= (a_p - a_q - a_r) + \sum_{k=1}^{n-2} (b_{pk} + b_{qk} + b_{rk}) i_k + (c_p - c_q - c_r) j$$

while

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$$u_p - u_q - u_r = (a_p - a_q) + [\sum_{k=1}^{n-2}(b_{pk} + b_{qk})i_k] + (c_p - c_q)j - u_r$$

$$= (a_p - a_q - a_r) + \sum_{k=1}^{n-2}(b_{pk} + b_{qk} + b_{rk})i_k + (c_p - c_q - c_r)j$$

So $u_p - (u_q + u_r) = u_p - u_q - u_r$.

About the multiplication of connection number, when $a_m + \sum_{k=1}^{n-2} b_{mk} + c_m = 1$, if $u_1 = a_1 + b_1 i + c_1 j$,

$u_2 = a_2 + b_2 i + c_2 j$, from the ternary multiplication rule we obtain a multiplication formula:

$$u_1 \times u_2 = (a_1a_2 + c_1c_2) + (a_1b_2 + b_1a_2 + b_1b_2 + c_1b_2 + b_1c_2)i + (a_1c_2 + c_1a_2)j$$

Definition 4: The general form of multiplication formula:

$$u_p \times u_q = (a_p a_q + c_p c_q) + \sum_{k=1}^{n-2}(b_{pk}u_q + b_{qk}u_p - b_{pk}b_{qk})i_k + (a_p c_q + a_q c_p)j$$

Theorem 2: The conclusions about the connection number

1) If $u_p = \sum_{k=1}^{n-2} b_{pk} i_k, u_q = \sum_{k=1}^{n-2} b_{qk} i_k$

then $u_p \times u_q = \sum_{k=1}^{n-2} b_{pk} b_{qk} i_k$;

2) Commutative law: If connection number

$$u_p = a_p + \sum_{k=1}^{n-2} b_{pk} i_k + c_p j, u_q = a_q + \sum_{k=1}^{n-2} b_{qk} i_k + c_q j$$

Then $u_p \times u_q = u_q \times u_p$.

Prove:

$$u_p \times u_q = (a_p a_q + c_p c_q) + \sum_{k=1}^{n-2}(b_{pk}u_q + b_{qk}u_p - b_{pk}b_{qk})i_k + (a_p c_q + a_q c_p)j$$

$$u_q \times u_p = (a_q a_p + c_q c_p) + \sum_{k=1}^{n-2}(b_{qk}u_p + b_{pk}u_q - b_{qk}b_{pk})i_k + (c_q a_p + c_p a_q)j$$

So $u_p \times u_q = u_q \times u_p$

3) Distributive law: If connection number

$$u_p = a_p + \sum_{k=1}^{n-2} b_{pk} i_k + c_p j, \; u_q = a_q + \sum_{k=1}^{n-2} b_{qk} i_k + c_q j,$$

$$u_r = a_r + \sum_{k=1}^{n-2} b_{rk} i_k + c_r j .$$

then $u_p \times (u_q + u_r) = u_p \times u_q + u_p \times u_r$

Prove:

$$u_p \times (u_q + u_r) = (a_p + \sum_{k=1}^{n-2} b_{pk} i_k + c_p j) \times [(a_q + a_r) + \sum_{k=1}^{n-2}(b_{qk} + b_{rk})i_k + (c_q + c_r)j]$$

$$= [a_p \times (a_q + a_r) + c_p(c_q + c_r)] + \sum_{k=1}^{n-2}[(b_{qk} + b_{rk})u_p + b_{pk}(u_q + u_r) - b_{pk}(b_{qk} + b_{rk})]i_k + [(c_q + c_r)a_p + c_p(a_q + a_r)]j$$

While $u_p \times u_q + u_p \times u_r = (a_p + \sum_{k=1}^{n-2} b_{pk} i_k + c_p j) \times (a_q + \sum_{k=1}^{n-2} b_{qk} i_k + c_q j) + (a_p + \sum_{k=1}^{n-2} b_{pk} i_k + c_p j) \times (a_r + \sum_{k=1}^{n-2} b_{rk} i_k + c_r j)$

$$= [(a_p a_q + c_p c_q) + \sum_{k=1}^{n-2}(b_{pk}u_q + b_{qk}u_p - b_{pk}b_{qk})i_k + (a_p c_q + a_q c_p)j] + [(a_p a_r + c_p c_r) + \sum_{k=1}^{n-2}(b_{pk}u_r + b_{rk}u_p - b_{pk}b_{rk})i_k + (a_p c_r + a_r c_p)j]$$

$$= [a_p \times (a_q + a_r) + c_p(c_q + c_r)] + \sum_{k=1}^{n-2}[(b_{qk} + b_{rk})u_p + b_{pk}(u_q + u_r) - b_{pk}(b_{qk} + b_{rk})]i_k + [(c_q + c_r)a_p + c_p(a_q + a_r)]j$$

Definition 5 Assume connection number

$$u_p = a_p + \sum_{k=1}^{n-2} b_{pk} i_k + c_p j, u_q = a_q + \sum_{k=1}^{n-2} b_{qk} i_k + c_q j \; ,$$

If there is a connection number

$u_m = a_m + \sum_{k=1}^{n-2} b_{mk} i_k + c_m j$ makes $u_p = u_m \times u_q$,then definite u_m is the quotient of u_p divided by u_q , Denoted as $u_p \div u_q$, At this time we say u_p able to be divided by u_q , otherwise s u_p cannot divided by u_q .

2) in theorem 1 explain the difference is not equal to zero between the same connection number . This is also in line with the objective reality : in the general case ,the difference of two uncertain amount is still an undetermined amount even with the same fluctuation range, it is not a certain

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amount 0. At the same time illustrate the addition of connection number that defined as above cannot constitute linear space, because the negative element does not exist.1) and2) in theorem 2 show that the multiplication is both certain and uncertain amount of the two uncertain amount. The operations that discussed in this article are more in line with the actual.

3.2. The Average Of Connection Number

The sum of n connection number expressed as the multiplication of the number n and the average of connection number：

$$\mu_1+\mu_2+\cdots+\mu_n=n\left(\frac{\mu_1+\mu_2+\cdots+\mu_n}{n}\right)$$
$$=n\left(\frac{a_1+a_2+\cdots+a_n}{n}+\frac{b_1+b_2+\cdots+b_n}{n}i+\frac{c_1+c_2+\cdots+c_n}{n}j\right)$$
$$=n(\bar{a}+\bar{b}i+\bar{c}j)$$

where $\bar{a},\bar{b},\bar{c}$ respectively denote the average of similarity degree, difference degree and contrary degree.

3.3. Compositional Operations Of Connection Number

Assume $\mu_1=a_1+b_1i+c_1j$, $\mu_2=a_2+b_2i+c_2j$, we defined the compositional operations of μ_1 and μ_2 :

$$\mu_1\circ\mu_2=(a_1+b_1i+c_1j)(a_2+b_2i+c_2j)$$
$$=a_1a_2+a_1b_2i+a_1c_2j+a_2b_1i+b_1b_2i+b_1c_2ij+a_2c_1j+b_2c_1ij+c_1c_2jj$$
$$=(a_1a_2+c_1c_2)+(a_1b_2+a_2b_1)i+(a_1c_2+b_1c_2+a_2c_1+b_2c_1)j$$

Mergered jj term to the similarity degree, the other terms that with j merge together. The composition operation makes the results more realistic and it portray problem more detailed and more reasonable.

4. APPLICATION EXAMPLES

Example 1:There is a program H consisted by 10 sub-programs, A is agreed with the five sub-programs of them, opposed three sub-programs and it held abstained attitude to the two sub-programs; B on A's attitude is half agree and half opposite. Find out the attitude that B on H .

Solution: According to the meaning of the questions, we can obtain the connection degree:

$\mu(A\text{-}H)=0.5+0.2i+0.3j$, $\mu(B\text{-}A)=0.5+0.5j$,

Using the compositional operations of connection number, so

$$\mu(B\text{-}H)=(0.5+0.2i+0.3j)\circ(0.5+0.5j)=0.4+0.1i+0.5j$$

That is to say the attitude that B on H is 0.4 agree, 0.5 oppose and 0.1 abstained. In addition, when $i=0$, it shows that B held abstained attitude of a sub-program is purely abstaining without prejudice.

Example 2:there are two idioms in China, three monks no water to drink and three smelly shoe makers are smarter than one Kong Ming, the cooperation effect of shoe makers and monk can be described like this : there is uncertainty due to differences in goal, thinking and understanding, personality, behavior, physical fitness of the three people ,this uncertainty make any role not only restrained by others but also by the impact of himself, it is uncertain whether the impact of the latter plays a "positive " or "negative" effect, thus the role of any one of the groups is $1+2i$, putting three people together is $3+6i$,when i takes a positive value, the value of $3+6i$ is larger than 3.When i<-0.5 , the value of $3+6i$ is smaller than 3.As $i=1$,then $3+6i=9$, that is three shoe makers are smarter than one Kong Ming. $i=-1$,then $3+6i=-3$ that equel to three monks no water to drink.

The example 1 is a problem of intermediary uncertainty, because abstention is an intermediary phenomenon that between agreement and disagreement, the abstention is an undividable problem in the traditional decision theory, but it can expand special analysis in set pair analysis. The form of 1+1+1>3 and 1+1+1<3 are not mathematical formula, but it has been given a better explanation in example 2.

5. CONCLUSION

This article gives the general form of the connection number's operation; it can be used in the more complex system. And we definite the compositional operations of connection number, these operations broaden the applications range of set pair analysis. This study extension the theory of connection number, but also provide new ideas for the next study, it has a certain application value and practical significance.

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