

QUANTITATIVE ANALYSIS ON INNOVATION INVESTMENT BASED ON GAME THEORY

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ABSTRACT

This paper focus on the issues on dynamic game of investment on new technology research and development in double-oligarchy monopoly market, attempting to reveal the internal mechanism of different enterprises in the research and development of new technologies. Assume that the market is of rigid demand and the demand quantity remains constant relatively. The result of research and development of new technologies manifests as reduction of enterprises' production cost. Two oligarchy enterprises play a price game without information sharing according to the research and development result to determine the optimal price and profit function, create the dynamic game model of investment on research and development of new technologies in double-oligarchy monopoly market, obtain the optimal price and income under the three conditions of research and development and follow-up research and development of two enterprises, research and development of single enterprise and no research and development for the two enterprises, conclude the range of sensitivity of enterprise research and development decision making to parameters such research and development investment amount, success probability and cost reduction and finally expand the model to the conditions of enterprises to prove that during research and development, enterprises should take into account of the market capacity to avoid excessive input in a market with saturated innovative enterprises.

Keywords: *Technical Innovation, R&D Investment, Double- oligarchy Monopoly*

1. INTRODUCTION

The modern science and technology have been developed rapidly and their relation with industrial development is increasingly close and competition among enterprises and countries in the research and development of new technologies have been increasingly fierce. The research and development of new technologies can make enterprises obtain continuous competition advantages in the market and improve the capability of sustainable development. The input of developed countries in the research and development of new technologies generally accounts for above 2% of GDP, for example, America is 2.68%, Japan 3.13% and Germany 2.49%, while China is only 1.34%. By virtue of this advantage, the developed industrial countries have obtained quite obvious core ability and international competition advantages, while the developing countries are in a dilemma of research and development and introduction in the research and development of new technologies, seriously weakening the research and development and innovative ability construction of the importing

countries, i.e. developing countries, which has been alerted by the governments of relevant countries. The technical innovation brought by research and development input might reduce the total cost of enterprises, so as to improve the enterprises' cost advantages and this is the source of enterprises' competition advantages. However, enterprises, as the main body of technical innovation, will face a dilemma choice, on one hand, the competition among enterprises is cut-throat and fierce and if they want to get profitable returns in the market and have a room to stand in the fierce competition, they must innovate continuously; on the other hand, enterprises also realize that due to the randomness of input effect and technical spillover, it is not that the more the input in technical innovation is, the better the effect will be, which has been widely concerned by scholars in various aspects and researched relatively systematically and deeply.

Enterprises are the main body and beneficiary of research and development of new technologies and the research and development of new technologies are one of the core contents of construction of

enterprises' core abilities. How enterprises make research and development investment decision, how the research and development investment decisions influence the change of industry market structure and how the improvement of common industrial technologies influence the development of industry have been widely concerned by scholars and relevant departments.

In order to get market shares, enterprises will often carry out research and development investment with patent competition as the form. While the activities of enterprises are not only competition, but also bring the result competition into the next market activities. The first who expounded the patent competition among enterprises were Dastupta and Stiglitz (1980), who thought that the research technologies are characterized by the following: the probability of an enterprise to make invention and obtain patent at a certain time point only depends on the current research and development expenses of the enterprise and is unrelated with the past research and development experience. Later, Fudenberg (1983) assumed that the invention probability in an industry increased with the increasing of R&D time to get a condition making the enterprise obtain the invention ahead of other enterprises, proving that the forerunners would obtain the monopoly of research and development indeed, even if it entered the competition a little earlier than the followers. He modeled the patent competition experience and used $X_i(t)$ to present at one time, or equal to 0 (enterprise i quits from the competition), or equal to 1 (enterprise i does not quit the competition) to prove that the forerunners would obtain the monopoly of research and development indeed, even if it entered the competition a little earlier than the followers. Harris and Vickers (1985) adopted a model with variable strength to obtain the same result in a weaker condition. Spence (1984) put forward that spillover effect might reduce the income of patent competition winner and increase that of the loser. Lee and Wilde (1980) revealed that the private residue from innovation is less than the social residue (when complete price discrimination is not adopted), which causes too few innovations; on the contrary, the business stealing effect revealed that an enterprise introducing new products will not internalize the profit loss caused to the competitors in the market, because it will lead to too many innovations. While Reinganum (1983) assumed that if the newcomers execute innovation first, they will become the market monopolist. In this assumption condition, there exists no profit scattering in the

product market and the potential comers have more impetus than the market monopolist.

Slightly different from researches of the above scholars, we assume that the market is of rigid demand and the demand quantity remains constant relatively, the result of research and development of new technologies manifests as reduction of enterprises' production cost, create the dynamic game model of investment on research and development of new technologies in double-oligarchy monopoly market, obtain the optimal price and income under the three conditions of research and development and follow-up research and development of two enterprises, research and development of single enterprise and no research and development for the two enterprises, conclude the range of sensitivity of enterprise research and development decision making to parameters such research and development investment amount, success probability and cost reduction; in addition to the reduction range R&D expenses and production cost, the forecast to the probability of R&D success of other enterprises is also an important factors. Based on this, we prove that the subjective judgment ability of decision maker can also influence the success and failure of actual investment and meanwhile prove under the condition of small batches of orders (small demand scale), even if the enterprises are in disadvantages in the degree of R&D and production cost improvement, they can also obtain market shares through the advantages in R&D expenses; similarly, under the condition of large batches of orders, enterprises are more willing to make large investment. Finally, we expand the model into the conditions of more enterprises, finding that enterprises, under the conditions of purchasing patents and developing new technology of the same type, should take into account of the existing authorized enterprises in the market and the number of enterprises which have already owned the new technology to avoid excessive input in a market with saturated innovative enterprises.

2. THE VARIABLE EXPLANATION AND PROBLEM ASSUMPTION

2.1 The variable description

The following notations will appear in the rest of the paper,

C : the cost of the two enterprises, which is a transparent cost information

α : the probability of successful implementation of new technology of enterprise 1

β : the probability of successful implementation of new technology of enterprise 2

C_1^+ : the decreasing production cost of enterprise 1 after successful research

C_2^+ : the decreasing production cost of enterprise 2 after successful research

R_1 : the research and development expense of enterprise 1

R_2 : the research and development expense of enterprise 2

Q : the demand in market(ordering quantity)

2.2 The problem assumption

1) Assume that the two enterprises have the same production cost and it is common knowledge, that is C_1^+ , C_2^+ , R_1 and R_2 are shared information, α and β are not shared information.

2) Assume that the market demand is of sealed bidding with fixed order and can be taken as the Bertrand model of rigid demand. When the two enterprises have the same pricing, they will share the market equally, otherwise, the party with low pricing will get the whole market.

3) Assume that enterprise 1 makes decisions in advance, it has two strategies according to the probability of research and development success, i.e. research and develop or not research and develop; while enterprise 2 make decisions later and it will have four strategies according to the strategies of enterprise 1

4) Both enterprises have the possibility to research and develop new technologies to reduce the production cost, but there will be risks in the process of research and development of new technologies; if the new technologies are researched and developed successfully, they can be applied in the production line to reduce the production cost; if fail, the production cost remains constant. To research and develop technologies, certain production cost must be required.

5) The reduction of production cost by new technologies and research and development cost of new technologies are both common information within the industry, while the probability of research and development success is determined by the own factors of the enterprises and is the enterprises' private information.

Based on the above assumptions, the cost of Enterprise 1 and Enterprise 2 can be obtained respectively:

$$\begin{cases} C_1 = \alpha C_1^+ + (1-\alpha)C \\ C_2 = \beta C_2^+ + (1-\beta)C \end{cases} \quad (1)$$

The revenue function of enterprise 1 is as follow,

$$\begin{cases} \Pi_1^1 = (p_1 - C_1)Q - R_1, & p_1 < p_2 \\ \Pi_1^2 = \frac{1}{2}(p_1 - C_1)Q - R_1, & p_1 = p_2 \\ \Pi_1^3 = -R_1, & p_1 > p_2 \end{cases} \quad (2)$$

The revenue function of enterprise 2 is as follow,

$$\begin{cases} \Pi_2^1 = (p_2 - C_2)Q - R_2, & p_1 > p_2 \\ \Pi_2^2 = \frac{1}{2}(p_2 - C_2)Q - R_2, & p_1 = p_2 \\ \Pi_2^3 = -R_2, & p_1 < p_2 \end{cases} \quad (3)$$

As α and β are private information, enterprise 1 estimates that the probability of successful research and development of enterprise 2 is β^+ which is a subjective judgment on the basis of previous experience of enterprise 1; similarly, enterprise 2 estimates that the probability of successful research and development of Enterprise 1 is α^+ which is a subjective judgment.

Based on the above assumptions, it can be that the cost forecast of enterprise 1 to enterprise 2 and that of enterprise 2 to enterprise 1 are respectively:

$$\begin{cases} \alpha^+ = \alpha^+ C_1^+ + (1-\alpha^+)C \\ \beta^+ = \beta^+ C_2^+ + (1-\beta^+)C \end{cases} \quad (4)$$

3. THE MODEL ANALYSIS

3.1 Research and development of enterprise 1, enterprise 2 follow-up

The precondition of research and development of enterprise 1 is that the probability of successful research and development of enterprise 2 is predicted by enterprise 1 and in this case, the production cost and research and development expense can be compensated under the circumstances of lowest bid after the research and development of enterprise 1, the following conditions are true:

$$\frac{1}{2}(p' - C_1)Q - R_1 \geq \frac{1}{2}(p' - C_2)Q - R_2 = 0 \quad (5)$$

p' is the bid of enterprise 1 under the circumstance that enterprise 1 holds that enterprise 2 is of no profit equilibrium, and if it is lower than this price, loss will occur and then enterprise 2 will

not follow up. As p' , the bid of enterprise 2, is of no profit, there is no motive power for follow-up. Then the p' will be obtained as follow,

$$p' = 2R_2/Q + C_2 \quad (6)$$

Therefore, enterprise 1 estimates that the bid of enterprise 2 is a distribution within $[2R_2/Q + C_2, C]$. If enterprise 1 estimates that the distribution is a uniform distribution, the bid of enterprise 2 estimated by enterprise 1 can be obtained.

$$p_2^* = \int_{p'}^C \frac{p - p'}{C - p'} dp = \frac{C + C_2}{2} + \frac{R_2}{Q} \quad (7)$$

The enterprise 1 holds that the probability of the bid of enterprise 1 lower than that of enterprise 2 is:

$$\text{prob}(p_1 < p_2^*) = \frac{C - p}{C - p_2^*} \quad (8)$$

The optimal quotation predicated by enterprise 1 and 2 can be obtained:

$$\begin{aligned} p_1^* &= \int_{p_2^*}^C \frac{C - p}{C - p_2^*} dp = \frac{p_2^* + C}{2} \\ p_2^* &= \int_{p_1^*}^C \frac{C - p}{C - p_1^*} dp = \frac{p_1^* + C}{2} \end{aligned} \quad (9)$$

As the enterprise adopts first-price-sealed bidding method, the opportunity of enterprise to get order is only related to their optimal pricing.

$$\begin{aligned} p_1^* - p_2^* &= \frac{p_2^* - p_1^*}{2} \\ p_2^* - p_1^* &= \frac{(\alpha^+ - \beta^+)(C - C_2^+) + \alpha^+(C_2^+ - C_1^+) + R_2 - R_1}{2} + \frac{R_2 - R_1}{Q} \end{aligned}$$

When , enterprise 1 can get the whole orders, the actual significance concluded is that the research and development can greatly reduce the production cost with a small research and development input, in addition, the evaluation of the opponent on its enterprise is also an important factor. This demonstrates that the high requirement of the subjective judgment of decision marker in actual investment is reasonable.

When $\alpha^+ > \beta^+$, $C_2^+ > C_1^+$, $R_2 > R_1$, the forecasts of enterprise 1 and enterprise 2 about the probability of success of its opponent are based on the existing facts, to simplify the discussion, the subjective factors are excluded. If the probability of success of its opponent estimated by enterprise 1 and enterprise 2 is a uniform distribution within $[0,1]$, the above formula can be turned into the following:

$$p_2^+ - p_1^+ = \frac{(C_2^+ - C_1^+)}{4} + \frac{R_2 - R_1}{Q} \quad (10)$$

When $C_2^+ > C_1^+$, for example, $R_2 > R_1$, enterprise 1 will get the whole orders.

When $C_2^+ < C_1^+$, $(C_2^+ - C_1^+)Q/4 + R_2 > R_1$, the enterprise 1 will get the whole orders, otherwise, enterprise 2 will obtain the whole orders. It is demonstrated that under in case of small order (small demanding scale), even the enterprise is at a disadvantage in research and development for improving production cost, it can gain market share by taking the advantages in research and development expense. In case of bulk order, the enterprise will be more willing to make big investment.

According to above formula, when enterprise 1 and enterprise 2 choose to invest in research and development, the probability of successful research of enterprise 1 and enterprise 2 is shown as follows:

$$\theta_j > \frac{\theta_j^+(C - C_j^+)}{C - C_i^+} + \frac{2(R_i - R_j)}{Q(C - C_i^+)}, \quad i, j = 1, 2; i \neq j$$

When θ_j^+ is a uniform distribution between $[0,1]$, the above formula can be turned into:

$$\theta_j > \frac{(C - C_j^+)Q - 4(R_i - R_j)}{2Q(C - C_i^+)} \quad (11)$$

It is proved that whether the enterprise invests or not is not only related to its investment, but also is closely related to the investment size, quantity of order and the degree of improvement of production cost of its opponent.

3.2 Expand to N enterprises model

The revenue function of enterprise j will remain unchanged, for enterprise i ,

$$\begin{cases} \Pi_i^1 = (p_i - C)Q, & p_j > p_i \\ \Pi_i^2 = \frac{1}{2}(p_i - C)Q, & p_i = p_j \\ \Pi_i^3 = 0, & p_i < p_j \end{cases} \quad (12)$$

Therefore, there are two ways as listed below:

- 1) enterprise i knows that the probability of successful research is too low and the bid of enterprise i under a circumstance of zero-profit equilibrium is still higher than the zero-profit bid of enterprise j . enterprise j predicts that the probability of successful research and development of enterprise i is low. According

to formula (11), $\theta_i \leq \frac{(C - C_i^+)Q - 4(R_i - R_j)}{2Q(C - C_i^+)}$.

At this moment, enterprise i will give up research and development and enterprise j will continue go on research and development.

- 2) The research and development of enterprise i is not enough to compensate for total cost.

If the two enterprises without research and development, there may be two reasons making the two enterprises fail to conduct research and development:

The two enterprises hold that their research and development will be successful by subjective judgment and their bid is higher than that of their opponent under the circumstances of zero-profit equilibrium.

According to above equations, the conditions that the two enterprises give up research and development are: $\theta_i \leq \frac{(C - C_i^+)Q - 4(R_i - R_j)}{2Q(C - C_i^+)}$. The

research and development of the two enterprises is unable to make up their cost, therefore, $\frac{Q(p_i - C_i)}{2} - R_i < 0$. If $p_i = C$, $\theta_i < \frac{2R_i}{(C - C_i^+)Q}$. It

is the reason that the two enterprises do not conduct research and development.

Under the conditions that the model expands to N enterprises, if the cost of N enterprises is C which is public knowledge, the probability of successful implementation of new technology of enterprise i is θ_i , the decreasing production cost of enterprise after successful research is C_i^+ and the research and development expense is R_i , when θ_i is the private information of enterprise i . If the demand in market demand is expressed as Q (quantity of order), based on the above assumptions, the cost of enterprise i is and the lowest bid of other enterprises $C_i = \theta_i C_i^+ + (1 - \theta_i)C$ predicted by enterprise i is $p_j = \min\{p_1, \dots, p_{i-1}, p_{i+1}, \dots, p_n\}$, so the optimal bid of enterprise i can be obtained:

$$p_i^* = \int_{p_j}^C p \text{Prob}\left\{\bigcup_{i=1}^{n-1} p \leq p_j\right\} dp \quad (13)$$

Where, the probability of bid of enterprise i lower than that of other enterprises is: The optimal bid after predication of enterprise i can be obtained. When $n \rightarrow \infty$, $p_i^* \rightarrow p_j^+$. It is demonstrated that when more and more enterprises conducts research

and development, the income of each enterprise will decrease.

When $n \rightarrow \infty$, the requirement for probability of successful research and development is high. It is proved that with more and more enterprises grasping the technology, the requirement for probability of successful research and development of each enterprise improves continuously. As the threshold of the technology is low, each enterprise is likely to succeed in research and development, therefore, the real production cost of enterprise after improvement can be better reflected in the price. On the other hand, due to the restriction of R_i, R_j, C, C_i^+, Q , the innovative enterprise will be constrained by the market capacity, so the number of innovative enterprise is not infinite. With same probability of each enterprise's innovation and same research and development expense, the maximum number of enterprise which can be contained in the order market is $n_{\max} = Q(C - C^0)/R$. It is stated that during purchasing patents and developing new technology of the same kind, the enterprise shall take into consideration the number of enterprises with authorization and owning this new technology in the market to avoid over-input in market of saturated innovative enterprise.

4. NUMERICAL ANALYSIS

To check the accuracy of computation by the formula, the following parameters are given.

According to above formula, the following conclusions can be made by programming in Matlab. With the other pre-conditions unchanged and α^+ and β^+ changed, for the income of enterprise 1, it is known that the subjective forecast of enterprise plays an important role in income after investing in new technology. With a similar probability of success, treating the opponent as one step ahead will be beneficial to its decision making.

5. CONCLUSION

By designing the research and development investment of enterprise as two-stage dynamic games of pricing after investment of two enterprises, a dynamic games model of investment in research and development of new technology in duopoly market is built in the text. The Nash Equilibrium obtained shows that the forecast on the probability of successful research and development of its opponent enterprise is also an important factor affecting the research and development

decision in addition to decrease in research and development expense and production cost. It is demonstrated that the subjective judgment of decision-maker can influence the real research and development investment and in case of small orders (small demanding scale), even the enterprise is at a disadvantage in research and development for improving production cost, it can gain market share by taking the advantages in research and development expense; in case of bulk order, the enterprise will be more willing to make great research and development investment to achieve greater and more efficient competitive advantaged in market. The conclusions drew under the conditions that the model expands to N enterprises shows that during purchasing patents and developing new technology of the same kind, the enterprise shall take into consideration the number of enterprises with authorization and owning this new technology in the market to avoid over-input in market of saturated innovative enterprise.

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