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### COORDINATING WITH DISCOUNT PRICING CONTRACT IN A TWO-LEVEL SUPPLY CHAIN

<sup>1</sup>WENGUO ZHANG, <sup>2\*</sup>WEI XU

<sup>1</sup>Beijing Institute of Technology, Zhuhai, China

<sup>2</sup>University of Macau, Macao SAR \*<sup>corresponding author</sup> E-mail: Jerryxw@live.com

#### ABSTRACT

The ordering model between single supplier and multi-retailers under the circumstance of non-cooperation supply chain is discussed in this paper, an improved linear price discount strategy is also provided. This strategy considers two situations of incremental discount and decremental discount at the same time. Moreover, Stackelberg game model of both parties' order on supply and demand is given in this paper. The numerical analysis result demonstrates that discount model is very effective in improving coordination on operation of supply chain.

Keywords: Price Discount, EOQ, Stackelberg Equilibrium, Supply Contract

#### 1. INTRODUCTION

Research on ordering quantity of supplier and retailer is one of the most important and the most fundamental contents in management of supply chain. In traditional economic order quantity model, the supplier and the retailer proceed from their own interests, and their pursuit is that their cost can reach the minimum order quantity. This order quantity is the optimal order quantity for the retailer, but it is not the optimum order quantity for the supplier. In other words, in traditional economic order quantity model, the supplier and the retailer do not coordinate with each other. In order to solve this problem, many scholars have conducted researches on it. However, in their research results, price discount always considers that when the retailer increase order quantity, the supplier gives price discount, or the supplier gives one-off price discount to make both parties obtain benefits.

First of all, under make-to-order production, noncooperative production order strategy when production or order flexibility of the supplier and is different from that of the retailer is considered. Then, cooperation model in which the seller and the buyer can obtain benefits through price discount is put forward, and the problem of decremental discount is considered.

Monahan provides determinative factor about increase of the optimal order quantity for the supplier and the retailer, i.e.  $k^* = \sqrt{S_2/S_1 + 1}$ ,

where  $S_1$  and  $S_2$  refer to order cost of the seller and order cost of the buyer respectively. Lee and Rosenblatt popularize this model, add the minimum marginal profit and allow the buyer to order any quantity. From the given model, the optimal order quantity increase factor K and the optimal order quantity of the seller can be found at the same time. It is an integer multiple k of the buyer's order quantity. Lee and Rosenblatt design an algorithm for situations of single buyer and single product to decide the profit's maximum quantity discount price list so as to calculate the biggest profit. In this method, it is assumed that order cost and cost of carry have been already known, EOQ strategy is adopted based on the buyer, according to quantity discount, the appropriate order quantity is given. As to single buyer and single seller. Baneriee assumes that production rate is limited, and the optimal cooperative production or order quantity model is given under the assumption that order was conducted before production. Goyal holds that economic production quantity should be integral multiple of the buyer's purchase quantity. Tersine and Barman provide a provisional price discount to make both parties obtain benefits. In addition, Anupindi, Akella, Kohli, Park and Lau discuss the reduction of both buyer and seller's cost through cooperative order strategy. Emmons and Gilbert focus on the effect that such policies have on both a retailer's and a manufacturer's profits when the retailer must commit prior to the selling season to both a stocking quantity and a price at which to sell an item. Manufacturers often use returns policies to

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encourage retailers to stock and price items more aggressively. The simultaneous pricing and procurement decisions associated with a one-period pure inventory model under deterministic or probabilistic demand were considered by Polatoglu. Furthermore, how options provide flexibility to a buyer to respond to market changes in the second period was illustrated. Dawn et al. study the implications of such arrangements between a buyer and a manufacturer for coordination of the channel, they also demonstrate the benefits of options in improving channel performance and evaluate the magnitude of loss due to lack of coordination. Returns policy is commitment by manufacturer or distributor upstream to retrieve excess inventories from downstream channel member. Tavur et al. provided major findings on returns policy in the supply contract literature.

# 2. ASSUMPTION AND SYMBOL OF MODEL

In this paper, only the relation between a single supplier and n retailers is studies, and each retailer has no difference in precedence order and they are coessential. Let's assume that the retailer's demand per unit time is subject to uniform distribution, and the supplier adopts production mode in order form, namely the supplier will produce the goods according to demand of the retailer.

P: the output of the supplier per unit time, i.e.

production capacity (P > nQ)

C: the supplier's production cost per unit

 $C_p$ : the setup cost and delivery expense of the

supplier

H: the supplier's cost of carry per unit within unit time

 $n_s$ : the supplier's number of order

c: the retailer's preparatory cost for each order

*h* : the retailer's cost of carry per unit product

 $n_r$ : the retailer's number of order

*T* : the retailer's order cycle time

*t* : the retailer's production time within T

 $P_d$ : market delivery price of both parties

## 3. THE SITUATION UNDER NO CONTRACT

### 3.1 The economic order quantity model of *n* retailers

The Retailers' total order cost within unit time can be classified into product's cost of carry, preparatory cost for purchase and product cost. Total cost within unit time is expressed by Tc, and it can be obtained that,

$$Tc = \frac{Q_r}{2}h + \frac{Q}{Q_r}c + P_dQ.$$
 (1)

The economic order quantity is obtained by  $\frac{dTc}{dTc} = 0$ 

$$dQ_r$$

$$Q_r = \sqrt{2cQ/h},\tag{2}$$

then the number of order is obtained by,

$$n_r^* = \frac{Q}{Q_r} = \sqrt{\frac{hQ}{2c}}.$$
 (3)

### 3.2 The optimal production lot size model of the supplier

The supplier adopts make-to-order mode. The total cost can be classified into preparatory cost (mainly including setup cost and order processing cost), cost of carry and production cost (production cost per unit product is *C*). Under the circumstance where the market delivery cost *c* does not change, the order cycle time is  $T = Q_s/nQ$ , the production time within *T* is  $t = Q_s/P$ , the number of order is  $n_s = nQ/Q_s$ , the average storage capacity is  $\frac{Q_s t}{2T} = \frac{nQQ_s}{2P}$ . Then the supplier's profit function which is expressed by  $\prod_s$ , and then it can be obtained that,

$$\prod_{s} = nQ(P_{d} - C) - \left[\frac{nQQ_{s}}{2P}H + \frac{nQ}{Q_{s}}C_{p}\right]$$
(4)

The economic order quantity is obtained by  $\frac{d\prod_{s}}{d\prod_{s}} = 0$ .

$$dQ_s = 1$$

$$Q_r = \sqrt{2C_p P/H} \tag{5}$$

$$n_s^* = \sqrt{\frac{nQH}{2cP}} \tag{6}$$

Then we can get the optimal profit as follow,

$$\prod_{s}^{*} = nQ(P_{d} - C) - nQ\sqrt{2C_{p}H/P}$$
(7)

Namely, the most expected result of the supplier is that EPQ can be obtained according to its own cost structure for delivery of goods so as to meet demands of the retailer.

However, this cannot meet the retailer's cost structure. When production is conducted according to making-to-order mode, production lot size of the supplier is equal to that of the retailer. Therefore, actual delivery quantity at each time is  $\sqrt{2cQ/h}$ .

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### **3.3** Two conditions of non-cooperation between the supplier and the retailer

When  $EPQ = n \cdot EOQ$ , the supplier and the retailer are in the optimal profit state, and this is a natural cooperative state which will not be discussed in the paper. However, generally,  $EPQ \neq n \cdot EOQ$ . Under this circumstance, the supplier's cost will deviate from the optimal point and then increase, and then the profit decreases. Therefore, the supplier is passive in independent non-cooperative relation. Under this circumstance, the supplier can be initiative to look for partners so as to minimize the cost of both parties and optimize the supply chain.

From the above analysis of the model, it can be known that supplier is in passive situation and the supplier makes decisions after the retailer makes decision. In this case, the supplier does not make optimal decision, and but the retailer makes optimal decision. When  $EPQ > n \cdot EOQ$ , if the retailer increases order quantity, the retailer's cost of carry increases, and then the retailer's interest will be damaged. However, this is beneficial for the supplier to increase profit. The supplier can stimulate the retailer to increase order quantity of each time through conventional incremental price discount method. When  $EPQ < n \cdot EOQ$ , if the retailer is required to decrease order quantity, the retailer's preparatory cost will increase and its interest will be damaged. However, this is beneficial for the supplier to increase profit. Therefore, in the optimization of supply chain, redistribution of interests of the supplier and the retailer shall be paid attention to so as to reduce or remain the retailer's cost when the supplier reduces cost.

#### 4. IMPROVEMENT OF PRICE DISCOUNT STRATEGY

Let's assume that both parties agree to adopt linear price discount, and the following price discount strategy model is put forward:

$$P' = P_d + b(Q - EOQ) \tag{8}$$

When b > 0, and then  $Q_r < n \cdot EOQ$ , this means that the retailer accepts discount policy when order quantity decreases, and the discount is decremental price discount, when b < 0, and then  $Q_r > n \cdot EOQ$ , this means that the retailer accepts discount policy when order quantity increases, and the discount is incremental price discount. In this case, from the above formula, the retailer's cost can be obtained that:

$$Tc^{\dagger} = \frac{Q_s}{2}h + \frac{Q}{Q_s}c + [P_d + b(Q - EOQ)]Q \qquad (9)$$

Combining single supplier model and multisupplier model discussed in this paper, the following improvements are conducted to the following price discount strategy model put forward in existing literatures:

$$P' = P_d - b \frac{Q - EOQ}{EPQ - n \cdot EOQ}$$
(10)

Where, Q means order quantity of each retailer, and b is constantly greater than 0.

This model has the following improvements:

(1) Discount thought of the original model is established based on the compensation for retailer's deviation from EOQ. Except for this compensation, this model also reflects reward for retailer's tendency towards EPQ. When EOQ < EPQ, formula (8) does not clearly indicate order quantity of the retailer along, and the discount can be obtained when 0 meets the condition that  $n \cdot EOQ < Q < EPQ$ . However, from formula (10), it can be seen that the retailer can only obtain discount when  $n \cdot EOO < O < EPO$ .

(2) When the supplier's information and the retailer's information are not symmetrical, compared to formula (8), formula (10) is more beneficial to preventing the retailer from making a false report about EOQ.

When  $EPQ > n \cdot EOQ$ , combining formula (9), when the retailer makes a false report about EOQ, making the false EOQ less than actual true value of EOQ, the retailer can still obtain relatively large discount even though the retailer order goods according to its true value of EOQ, combining formula (10), when the retailer makes a false report about EOQ, making the false EOQ less than true value of EOQ, as to  $\frac{Q - EOQ}{EPQ - n \cdot EOQ}$ , although the

numerator is greater, the denominator also increases. Therefore, benefits obtained by the retailer through making false report about EOQ by combining formula (10) is less than that by combining formula (8).

(3) From formula (10), it can be seen that, when the retailer's order quantity is EPQ, it is the most beneficial to the supplier. Therefore, in the case, the

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discount obtained by the retailer is the largest, which cannot be reflected in formula (8).

When  $k = EPQ - n \cdot EOQ$ , the ordering cost of retailer is given by,

$$Tc^{i} = \frac{Q_{s}}{2}h + \frac{Q}{Q_{s}}c + [P_{d} - \frac{b}{k}(Q - n \cdot EOQ)]Q, \quad i = 1, 2, ..., n$$

So we can easily get the following equations,

$$\begin{cases} Q_r^* \ge \sqrt{2cQ/h}, & EPQ > n \cdot EOQ \\ Q_r^* \le \sqrt{2cQ/h}, & EPQ < n \cdot EOQ \end{cases}$$
(10)

Where  $Q_r = Q_s/n$ , also we can get  $Tc_r^{i^*} = Tc_r^i(Q^*)$ .

As shown from above sets of equations, when  $EPQ > n \cdot EOQ$ ,  $\frac{dTc_r^{i^*}}{dh} < 0$ , that is  $Q_r^* < EOQ$ .

When 
$$EPQ < n \cdot EOQ$$
,  $\frac{\mathrm{d}Tc_r^{i^*}}{\mathrm{d}b} < 0$ . And because of

 $\frac{\mathrm{d}Tc_r^{i^*}}{\mathrm{d}b} = -\frac{Q_r - EOQ}{EPQ - EOQ}Q \quad . \quad Tc_r^{i^*} \quad \text{is} \quad \text{decreasing}$ 

function of b, that is, at that point b = 0, we can get the maximum cost.

Proposition 1. Under the improvement of price discount contract, if b > 0, then  $Tc_r^{*} < Tc_r^{*}$ .

In order to make the retailer collaboration actively, the supplier has to minimize the cost of retailer under the price discount contract, at the same time make the biggest profits. So we utilize the Stackelberg equilibrium. We can get,

$$\begin{cases} \max \prod_{s}^{\cdot} = nQ[P_{d} - C - \frac{b}{k}(Q_{r} - EOQ)] \\ -\left[\frac{n^{2}Q_{r}Q}{2P}H + \frac{Q}{Q_{r}}C_{p}\right] \\ Q_{r} = \arg \min Tc_{r}^{i} = \frac{Q_{r}}{2}Q + \frac{Q}{Q_{r}}c \\ +\left[P_{d} + \frac{b}{k}(Q_{r} - EOQ)\right]Q, \quad i = 1, 2...n \\ Q_{r} \ge 0 \end{cases}$$
(11)

Where  $Q_r$  is the ordering quantity of the retailer.

$$\max \prod_{s} = nQ[P_{d} - C - \frac{b}{k}(Q_{r}^{*} - EOQ)]$$
$$-\left[\frac{n^{2}Q_{r}Q}{2P}H + \frac{Q}{Q_{r}^{*}}C_{p}\right], \text{ where the}$$

optimal quantity of the retailer is  $Q_r^* = \sqrt{\frac{2kcQ}{kh - 2bQ}}$ .

As 
$$b = 0$$
,  $Q_r^* = \sqrt{\frac{2kcQ}{kh - 2bQ}} \Rightarrow \sqrt{\frac{2cQ}{h}}$ , so we can

easily get the result of formula  $\max \prod_{s}^{\prime} (b=0)$ .

Proposition 2. When max  $\prod_{s}$  satisfy the condition,

$$\max \prod_{s} = nQ(P_d - C) - \frac{C_p hP + cHn^2 Q}{P} \sqrt{Q/2ch}$$

the benefits of co-operation outweigh the risks for the supplier.

We can calculate the optimal solution  $b^*$  by means of MATLAB, then get the corresponding price discount. As can be shown from the above proposition, when  $\max \prod_s = \prod^r (b^*)$ , the Stackelberg - Nash equilibrium is  $(b^*, Q_r^*)$ . At last, the delivery price is  $P' = P_d + b/k (Q_r^* - EOQ)$ .

#### 5. NUMERICAL ANALYSIS

The following is numerical analysis of above basic conclusions based on two situations. We will calculate the EOQ of retailer, the minimum cost, the actual profit of the supplier and the EPQ of supplier under the cooperation or non-cooperation situation.

If we set Q = 1000, P = 5000,  $C_p = 180$ , H = 2, c = 80, h = 0.1, C = 5,  $P_d = 8$  and n = 3. The economic order quantity of retailer EOQ=1264,  $Tc_r^* = 8126$ , EPQ=948 without cooperation. However, under the improvement of price discount contract, k = EPQ - 3EOQ = -2846, we can get the optimal b, that is  $b^* = 0.455$ . The delivery price

$$P = 7.89$$
,  $Tc_r^{i^*} = 8089$ ,  $\prod_{s}^{i^*} = 7557$ .

From the above examples can be found,  $n \cdot EOQ = 1264 > EPQ = 948$ , but  $b^* = 0.455 > 0$ . Some discount-off price would benefit both parties.

#### 6. CONCLUSION

The coordination ordering model under noncooperative relation between a single supplier and multiple coessential retailers is discussed in this paper. As to the phenomenon where the supply is not coordinated with the demand, an improved price discount strategy is put forward in this paper. Compared to the original price discount strategy, this improve strategy is more reasonable and more practical. In the improved price discount strategy, two conditions of incremental discount and

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decremental discount are considered. Moreover, the Stackelberg game model of both parties' order on supply and demand is given, and examples are given for further explanation and analysis, which has a guiding significance for enterprises. The following orientations are key research orientations in the future: research on coordination order model and discrete model under multiple different retailers as well as coordination order model under multilayer supply chain, as well as the research on supply-demand cooperation under random demand as well as coordination model under multiple retailers and multi-layer supply chain.

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