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# ROBUST CONTROL ALGORITHM BY ADJUSTING INPUT VOLTAGE FOR DC-DC CONVERTERS

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## ABSTRACT

In view of the switching property and the existence of disturbance in DC-DC converters, a robust control algorithm by adjusting input voltage is studied based on the switched linear model of DC-DC converters. Considering the output disturbance and weighted functions chosen for perturbation input and controlled output, the augmented switched linear model of DC-DC converters is established, in which the input voltage is as control variable. According to linear matrix inequality (LMI) and $H_\infty$ control theories, the sufficient condition of controller existence and the design method of $H_\infty$ controller for the proposed switched system are derived in the form of LMI. The obtained controller is such that the closed loop system is asymptotically stable under arbitrary switch conditions and has an $H_\infty$ disturbance attenuation bound. The simulation results for buck and boost converters are presented to show that the good robust stability can be obtained with respect to the output disturbance. By adjusting the input voltage, the output voltage can be stabilized on the expected value with the parameters perturbation under arbitrary constant duty cycle, which demonstrates the effectiveness of the proposed algorithm.

**Keywords:** *$H_\infty$ Control, Switched System, Linear Matrix Inequality(LMI), DC-DC Converter*

## 1. INTRODUCTION

DC-DC switch converters are the core of power supply and have been used extensively in many fields. The classical control technique for such converters is PID control, which is derived based on small-signal models of converters. However, DC-DC converters are nonlinear systems in essence, whose small-signal models obtained by linearization and averaging procedure can not describe their nonlinear behavior entirely. Hence, it is primary to find more exact models for controller design of the converters. Some models of DC-DC converters have been proposed. The sampled-data model[1] of PWM systems was proposed, which accurately describes the robustness constraint. Base on the linear approximation of the model that captures the primary switching behavior of the system, some discrete time control methods were derived such as LQ optimal control[2] and $H_\infty$ control[3]. The switched linear system model is also proposed to describe dynamical performance of hybrid dynamic systems. This model includes several linear subsystems, and can achieve a certain performance requirements by appropriately switching between these linear subsystems. However, DC-DC switch converters are typically switched linear system due to their operating characteristics as follows: a converter operates on different states when switches are on or off; each operating state will correspond to a continuous linear time-invariant system; the converter switches between these different linear systems. In recent years, a considerable number of study results were reported for the stability of switched systems[4~6]. In [6], the periodic switched linear system models were built for the buck and boost converter. Furthermore, the controllability and reachability of DC-DC converters were proved based on the switched linear system models. However, the interference problem is not considered in the modeling for the above systems.

$H_\infty$ control strategy has been introduced to control switch converters due to its linear characteristics partially. The derived controller can also be used in large signal applications. Moreover, $H_\infty$ control strategy is superior to the conventional control approach in robustness and interference rejection. Naim et al. applied $H_\infty$ control to a boost converter, which show the good agreement between

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simulation and experimental results[7]. Vidal et al. designed an H∞ controller for boost and buck-boost converters that maximizes the bandwidth control loop with good tracking and high rejection capability of disturbances introduced by changes of load and input voltage[8]. An H∞ loop-shaping controller is proposed that not only retains the advantages of the existing H∞ control strategy but also affords an additional tuning parameter which reflects a trade-off between stability robustness and the performance in time domain[9]. Nevertheless, the above mentioned H∞ controllers were designed based on linear and averaged models of converters. The H∞ controllers were obtained by solving two algebraic Riccati equations, which requires controlled systems satisfy several assumptions. Thus the modeling for the system will become more difficult. As the H∞ control problem can be described in terms of LMIs, the robust control strategies based on LMI approach[10~13] have come out with more and more methods were presented to solve the convex optimization problem. The controller design based on LMI only requires the controlled objects are stabilizable and detectable, so that the modeling for the system and computing for the controller get easier and more reasonable.

In this report, choosing input voltage as control variable, a switched linear error model with perturbations is built for DC-DC switch converters. Based on the proposed model, an H∞ output feedback controller is developed, achieving output voltage tracking by regulating input voltage. The sufficient condition of the existence of the aforementioned controller and the controller algorithm are presented by LMI approach. H∞ output feedback controllers for buck and boost converters operating in continuous conduction mode are derived. Finally, simulations are performed to demonstrate the dynamic performance and robustness of the H∞ controllers derived.

## 2. SWITCHED LINEAR MODEL'S DERIVATION

In DC-DC switch converters, there are two kinds of storage elements, namely capacitors and inductors. The inductor current and capacitor voltage are chosen as state variables as usually during modeling system. Here, input voltage is chosen as control input signal, the load voltage as measurement output and output disturbance as one input. According to on or off state of power switches, a converter presents different circuit topologies meaning different states during a switching period. Each state will lead to a set of linear equations that describe one linear subsystem. Suppose that a converter switches between its linear subsystem on a switch sequence denoted by $\{t_k, i_k\}$, $k$=1, 2, … , where $t_1 < t_2 < \ldots < t_k < \ldots$, $\lim_{k\to\infty} t_k = \infty$, $i_k \in \boldsymbol{M}=\{1, 2, \ldots, N\}$. The converter comes into subsystem $i_k$ at time $t_k$. Further, suppose state variables of a converter are continuous, given $x(t_k^-) = x(t_k^+)$. Then the switched linear model for DC-DC switch converter is defined as:

$$\begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{A}_i \boldsymbol{x}(t) + \boldsymbol{B}_{i1}\boldsymbol{w}(t) + \boldsymbol{B}_{i2}\boldsymbol{u}(t) \\ \boldsymbol{z}(t) = \boldsymbol{C}_{i1}\boldsymbol{x}(t) + \boldsymbol{E}_{i1}\boldsymbol{w}(t) + \boldsymbol{E}_{i2}\boldsymbol{u}(t) \\ \boldsymbol{y}(t) = \boldsymbol{C}_{i2}\boldsymbol{x}(t) + \boldsymbol{D}_{i1}\boldsymbol{w}(t) + \boldsymbol{D}_{i2}\boldsymbol{u}(t) \end{cases} \quad (1)$$

where $\boldsymbol{x}(t) \in \mathbf{R}^n$ is the state vector of the converter, $\boldsymbol{w}(t) \in \mathbf{R}^q$ perturbation input, $\boldsymbol{u}(t) \in \mathbf{R}^m$ control input, $\boldsymbol{y}(t) \in \mathbf{R}^p$ measurement output, and $\boldsymbol{z}(t) \in \mathbf{R}^r$ controlled output. $\boldsymbol{A}_i$, $\boldsymbol{B}_{i1}$, $\boldsymbol{B}_{i2}$, $\boldsymbol{C}_{i1}$, $\boldsymbol{C}_{i2}$, $\boldsymbol{D}_{i1}$, $\boldsymbol{D}_{i2}$, $\boldsymbol{E}_{i1}$, $\boldsymbol{E}_{i2}$ are real constant matrixes with appropriate dimension, $i \in \boldsymbol{M}=\{1, 2, \ldots, N\}$.

Define error variables as follows: $\boldsymbol{x}_e=\boldsymbol{x}-\boldsymbol{x}_d$, $\boldsymbol{y}_e=\boldsymbol{y}-\boldsymbol{y}_d$, $\boldsymbol{u}_e=\boldsymbol{u}-\boldsymbol{u}_d$, $\boldsymbol{w}_e=\boldsymbol{w}-\boldsymbol{w}_d$, where $\boldsymbol{x}_d$, $\boldsymbol{y}_d$, $\boldsymbol{u}_d$ and $\boldsymbol{w}_d$ are desirable operating point. System (1) can be described by error variables as:

$$\begin{cases} \dot{\boldsymbol{x}}_e(t) = \boldsymbol{A}_i \boldsymbol{x}_e(t) + \boldsymbol{B}_{i1}\boldsymbol{w}_e(t) + \boldsymbol{B}_{i2}\boldsymbol{u}_e(t) \\ \boldsymbol{z}_e(t) = \boldsymbol{C}_{i1}\boldsymbol{x}_e(t) + \boldsymbol{E}_{i1}\boldsymbol{w}_e(t) + \boldsymbol{E}_{i2}\boldsymbol{u}_e(t) \\ \boldsymbol{y}_e(t) = \boldsymbol{C}_{i2}\boldsymbol{x}_e(t) + \boldsymbol{D}_{i1}\boldsymbol{w}_e(t) + \boldsymbol{D}_{i2}\boldsymbol{u}_e(t) \end{cases} . \quad (2)$$

To obtain high rejection capability of disturbances and good tracking performance, weighted functions are appropriately chosen for perturbation input and controlled output.

Define $\boldsymbol{w}_e(s) = \boldsymbol{W}_w(s)\tilde{\boldsymbol{w}}_e(s)$, $\tilde{\boldsymbol{z}}_e(s) = \boldsymbol{W}_z(s)\boldsymbol{z}_e(s)$. Then the aim of H∞ control can be transformed into design an output feedback controller of the augmented error system $P(s)$: $[\tilde{\boldsymbol{w}}_e\ \boldsymbol{u}_e]^{\mathrm{T}} \to [\tilde{\boldsymbol{z}}_e\ \boldsymbol{y}_e]^{\mathrm{T}}$, satisfying the prescribed H∞ norm bound of the closed-loop system transfer function from the input $\tilde{\boldsymbol{w}}_e$ to the output $\tilde{\boldsymbol{z}}_e$ denoted by $T_{zw}$. $P(s)$ can be obtained as:

$$P(s) = \begin{bmatrix} \boldsymbol{W}_z & 0 \\ 0 & \boldsymbol{I} \end{bmatrix} G(s) \begin{bmatrix} \boldsymbol{W}_w & 0 \\ 0 & \boldsymbol{I} \end{bmatrix}$$

where $G(s)$ is the transfer function form of (2). The above system can be written easily as state-space representation:

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$$\begin{cases}\dot{\boldsymbol{x}}_e(t)=\tilde{\boldsymbol{A}}_i\boldsymbol{x}_e(t)+\tilde{\boldsymbol{B}}_{i1}\tilde{\boldsymbol{w}}_e(t)+\tilde{\boldsymbol{B}}_{i2}\boldsymbol{u}_e(t)\\ \tilde{\boldsymbol{z}}_e(t)=\tilde{\boldsymbol{C}}_{i1}\boldsymbol{x}_e(t)+\tilde{\boldsymbol{E}}_{i1}\tilde{\boldsymbol{w}}_e(t)+\tilde{\boldsymbol{E}}_{i2}\boldsymbol{u}_e(t)\\ \boldsymbol{y}_e(t)=\tilde{\boldsymbol{C}}_{i2}\boldsymbol{x}_e(t)+\tilde{\boldsymbol{D}}_{i1}\tilde{\boldsymbol{w}}_e(t)+\tilde{\boldsymbol{D}}_{i2}\boldsymbol{u}_e(t)\end{cases} \quad (3)$$

Equation (3) is the augmented error switched linear model for DC-DC converters, which is also the controlled object.

## 3. H∞ OUTPUT FEEDBACK CONTROLLER BASED ON LMI

Assuming that the controlled object described as (3) satisfies the following condition: ( $\tilde{\boldsymbol{A}}_i$ , $\tilde{\boldsymbol{B}}_{i2}$ , $\tilde{\boldsymbol{C}}_{i2}$ ) is stabilizable and detectable. The output feedback H∞ control aims to design a controller with state-space expression as:

$$\begin{cases}\dot{\hat{\boldsymbol{x}}}(t)=\boldsymbol{A}_i^k\hat{\boldsymbol{x}}(t)+\boldsymbol{B}_i^k\boldsymbol{y}_e(t)\\ \boldsymbol{u}_e(t)=\boldsymbol{C}_i^k\hat{\boldsymbol{x}}(t)+\boldsymbol{D}_i^k\boldsymbol{y}_e(t)\end{cases} \quad (4)$$

where $\hat{\boldsymbol{x}}(t)\in\mathbf{R}^{n_k}$ is state vector of controller, $\boldsymbol{A}_i^k$, $\boldsymbol{B}_i^k,\boldsymbol{C}_i^k$, $\boldsymbol{D}_i^k$ ($i$=1, 2, …, $N$) are parameter matrixes to be solved. The controller (4) should assure that the closed-loop system made of (3) and (4) is asymptotically stable and satisfies the prescribed H∞ norm bound of $T_{zw}$.

### 3.1. Sufficient Condition of Controller Existence

Applying (4) to (3), the closed-loop system is obtained as:

$$\begin{cases}\dot{\bar{\boldsymbol{x}}}(t)=\bar{\boldsymbol{A}}_i\bar{\boldsymbol{x}}(t)+\bar{\boldsymbol{B}}_i\tilde{\boldsymbol{w}}_e(t)\\ \tilde{\boldsymbol{z}}_e(t)=\bar{\boldsymbol{C}}_i\bar{\boldsymbol{x}}(t)+\bar{\boldsymbol{D}}_i\tilde{\boldsymbol{w}}_e(t)\end{cases} \quad (5)$$

where $\bar{\boldsymbol{x}}=[\boldsymbol{x}_e \quad \hat{\boldsymbol{x}}]^{\mathrm{T}}$

$$\bar{\boldsymbol{A}}_i=\begin{bmatrix}\tilde{\boldsymbol{A}}_i+\tilde{\boldsymbol{B}}_{i2}\boldsymbol{D}_i^k\tilde{\boldsymbol{C}}_{i2} & \tilde{\boldsymbol{B}}_{i2}\boldsymbol{C}_i^k\\ \boldsymbol{B}_i^k\tilde{\boldsymbol{C}}_{i2} & \boldsymbol{A}_i^k\end{bmatrix}$$

$$\bar{\boldsymbol{B}}_i=\begin{bmatrix}\tilde{\boldsymbol{B}}_{i1}+\tilde{\boldsymbol{B}}_{i2}\boldsymbol{D}_i^k\tilde{\boldsymbol{D}}_{i1}\\ \boldsymbol{B}_i^k\tilde{\boldsymbol{D}}_{i1}\end{bmatrix}$$

$$\bar{\boldsymbol{C}}_i=\begin{bmatrix}\tilde{\boldsymbol{C}}_{i1}+\tilde{\boldsymbol{E}}_{i2}\boldsymbol{D}_i^k\tilde{\boldsymbol{C}}_{i2} & \tilde{\boldsymbol{E}}_{i2}\boldsymbol{C}_i^k\end{bmatrix}$$

$$\bar{\boldsymbol{D}}_i=\tilde{\boldsymbol{E}}_{i1}+\tilde{\boldsymbol{E}}_{i2}\boldsymbol{D}_i^k\tilde{\boldsymbol{D}}_{i1}.$$

**Theorem:** The system (5) is robust stable under arbitrary switch conditions and the infinite norm of $T_{zw}$ is smaller than $\gamma$ ( $\gamma$ >0) if there exists a series of symmetric positive definite matrixes $\boldsymbol{P}_i$ ($i$=1, 2, …, $N$) such that:

$$\begin{bmatrix}\bar{\boldsymbol{A}}_i^{\mathrm{T}}\boldsymbol{P}_i+\boldsymbol{P}_i\bar{\boldsymbol{A}}_i & \boldsymbol{P}_i\bar{\boldsymbol{B}}_i & \bar{\boldsymbol{C}}_i^{\mathrm{T}}\\ \bar{\boldsymbol{B}}_i^{\mathrm{T}}\boldsymbol{P}_i & -\gamma\boldsymbol{I} & \bar{\boldsymbol{D}}_i^{\mathrm{T}}\\ \bar{\boldsymbol{C}}_i & \bar{\boldsymbol{D}}_i & -\gamma\boldsymbol{I}\end{bmatrix}<0$$

$$\forall i\in \boldsymbol{M}=\{1,2,\cdots,N\}. \quad (6)$$

**Proof:** Define a performance function denoted by $\xi=[\xi_1,\xi_2,\cdots,\xi_N]^{\mathrm{T}}$. So that $\xi_i=\gamma$ when switched system is on the subsystem $i_k$ represented by matrix coefficient $\tilde{\boldsymbol{A}}_i$, while other subsystem $\xi_i=0$. Note that $\Lambda=\mathbf{diag}\{\xi_i^{1/2}I,\xi_i^{1/2}I,\xi_i^{-1/2}I\}$, there is

$$\Lambda\begin{bmatrix}\bar{\boldsymbol{A}}_i^{\mathrm{T}}\boldsymbol{P}_i+\boldsymbol{P}_i\bar{\boldsymbol{A}}_i & \boldsymbol{P}_i\bar{\boldsymbol{B}}_i & \bar{\boldsymbol{C}}_i^{\mathrm{T}}\\ \bar{\boldsymbol{B}}_i^{\mathrm{T}}\boldsymbol{P}_i & -\gamma\boldsymbol{I} & \bar{\boldsymbol{D}}_i^{\mathrm{T}}\\ \bar{\boldsymbol{C}}_i & \bar{\boldsymbol{D}}_i & -\gamma\boldsymbol{I}\end{bmatrix}\Lambda=$$

$$\begin{bmatrix}\xi_i(\bar{\boldsymbol{A}}_i^{\mathrm{T}}\boldsymbol{P}_i+\boldsymbol{P}_i\bar{\boldsymbol{A}}_i) & \xi_i\boldsymbol{P}_i\bar{\boldsymbol{B}}_i & \bar{\boldsymbol{C}}_i^{\mathrm{T}}\\ \xi_i\bar{\boldsymbol{B}}_i^{\mathrm{T}}\boldsymbol{P}_i & -\gamma^2\boldsymbol{I} & \bar{\boldsymbol{D}}_i^{\mathrm{T}}\\ \bar{\boldsymbol{C}}_i & \bar{\boldsymbol{D}}_i & -\boldsymbol{I}\end{bmatrix}=$$

$$\begin{bmatrix}\bar{\boldsymbol{A}}_i^{\mathrm{T}}\bar{\boldsymbol{P}}_i+\bar{\boldsymbol{P}}_i\bar{\boldsymbol{A}}_i & \bar{\boldsymbol{P}}_i\bar{\boldsymbol{B}}_i & \bar{\boldsymbol{C}}_i^{\mathrm{T}}\\ \bar{\boldsymbol{B}}_i^{\mathrm{T}}\bar{\boldsymbol{P}}_i & -\gamma^2\boldsymbol{I} & \bar{\boldsymbol{D}}_i^{\mathrm{T}}\\ \bar{\boldsymbol{C}}_i & \bar{\boldsymbol{D}}_i & -\boldsymbol{I}\end{bmatrix}$$

where $\bar{\boldsymbol{P}}_i=\xi_i\boldsymbol{P}_i$. Then there exists a series of symmetric positive definite matrixes $\boldsymbol{P}_i$ such that (6) holds if there exists a series of symmetric positive definite matrixes $\bar{\boldsymbol{P}}_i$ such that:

$$\begin{bmatrix}\bar{\boldsymbol{A}}_i^{\mathrm{T}}\bar{\boldsymbol{P}}_i+\bar{\boldsymbol{P}}_i\bar{\boldsymbol{A}}_i & \bar{\boldsymbol{P}}_i\bar{\boldsymbol{B}}_i & \bar{\boldsymbol{C}}_i^{\mathrm{T}}\\ \bar{\boldsymbol{B}}_i^{\mathrm{T}}\bar{\boldsymbol{P}}_i & -\gamma^2\boldsymbol{I} & \bar{\boldsymbol{D}}_i^{\mathrm{T}}\\ \bar{\boldsymbol{C}}_i & \bar{\boldsymbol{D}}_i & -\boldsymbol{I}\end{bmatrix}<0. \quad (7)$$

According to (7), the following inequality holds: $\bar{\boldsymbol{A}}_i^{\mathrm{T}}\bar{\boldsymbol{P}}_i+\bar{\boldsymbol{P}}_i\bar{\boldsymbol{A}}_i<0$. Moreover, $\bar{\boldsymbol{P}}_i$ >0. So it is proved that the system (5) is asymptotically stable under arbitrary switch conditions.

As (7) holds strictly, a suitable constant $\varepsilon$ ($0<\varepsilon<1$) always can be chosen so that

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$$\begin{bmatrix} \overline{A}_i^{\mathrm{T}}\overline{P}_i + \overline{P}_i\overline{A}_i & \overline{P}_i\overline{B}_i & \overline{C}_i^{\mathrm{T}} \\ \overline{B}_i^{\mathrm{T}}\overline{P}_i & -\gamma^2(1-\varepsilon)I & \overline{D}_i^{\mathrm{T}} \\ \overline{C}_i & \overline{D}_i & -I \end{bmatrix} < 0 .$$

Using properties of Schur complement of matrix, the above inequality can be equivalent to the following inequality:

$$\begin{bmatrix} \overline{C}_i^{\mathrm{T}} \\ \overline{D}_i^{\mathrm{T}} \end{bmatrix}\begin{bmatrix} \overline{C}_i & \overline{D}_i \end{bmatrix} + \begin{bmatrix} \overline{A}_i^T\overline{P}_i + \overline{P}_i\overline{A}_i & \overline{P}_i\overline{B}_i \\ \overline{B}_i^T\overline{P}_i & -\gamma^2(1-\varepsilon)I \end{bmatrix} < 0 \quad (8)$$

For any $T$>0, considering the performance function: $J = \int_0^T \|\tilde{z}_e(t)\|^2 \mathbf{d}t - (1-\varepsilon)\gamma^2 \int_0^T \|\tilde{w}_e(t)\|^2 \mathbf{d}t$, under the condition of zero initial state, $J$ can be calculated

$$\begin{aligned} J &= \int_0^T [\tilde{z}_e^T(t)\tilde{z}_e(t) - (1-\varepsilon)\gamma^2\tilde{w}_e^T(t)\tilde{w}_e(t) + \dot{V}(\overline{x}(t))]\mathbf{d}t - V(\overline{x}(T)) \\ &= \int_0^T [\tilde{z}_e^T(t)\tilde{z}_e(t) - (1-\varepsilon)\gamma^2\tilde{w}_e^T(t)\tilde{w}_e(t) + 2\overline{x}^{\mathrm{T}}(t)P_{ik}(\overline{A}_{ik}\overline{x}(t) + \overline{B}_{ik}\tilde{w}_e(t))]\,\mathbf{d}t - V(\overline{x}(T)) \\ &= \int_0^T [\overline{x}^{\mathrm{T}}(t)\ \ \tilde{w}_e^T(t)]\left(\begin{bmatrix} \overline{C}_i^{\mathrm{T}} \\ \overline{D}_i^{\mathrm{T}} \end{bmatrix}\begin{bmatrix} \overline{C}_i & \overline{D}_i \end{bmatrix} + \begin{bmatrix} \overline{A}_{ik}^T P_{ik} + P_{ik}\overline{A}_{ik} & P_{ik}\overline{B}_{ik} \\ \overline{B}_{ik}^T P_{ik} & -\gamma^2(1-\varepsilon)I \end{bmatrix}\right)\begin{bmatrix} \overline{x}(t) \\ \tilde{w}_e(t) \end{bmatrix}\mathbf{d}t - V(\overline{x}(T)) \end{aligned}$$

. According to (8), it can be seen that the integral item of $J$ is less than zero. Further using zero initial conditions, there is

$$\overline{x}^{\mathrm{T}}(T)\overline{P}_i\overline{x}(T) + \int_0^T \tilde{z}_e^T(t)\tilde{z}_e(t)\mathbf{d}t < (1-\varepsilon)\gamma^2\int_0^T \tilde{w}_e^T(t)\tilde{w}_e(t)\mathbf{d}t .$$

For $\tilde{w}_e(t) \in L_2[0\ \infty)$ and asymptotic stability of the system (5), if making $T\to\infty$ on both sides of the above inequality, the formula can be obtained as follows: $\|\tilde{z}_e(t)\|^2 \le (1-\varepsilon)\gamma^2\|\tilde{w}_e(t)\|^2 < \gamma^2\|\tilde{w}_e(t)\|^2$, which implies that the system (5) has the $H_\infty$ robust performance $\gamma$. The proof is complete.

### 3.2 Controller Algorithm

In the existence condition (6) of the controller, the matrix variables $P_i$ and the controller parameter matrixes emerge in a nonlinear way. So the controller is hard to be computed from (6). Here variable substitution method is applied. By introducing new variables, (6) can be transformed into a new form which is a LMI expressed by new variables.

**Lemma**[14] A necessary and sufficient condition is

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0 \quad (9)$$

for existing a symmetric positive definite matrixes $P$ such that the following equations: $P = \begin{bmatrix} Y & N \\ N^{\mathrm{T}} & * \end{bmatrix}$, $P^{-1} = \begin{bmatrix} X & M \\ M^{\mathrm{T}} & * \end{bmatrix}$ hold, where * is any possible matrix.

Two new matrix variables $X_i$ and $Y_i$ are introduced to deduce the below formula according the above lemma: $P_i\begin{bmatrix} X_i & I \\ M_i^{\mathrm{T}} & 0 \end{bmatrix} = \begin{bmatrix} I & Y_i \\ 0 & N_i^{\mathrm{T}} \end{bmatrix}$, $P_iP_i^{-1} = I$. The $M_i$ and $N_i$ can be calculated by decomposing the matrix $I$-$X_iY_i$ through singular value decomposition. Here denote: $F_i = \begin{bmatrix} X_i & I \\ M_i^{\mathrm{T}} & 0 \end{bmatrix}$.

Define the following variable substitution formula:

$$\begin{cases} \hat{A}_i = Y_i(\tilde{A}_i + \tilde{B}_{i2}D_i^k\tilde{C}_{i2})X_i + N_iB_i^k\tilde{C}_{i2}X_i + Y_i\tilde{B}_{i2}C_i^kM_i^{\mathrm{T}} + N_iA_i^kM_i^{\mathrm{T}} \\ \hat{B}_i = Y_i\tilde{B}_{i2}D_i^k + N_iB_i^k \\ \hat{C}_i = D_i^k\tilde{C}_{i2}X_i + C_i^kM_i^{\mathrm{T}} \\ \hat{D}_i = D_i^k \end{cases} \quad (10)$$

The left of (6) multiplies diag{ $F_i^{\mathrm{T}}$ , $I$, $I$} on the left and diag{ $F_i$ , $I$, $I$} on the right respectively. Then an equivalent inequality of (6) is obtained as (11)

$$\begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} & \Theta_{14} \\ * & \Theta_{22} & \Theta_{23} & \Theta_{24} \\ * & * & -\gamma I & \Theta_{34} \\ * & * & * & -\gamma I \end{bmatrix} < 0 \quad (11)$$

where $\Theta_{11} = \tilde{A}_iX_i + X_i\tilde{A}_i^{\mathrm{T}} + \tilde{B}_{i2}\hat{C}_i + (\tilde{B}_{i2}\hat{C}_i)^{\mathrm{T}}$

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$$\boldsymbol{\Theta}_{12} = \hat{\boldsymbol{A}}_i^{\mathrm{T}} + \tilde{\boldsymbol{A}}_i + \tilde{\boldsymbol{B}}_{i2}\hat{\boldsymbol{D}}_i\tilde{\boldsymbol{C}}_{i2}$$
$$\boldsymbol{\Theta}_{13} = \tilde{\boldsymbol{B}}_{i1} + \tilde{\boldsymbol{B}}_{i2}\hat{\boldsymbol{D}}_i\tilde{\boldsymbol{D}}_{i1}$$
$$\boldsymbol{\Theta}_{14} = (\tilde{\boldsymbol{C}}_{i1}\boldsymbol{X}_i + \tilde{\boldsymbol{E}}_{i2}\hat{\boldsymbol{C}}_i)^{\mathrm{T}}$$
$$\boldsymbol{\Theta}_{22} = \tilde{\boldsymbol{A}}_i^{\mathrm{T}}\boldsymbol{Y}_i + \boldsymbol{Y}_i\tilde{\boldsymbol{A}}_i + \hat{\boldsymbol{B}}_i\tilde{\boldsymbol{C}}_{i2} + (\hat{\boldsymbol{B}}_i\tilde{\boldsymbol{C}}_{i2})^{\mathrm{T}}$$
$$\boldsymbol{\Theta}_{23} = \boldsymbol{Y}_i\tilde{\boldsymbol{B}}_{i1} + \hat{\boldsymbol{B}}_i\tilde{\boldsymbol{D}}_{i1}$$
$$\boldsymbol{\Theta}_{24} = (\tilde{\boldsymbol{C}}_{i1} + \tilde{\boldsymbol{E}}_{i2}\hat{\boldsymbol{D}}_i\tilde{\boldsymbol{C}}_{i2})^{\mathrm{T}}$$
$$\boldsymbol{\Theta}_{34} = (\tilde{\boldsymbol{E}}_{i1} + \tilde{\boldsymbol{E}}_{i2}\hat{\boldsymbol{D}}_i\tilde{\boldsymbol{D}}_{i1})^{\mathrm{T}}.$$

The symbol * in (11) represents symmetric terms in the LMI. From (11), it can be seen that it is a LMI about matrix variables $\hat{\boldsymbol{A}}_i$, $\hat{\boldsymbol{B}}_i$, $\hat{\boldsymbol{C}}_i$, $\hat{\boldsymbol{D}}_i$, $\boldsymbol{X}_i$ and $\boldsymbol{Y}_i$, which can be solved using the LMI tools of Matlab.

Designing output feedback $H_\infty$ controller concludes following steps.

Step 1: Set $\gamma$.

Step 2: Choose weighted functions.

Step 3: Solve the LMI (11), if no solution, go to step 2.

Step 4: Deriving $\boldsymbol{M}_i$ and $\boldsymbol{N}_i$ by solving singular value decomposition of $\boldsymbol{I}\text{-}\boldsymbol{X}_i\boldsymbol{Y}_i$

Step 5: Deriving the controller parameter matrixes by (10).

## 4. DESIGNED EXAMPLE

### 4.1 Buck Converter

The topology of the buck converter is shown in Figure 1, where $V_{in}$ is DC source voltage, $R_s$, $R_1$, $R_d$,

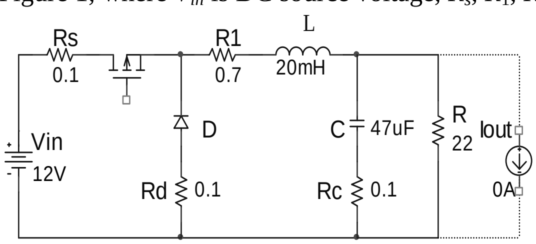


*Figure 1. Topology of Buck Converter*

$R_c$ the internal resistances of the switch transistor, inductor $L$, diode $D$, and capacitor $C$. The symbol $R$ represents the load resistance. The output current source is inserted in the circuit to introduce load perturbation. In this paper, the buck converter operating continuous conduction mode is only considered, where the circuit can be identified two switched subsystems. The error signals $y_e$, $u_e$ of measurement output and control input are chosen as controlled output. The coefficient matrix expressions for the two switched subsystems can be put in the form shown below:

$$\boldsymbol{A}_1 = \begin{bmatrix} -(R_s + R_1 + aR_c)/L & -a/L \\ a/C & -1/RC \end{bmatrix},$$

$$\boldsymbol{B}_{11} = \begin{bmatrix} aR_c/L \\ -a/C \end{bmatrix},\ \boldsymbol{B}_{12} = \begin{bmatrix} 1/L \\ 0 \end{bmatrix},\ \boldsymbol{C}_{11} = \begin{bmatrix} aR_c & a \\ 0 & 0 \end{bmatrix},$$

$$\boldsymbol{E}_{11} = \begin{bmatrix} \text{-}aR_c \\ 0 \end{bmatrix},\ \boldsymbol{E}_{12} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ \boldsymbol{C}_{12} = \begin{bmatrix} aR_c & a \end{bmatrix},$$

$$\boldsymbol{D}_{11} = -aR_c,\ \boldsymbol{D}_{12} = 0;$$

$$\boldsymbol{A}_2 = \begin{bmatrix} -(R_d + R_1 + aR_c)/L & -a/L \\ a/C & -1/RC \end{bmatrix},$$

$$\boldsymbol{B}_{21} = \boldsymbol{B}_{11},\ \boldsymbol{B}_{22} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ \boldsymbol{C}_{21} = \boldsymbol{C}_{11},\ \boldsymbol{E}_{21} = \boldsymbol{E}_{11},$$

$$\boldsymbol{E}_{22} = \boldsymbol{E}_{12},\ \boldsymbol{C}_{22} = \boldsymbol{C}_{12},\ \boldsymbol{D}_{21} = \boldsymbol{D}_{11},\ \boldsymbol{D}_{22} = \boldsymbol{D}_{12},$$

where $a = R/(R + R_c)$.

Setting weighted functions for perturbations input and controlled output respectively as follows: $W_w$=0.1, $W_u$=0.31, $W_y$=$(0.01s+1500)/(s^2+10^4s+180)$, and $\gamma$=1. The desirable operating point is set as $x_d$=$[0.14\text{A}\ 3\text{V}]^{\mathrm{T}}$, $y_d$=3V, $u_d$=$y_d$ /$d$, $d$ is duty cycle, $w_d$=0.5A. The parameter matrixes of output feedback $H_\infty$ controller for buck converter are calculated by the LMI tools of Matlab as follows:

$$\boldsymbol{A}_1^k = \begin{bmatrix} 7152.8 & \text{-}399.44 & 1241.5 & \text{-}993320 \\ \text{-}404.97 & \text{-}212760 & 518.72 & 473.77 \\ 2182.9 & 518.71 & \text{-}9843.6 & \text{-}126360 \\ 59.648 & \text{-}3.3215 & 9.1827 & \text{-}8263.2 \end{bmatrix},$$

$$\boldsymbol{B}_1^k = \begin{bmatrix} 349.51 \\ 212770 \\ \text{-}520.9 \\ 2.9501 \end{bmatrix},\ \boldsymbol{C}_1^k = \begin{bmatrix} 148.57 \\ 2.8822e\text{-}5 \\ 9.7491e\text{-}4 \\ \text{-}20074 \end{bmatrix}^T,\ \boldsymbol{D}_1^k = 0;$$

$$\boldsymbol{A}_2^k = \begin{bmatrix} \text{-}9999.6 & \text{-}6.0469 & 76.568 & 53.167 \\ \text{-}6.0456 & \text{-}212760 & 1487 & \text{-}0.43897 \\ 76.585 & 1487 & \text{-}50.985 & \text{-}0.4381 \\ \text{-}34.801 & \text{-}0.09963 & 0.25911 & 0.054638 \end{bmatrix},$$

$$\boldsymbol{B}_2^k = \begin{bmatrix} \text{-}25.476 \\ 212760 \\ \text{-}1537.1 \\ \text{-}9.7834e\text{-}3 \end{bmatrix},\ \boldsymbol{C}_2^k = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},\ \boldsymbol{D}_2^k = 0.$$

The buck converter under the above-mentioned controller is simulated by switching its two subsystems on a pulse signal with 100kHz frequency and 0.5 duty cycle. The error waveform

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of output voltage and reference output voltage is shown in Figure 2. In the case of reference input voltage $V_{in}$ step changes from 12V to 18V as start up performance at 0.3s and from 18V to 6V at 0.6s, the behaviors of output voltage and control signal are shown in Figure 3 and Figure 4. It can be seen in Figure 3 that the closed-loop system has good damping response to the wide change in the input voltage, and the output voltage is stabilized at the desirable voltage of 3V. Figure 4 depicts that the designed controller have a good adjustment ability following the input voltage variation.

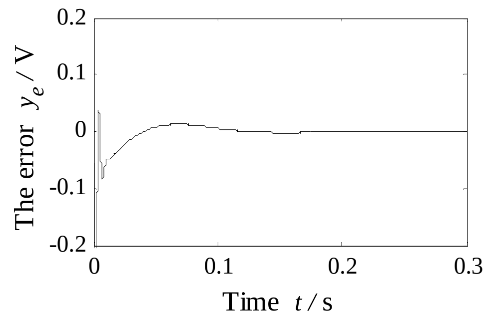


*Figure 2. The Error Waveform of Output Voltage for the Buck Converter*

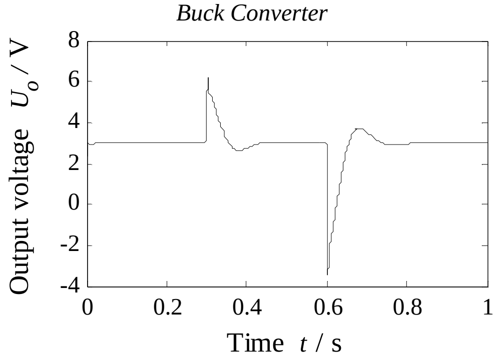


*Figure 3. Output Voltage Response of Buck Converter for Input Voltage Variations*

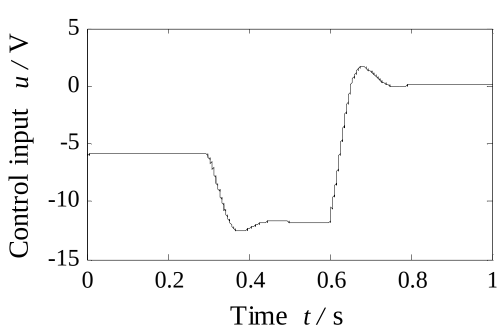


*Figure 4. Control Input Response of Buck Converter for Input Voltage Variations*

### 4.2 Boost Converter

The boost converter is depicted in Figure 5. The

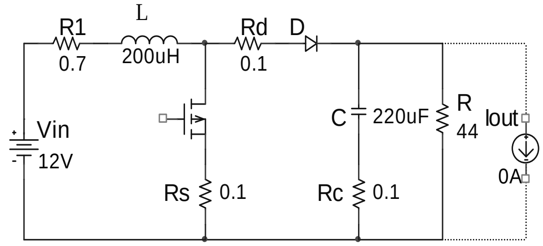


*Figure 5. Topology of Boost Converter*

circuit is also considered working in the continuous conduction mode. The coefficient matrix expressions of its two switched subsystems are given as follows:

$$\boldsymbol{A}_1=\begin{bmatrix}-(R_s+R_1)/L & 0\\ 0 & -a/RC\end{bmatrix},\quad \boldsymbol{B}_{11}=\begin{bmatrix}0\\ -a/C\end{bmatrix},$$

$$\boldsymbol{B}_{12}=\begin{bmatrix}1/L\\ 0\end{bmatrix},\quad \boldsymbol{C}_{11}=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix},\quad \boldsymbol{E}_{11}=\begin{bmatrix}0\\ 0\end{bmatrix},$$

$$\boldsymbol{E}_{12}=\begin{bmatrix}\mathbf{0}\\ 1\end{bmatrix},\ \boldsymbol{C}_{12}=\begin{bmatrix}0 & a\end{bmatrix},\ \boldsymbol{D}_{11}=-aR_c,\ \boldsymbol{D}_{12}=0;$$

$$\boldsymbol{A}_2=\begin{bmatrix}-(R_d+R_1+aR_c)/L & -a/L\\ a/C & -1/RC\end{bmatrix},$$

$$\boldsymbol{B}_{21}=\begin{bmatrix}aR_c/L\\ -a/C\end{bmatrix},\quad \boldsymbol{B}_{22}=\begin{bmatrix}1/L\\ 0\end{bmatrix},\quad \boldsymbol{C}_{21}=\boldsymbol{C}_{11},$$

$$\boldsymbol{E}_{21}=\boldsymbol{E}_{11},\quad \boldsymbol{E}_{22}=\boldsymbol{E}_{12},\quad \boldsymbol{C}_{22}=\begin{bmatrix}aR_c & a\end{bmatrix},$$

$$\boldsymbol{D}_{21}=\boldsymbol{D}_{11},\ \boldsymbol{D}_{22}=\boldsymbol{D}_{12}.$$

Setting weighted functions and the desired operating point respectively as follows: $W_w$=0.01/(400$\pi s$+200$\pi$), $W_y$=(0.3$s$+1200$\pi$)/($s$+100$\pi$), $W_u$=0.4. $x_d$=[1.14A 50V]$^{\mathrm{T}}$, $y_d$=50V, $u_d$=$d\cdot y_d$, $w_d$=0.5A, $\gamma$ =1, the parameter matrixes of output feedback H$_\infty$ controller for boost converter are calculated as follows:

$$\boldsymbol{A}_1^k=\begin{bmatrix}-5571.6 & 0 & 0 & 0\\ 0 & -413.4 & -146.72 & -199.21\\ 0 & -160.14 & -62.216 & -82.384\\ 0 & -140.34 & -93.125 & -580.23\end{bmatrix},$$

$$\boldsymbol{B}_1^k=\begin{bmatrix}0\\ 5.12\\ -14.541\\ 63.948\end{bmatrix},\quad \boldsymbol{C}_1^k=\begin{bmatrix}-0.314\\ 0\\ 0\\ 0\end{bmatrix}^{\mathbf{T}},\quad \boldsymbol{D}_1^k=0;$$

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$$A_2^k = \begin{bmatrix} 39491 & -1038.3 & 1191.8 & 125180 \\ 2584.5 & 392.04 & 1601.4 & -1193.4 \\ 195270 & -1853.4 & -5329.5 & 606710 \\ -17333 & 430.52 & -480.34 & -54741 \end{bmatrix},$$

$$B_2^k = \begin{bmatrix} -766.29 \\ 986.36 \\ 2171.6 \\ 456.13 \end{bmatrix}, \ C_2^k = \begin{bmatrix} -42.725 \\ 0.159 \\ 0.239 \\ -127.6 \end{bmatrix}^T, \ D_2^k = 0 .$$

The boost converter under the above-mentioned controller is simulated by switching its two subsystems on a pulse signal with 100kHz frequency and 0.2 duty cycle. The input voltage variation for boost converter are chosen as same as in section 4.1. The error waveform of output voltage and reference output, output voltage and control input responses of the controlled boost circuit are shown respectively in Figure 6~Figure 8, which depict the closed-loop system has good stability and good tracking performance.

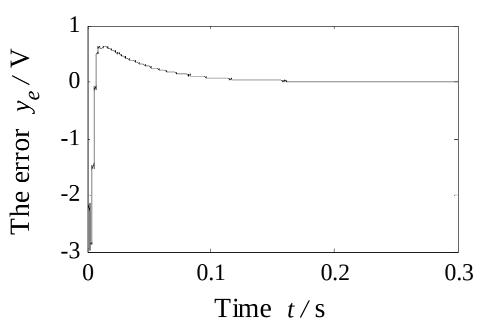


*Figure 6. The Error Waveform of Output Voltage for the Boost Converter*

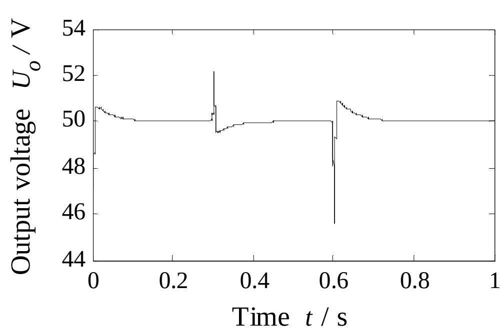


*Figure 7. Output Voltage Response of Boost Converter for Input Voltage Variations*

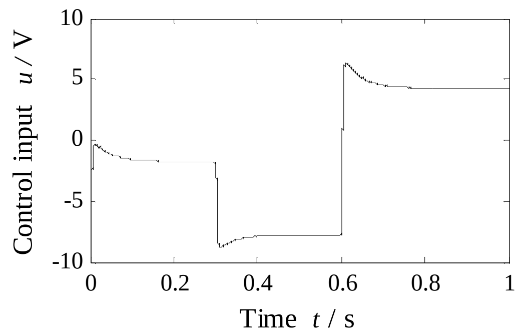


*Figure 8. Control Input Response of Boost Converter for Input Voltage Variations*

In the above two application examples, observing the derived controllers, it can be found that the controller for the second subsystem of buck converter is zero, and the controller for the first subsystem of boost converter is too small to be ignored its function. It means that only one of controllers of switched subsystems will control the closed-loop switched system. Therefore, in the design of the control for actual circuits, it is not need to switch between multiple controllers to control the converter. It will not increase the complexity of the control circuit.

## 5. CONCLUSION

Based on LMI and $H_\infty$ control theory, a control strategy of adjusting input voltage to stabilize output is study in this paper by building switched linear model under perturbations for DC-DC converters. The designed controller guarantees the closed-loop system asymptotically stability and shows good robust performance to perturbations. The method in this paper has the following advantages:

(1) The derivation of a switched linear model does not require any approximation. Comparing to the linear average model, the switched linear model more accurately represents nonlinear characteristics of DC-DC converter.

(2) The number of controllers for closed-loop system can be less than the number of switched subsystems, which can reduce the size and complexity of control circuit.

(3) It is different from usually pulse-width-modulation switching concepts, where the switch position acts as the control input and the duty cycle is solved to adjust the output voltage. The output feedback $H_\infty$ control method here use input voltage as control input and can use an arbitrary constant duty cycle to control converter tracking to the expected value of the output voltage by adjusting

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the control input. It is no need to modulate PWM signal.

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## REFRENCES:

[1] S. Almér, U. Jönsson, C.Y. Kao and J. Mari, "Stability Analysis of a Class of PWM Systems", *IEEE Transactions on Automatic Control*, Vol. 52, No. 6, 2007, pp. 1072-1078.

[2] H. Fujioka, C.Y. Kao, S. Almér and U. Jönsson, "LQ Optimal Control for a Class of Pulse Width Modulated Systems", *Automatica*, Vol. 43, No. 6, 2007, pp. 1009-1020.

[3] H. Fujioka, C.Y. Kao, S. Almér and U. Jönsson, "Robust Tracking with $H_\infty$ Performance for PWM Systems", *Automatica*, Vol. 45, No. 8, 2009, pp. 1808-1818.

[4] Shilong Liu, Zhengrong Xiang and Qingwei Chen, "Stability and Stabilization of a Class of Switched Nonlinear Systems with Time-Varying Delay", *Applied Mathematics and Computation*, Vol. 218, No. 23, 2012, pp. 11534-11546.

[5] M.S. Mahmoud, "Interconnected Continuous-Time Switched Systems: Robust Stability and Stabilization", *Nonlinear Analysis*, Vol. 4, No. 3, 2010, pp. 531-542.

[6] Zongbo Hu, Bo Zhang, Shaofu Hu and Weihua Deng, "Output Controllability Study on DC-DC Converters as Switched Linear Systems", *Acta Electronica Sinica*, Vol. 33, No. 2, 2005, pp. 370-374.

[7] R. Naim, G. Weiss. and B.Y. Shmuel, "$H_\infty$ Control Applied to Boost Power Converters", *IEEE Transactions on Power Electronics*, Vol. 12, No. 4, 1997, pp. 677-683.

[8] I.E. Vidal, S.L Martinez, B.H Valderrama, F. Guinjoan and J. Maixé, "Analysis and Design of $H_\infty$ Control of Nonminimum Phase-Switching Converters", *IEEE Transactions on Circuits and Systems-I*, Vol. 50, No. 10, 2003, pp. 1316-1323.

[9] S.K. Patnaik and B. Umamaheswari, "$H_\infty$ Loop-Shaping Controller for a Boost Converter with Harmonic Reduction", *Journal of Circuits, Systems, and Computers*, Vol. 17, No. 2, 2008, pp. 279-295.

[10] H.C. Sung, J.B. Park and Y.H. Joo, "Robust Observer-Based Fuzzy Control for Variable Speed Wind Power System: LMI approach", *International Journal of Control, Automation and Systems*, Vol. 9, No. 6, 2011, pp. 1103-1110.

[11] Bing Li and Junfeng Wu, "A New Delay-Dependent Stability Criteria for Networked Control Systems", *Journal of Theoretical and Applied Information Technology*, Vol. 43, No. 1, 2012, pp. 127-132.

[12] R. Msjumder, B. Chaudhuri, H.E. Zobaidi, B.C. Pal and I.M. Jaimoukha, "LMI Approach to Normalized $H_\infty$ Loop-Shaping Design of Power System Damping Controllers", *IEE Proceedings Generation Transmission and Distribution*, Vol. 152, No. 6, 2005, pp. 952-960.

[13] A. A. Modarres, H. Momeni, "Unknown Input Sliding-Mode Observer Design for a Drum-Typeâ-Boiler", *Journal of Theoretical and Applied Information Technology*, Vol. 24, No. 1, 2011, pp. 11-16.

[14] T. Iwasaki and R.E. Skelton, "All Controllers for the General $H_\infty$ Control Problem: LMI Existence Condition and State Space Formulas", *Automatica*, Vol. 30, No. 8, 1994, pp. 1307-1317.