



STABILITY CONTROL FOR SIX-WHEEL DRIVE ARTICULATED VEHICLE BASED ON DIRECT YAW MOMENT CONTROL METHOD

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ABSTRACT

Six-wheel drive (6WD) articulated dump truck has excellent passing ability and high productivity, but its stability should be improved. A dynamic model of 6WD articulated vehicle with 3 degree-of-freedom (DOF) is established. With the control variables of yaw velocity and side-slip angle at mass center of front body, a feedforward-feedback control system is designed by linear quadratic regulator (LQR) theory based on direct yaw-moment control (DYC) method. A simulation with a step input of articulated angle is conducted, and the result shows that the value of side-slip angle at mass center, yaw velocity and lateral acceleration all drop after control, and the control system is effective to improve the vehicle stability. The weight coefficients are regulated to analyze their influence on control effect, and the result indicates that it is necessary to regulate them to reach the optimum of both energy and error of control.

Keywords: *Articulated Vehicle, Vehicle Stability Control, Direct Yaw Moment Control (DYC), Linear Quadratic Regulator (LQR)*

1. INTRODUCTION

Articulated dump truck is a kind of off-highway vehicle with an articulated body, which allows front body rotate independently relative to rear body. Articulated dump truck is a 6x6 drive vehicle with 3 drive axles [1, 2]. Its steering angle and pitch angle reach 45° and 15° respectively, which can ensure that all tires contact ground simultaneously, so it has good passing ability and high productivity. In spite of the advantages mentioned above, the lateral and longitudinal stability of articulated dump truck get worse because of the increase of vehicle DOF, so its stability should be improved. Increasing the friction of articulated body and regulating the damping of steering hydraulic cylinder are the conventional methods to improve the vehicle stability, but these methods waste too much energy and have low reliability. With the development of control technology, DYC is applied in controlling vehicle stability. DYC is to change longitudinal forces on left and right wheels to generate an additional yaw moment to improve vehicle stability [3].

The direct yaw moment control using in wheel-motors and the active front steering (AFS) control algorithm are described in Ref.4, and the combination of DYC control and AFS control is researched in [5, 6]. Feedforward-feedback control is used to enhance the stability of a 4WD vehicle, and the feedback coefficient is decided by optimal control theory in [7]. A fuzzy feedforward-feedback control system is designed and tested with ABS to enhance the lateral stability of vehicle in [8], and a feedback control system based on H_∞ theory is studied in [9]. Massto Abe and Ossama Mokhiamar [10-13] have researched and developed a DYC control system based on slip mode control method, and the system has been tested on real vehicle and proved to be effective.

Electric drive is a new drive form of articulated dump truck, which utilizes 6 wheel-motors to drive wheels respectively. Based on the precise and fast response characters of wheel-motor, drive torques on 6 wheels can be controlled independently to generate an additional yaw moment [14,15], which can improve the vehicle stability.

In order to enhance the stability of 6WD articulated vehicle, a 3-DOF dynamic model of 6WD articulated vehicle is firstly established. Moreover, a feedforward-feedback control system based on LQR theory is designed. In order to verify the effectiveness of the control system, a simulation with a step input of articulated angle is conducted, and the weight coefficients are regulated to analyze their influence on control effect.

2. DYNAMIC MODEL

In order to study the dynamic character of articulated vehicle, the vehicle model is simplified to a system with 3 DOF, which includes rotations of front body and rear body along vertical axis, and the lateral translation of vehicle (Fig.1).

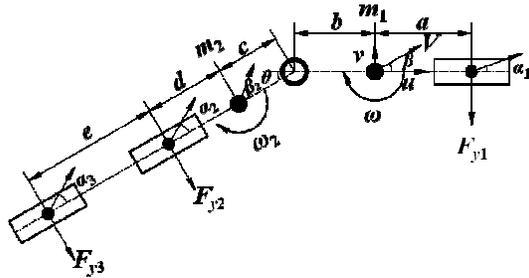


Figure 1. 3 DOF model of a 6WD articulated vehicle

The mass and rotational inertia of front and rear body are m_1, J_1 and m_2, J_2 . The side-slip angles of front, middle and rear tires are respectively α_1, α_2 , and α_3 . Lateral velocity of front body is v , and articulated angle between front and rear body under steering is θ .

Firstly, two components of the absolute acceleration at mass center of front body are decided. As shown in Fig.2, XY is the absolute coordinate system, and xy is the vehicle coordinate system fixed on the front body. The x-component and y-component of the velocity at mass center of front body are respectively u and v at t . When the vehicle is steering, its motion includes translation and rotation, so both value and direction of the velocity at mass center of front body have changed at $t+\Delta t$. The directions of horizontal axis and vertical axis in the vehicle coordinate system also have changed at the same time.

Variation of velocity along axis y is

$$\Delta V_{y1} = (v + \Delta v) \cos \Delta\phi - v + (u + \Delta u) \sin \Delta\phi \quad (1)$$

$\Delta\phi$ is very small, and the small quantity in the second order is ignored. Eq.1 is transformed to

$$\Delta V_{y1} = \Delta v + u\Delta\phi \quad (2)$$

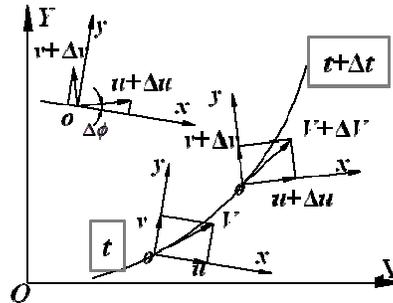


Figure 2. Motion analysis based on the coordinate system fixed on the vehicle

Yaw velocity of front body is ω . Eq.2 is divided by Δt and then taken to the limit. The y-component of absolute acceleration at mass center of front body in the vehicle coordinate system is

$$a_{y1} = \dot{v} + u\omega = u\dot{\beta} + u\omega \quad (3)$$

Similarly, the variation of y-component of absolute acceleration at mass center of rear body in the vehicle coordinate system is

$$a_{y2} = \dot{u} + \omega - (b+c)\dot{\omega} + c\ddot{\theta} \quad (4)$$

Equilibrium equations of force and moment of the vehicle are

$$\begin{cases} m_1 a_{y1} + m_2 a_{y2} = F_{y1} + F_{y2} + F_{y3} \\ J_1 \dot{\omega} - m_2 a_{y2} b = aF_{y1} - bF_{y2} - bF_{y3} \\ J_2 (\ddot{\theta} - \dot{\omega}) + m_2 a_{y2} c = (c+d)F_{y2} + (c+d+e)F_{y3} \end{cases} \quad (5)$$

There is a linear relationship between side-slip angle and lateral force when side-slip angle is relatively small. The articulated dump truck always travels in a low speed, and the side-slip angle of tire is generally no more than 3°. So the lateral force and side-slip angle are considered to satisfy the linear relationship:

$$F_y = k\alpha \quad (6)$$

where k is cornering stiffness, and α is side-slip angle of tire.

The side-slip angles of front, middle, and rear axles can be expressed by v, ω , and θ as

$$\begin{cases} \alpha_1 = -\beta - \frac{a\omega}{u} \\ \alpha_2 = -\theta - \beta + \frac{(b+c+d)\omega - (c+d)\dot{\theta}}{u} \\ \alpha_3 = -\theta - \beta + \frac{(b+c+d+e)\omega - (c+d+e)\dot{\theta}}{u} \end{cases} \quad (7)$$

As shown in Fig.3, two additional driving forces, $+F_{X1}$ and $-F_{X1}$, are respectively loaded on left and right tires in the front, which means a yaw moment M is loaded on the front body.

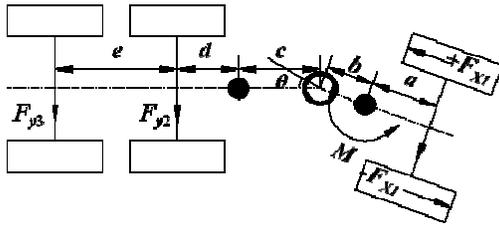


Figure 3. Force Equilibrium Of Vehicle Under Steady-State Steering

In this case, equilibrium equation of force and moment of the vehicle is

$$\begin{cases} m_1 a_{y1} + m_2 a_{y2} = F_{y1} + F_{y2} + F_{y3} \\ J_1 \dot{\omega} - m_2 a_{y2} b = aF_{y1} - bF_{y2} - bF_{y3} + M \\ J_2 (\ddot{\theta} - \dot{\omega}) + m_2 a_{y2} c = (c+d)F_{y2} + (c+d+e)F_{y3} \end{cases} \quad (8)$$

Eq.6 and Eq.7 are taken into Eq.8, and then the following equation can be obtained.

$$\begin{bmatrix} \dot{\beta} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} \begin{bmatrix} \beta \\ \omega \end{bmatrix} + \begin{bmatrix} a_3 & a_4 \\ b_3 & b_4 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \theta \end{bmatrix} + \begin{bmatrix} a_5 \\ b_5 \end{bmatrix} M \quad (9)$$

where $a_1, a_2, a_3, a_4, a_5, b_1, b_2, b_3, b_4,$ and b_5 can be expressed by $m_1, J_1, m_2, J_2, a, b, c, d, e, c_1, c_2,$ and c_3 .

Eq.9 is transformed into Eq.10 and Eq.11 by Laplace transform.

$$\beta(s) = \frac{a_2 b_3 s + a_2 b_4 - (b_2 - s)(a_3 s + a_4)}{(a_1 - s)(b_2 - s) - a_2 b_1} \theta(s) + \frac{a_2 b_3 - a_3 b_2 + a_5 s}{(a_1 - s)(b_2 - s) - a_2 b_1} M(s) \quad (10)$$

$$\omega(s) = \frac{a_3 b_1 s + a_4 b_1 - (a_1 - s)(b_3 s + b_4)}{(a_1 - s)(b_2 - s) - a_2 b_1} \theta(s) + \frac{a_5 b_1 - a_1 b_5 + b_5 s}{(a_1 - s)(b_2 - s) - a_2 b_1} M(s) \quad (11)$$

3. DYNAMIC MODEL

Yaw velocity and side-slip angle of vehicle at mass center are two important parameters to describe the vehicle motion status. The yaw velocity and side-slip angle at mass center of front body are selected to describe the stability of 6WD articulated vehicle, because that of rear body can be expressed by the front ones.

The control system is designed by LQR theory based on DYC method. Feedforward-feedback control is applied in control structure, and the combination control for yaw velocity and side-slip angle of front body are realized. The direct yaw moment as the controller input should make side-slip angle and yaw velocity keep up with their ideal value.

3.1. Feed-Forward Controller

The control objective is to make the side-slip angle at mass center of front body approach to zero. The relationship between the controller input and the articulated angle is

$$M_{ff}(s) = G_{ff} \theta(s) \quad (12)$$

where G_{ff} is feedback gain, and M_{ff} is yaw moment of feedforward control.

When the vehicle is under steady-state steering ($s=0$), Eq.12 can be transformed to

$$M_{ff0} = G_{ff} \theta_0 \quad (13)$$

The following equation is obtained by taking Eq.13 into Eq.10.

$$\beta_0 = \frac{a_2 b_4 - b_2 a_4 + (a_2 b_5 - a_5 b_2) G_{ff}}{a_1 b_2 - a_2 b_1} \theta_0 \quad (14)$$

Because $\beta_0=0$, the feedforward gain is

$$G_{ff} = \frac{b_2 a_4 - a_2 b_4}{a_2 b_5 - a_5 b_2} \quad (15)$$

By taking Eq.13 and Eq.15 into Eq.11, transfer function from articulated angle to yaw velocity can be obtained as

$$\frac{\omega(s)}{\theta(s)} = \frac{[b_1 a_3 s + a_4 b_1 - (a_1 - s)(b_3 s + b_4)]}{[(a_1 - s)(b_2 - s) - a_2 b_1]} + \frac{(b_2 a_4 - a_2 b_4)(b_1 a_5 - b_5 a_1 + b_5 s)}{[(a_1 - s)(b_2 - s) - a_2 b_1](a_2 b_5 - a_5 b_2)} \quad (16)$$

3.2. Reference Model

The side-slip angle at mass center of front body should approach to zero (control objective). Yaw velocity is a second-order delayed-time system about articulated angle θ and approximated as a

first-order delayed-time system for simplicity, which is shown in Eq.17.

$$\frac{\omega_d(s)}{\theta(s)} = \frac{k_{od}(u)}{1 + \tau_{od}(u)s} \quad (17)$$

where $k_{od}(u)$ is steady-state value, and $\tau_{od}(u)$ is time constant.

The response value $s=0$ is taken into Eq.17, $k_{od}(u)$ can be expressed as

$$k_{od}(u) = \frac{a_4 b_1 - a_1 b_4}{a_1 b_2 - a_2 b_1} + \frac{(b_2 a_4 - a_2 b_4)(b_1 a_5 - b_5 a_1)}{(a_1 b_2 - a_2 b_1)(a_2 b_5 - a_5 b_2)} \quad (18)$$

Similarly, $\tau_{od}(u)$ can be obtained:

$$\tau_{od}(u) = \frac{k_{od}(u)}{b_1 a_3 + b_3 + b_4 - a_1 b_3 + (b_2 a_4 - a_2 b_4) b_5} \quad (19)$$

The state equation of reference model is

$$\dot{X}_d = A_d X_d + H_d \theta_f \quad (20)$$

where $X_d = \begin{bmatrix} \beta_d \\ \omega_d \end{bmatrix}$, $A_d = \begin{pmatrix} 0 & 0 \\ 0 & -1/\tau_{od}(u) \end{pmatrix}$, and

$$H_d = \begin{pmatrix} 0 \\ k_{od}(u) \\ \tau_{od}(u) \end{pmatrix}.$$

3.3. Feedback Controller

Feedforward compensation is not enough when the system is suffering external disturbance or has uncertain factors. Feedback compensation is very effective to track the ideal system characteristic, so feedforward-feedback combination control is adopted.

The state equation of real model is

$$\dot{X} = AX + BM_{fb} \quad (21)$$

E is the error between reference value X_d and real value X , so

$$\dot{E} = \dot{X} - \dot{X}_d \quad (22)$$

By taking Eq.20 and Eq. 21 into Eq.22, the following equation can be obtained:

$$\begin{aligned} \dot{E} &= AE + BM_{fb} + (A - A_d)X_d + (H - H_d)\theta_f \\ &= AE + BM_{fb} + W \end{aligned} \quad (23)$$

The last term W in Eq.23 is a disturbance and can be ignored ($W=0$), so

$$\dot{E} = AE + BM_{fb} \quad (24)$$

Optimal control theory is applied into the design of feedback controller, so

$$M_{fb} = -G_{fb} E = g_{fb1}(\beta - \beta_d) - g_{fb2}(\omega - \omega_d) \quad (25)$$

where g_{fb1} and g_{fb2} are feedback gains of state errors between real model and ideal model.

The performance index function is

$$J = \int_0^{\infty} (E^T Q E + M_{fb}^T R M_{fb}) dt \quad (26)$$

where Q and R are respectively state weight matrix and control weight matrix and can be regulated.

Feedback gain of controller can be obtained by solving the Riccati Equation by means of LQR algorithm.

Total additional yaw moment is

$$M_Z = M_{ff} + M_{fb} \quad (27)$$

3.4. Control System

The control system of 6WD articulated vehicle which includes feedforward controller, feedback controller, reference model and dynamic model is established in SIMULINK software (Fig.5). The yaw velocity and side-slip angle at mass center of front body are under combination control.

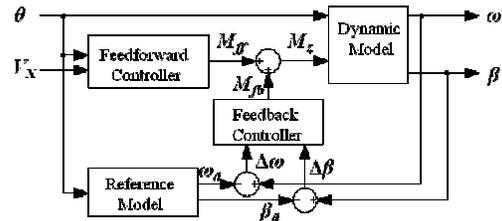
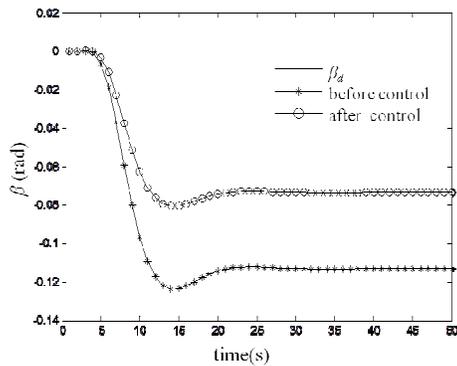


Figure 4. Control System

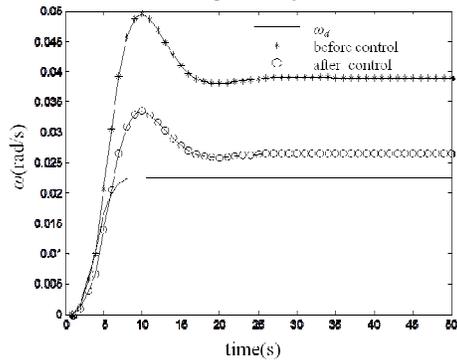
4. DYNAMIC MODEL

4.1. Step Simulation Result

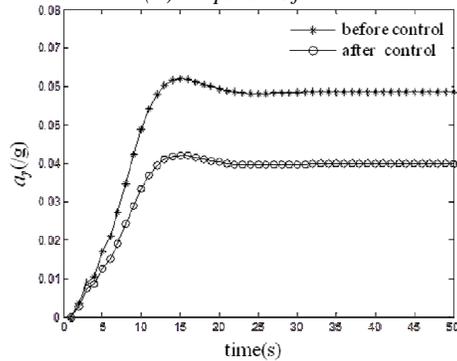
A simulation with step input of θ is conducted on the 3-DOF dynamic model to analyze the effectiveness of the control system. The stability and following characteristic are studied by the step simulation where vehicle velocity is 15m/s, and initial articulated angle is 2°. The variation of side-slip angle at mass center β , yaw velocity ω , lateral acceleration a_y and additional yaw moment M_Z are the key point of simulation analysis. Fig.5 is the response curves of β , ω , a_y , and M_Z in step simulation respectively.



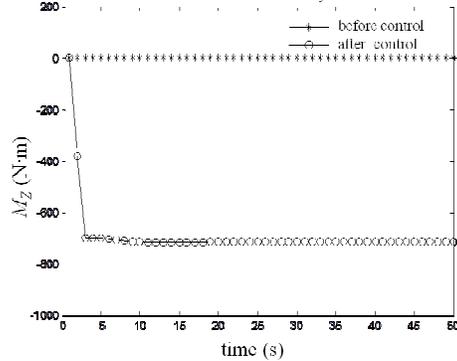
(A) Response Of β



(B) Response Of Ω



(C) Response Of A_y



(D) Response Of M_z

Figure 5. Simulation Results

The ideal value of β is 0. β increases from 0 to 0.125rad and stabilizes at 0.12rad before control, while β raises from 0 and then tends to be stable at

0.08rad after control (Fig.5 (a)). The ideal value of ω finally stabilizes at 0.0225rad/s. ω increases from 0 to 0.05rad/s and stabilizes at 0.04rad/s before control, while the stable value of ω reduces from 0.04rad/s to 0.025rad/s after control (Fig.5 (b)). Similarly, the stable value of a_y also has decreased from 0.06g to 0.04g after control (Fig.5 (c)). M_z sharply rises from 0 to 700Nm after control (Fig.5 (d)). It can be seen that β and ω get closer to their ideal values with the effect of M_z , and the vehicle stability is improved obviously.

4.2. Influence Of Weight Coefficient

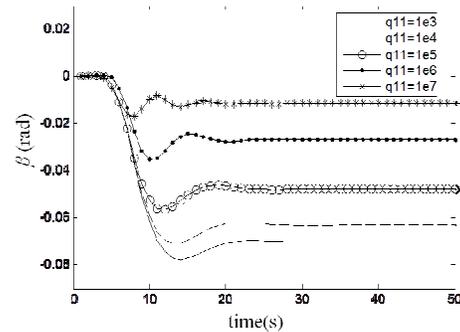
The diagonal elements of state weighted matrix Q (Eq.28) in feedback controller represent the levels of error components.

$$Q = \begin{pmatrix} q_{11} & 0 \\ 0 & q_{22} \end{pmatrix} \quad (28)$$

R indicates the consuming energy of control in performance function. The objective of optimal control is to make the performance function close to its minimum, so regulating q_{11} , q_{22} and R can lower the control energy as well as keep a relative small error, which will finally reach the optimum of both energy and error.

4.2.1. Influence of q_{11}

The response curves of β , ω , a_y , and M_z when q_{11} varies are shown in Fig.6. β gets closer to its ideal value ($\beta_d=0$) and a_y drops with the increase of q_{11} . Although ω is smaller and smaller, the error between ω and ω_d gets bigger with the increase of q_{11} . The overlarge q_{11} causes the fluctuations of M_z and ω .



(A) Response Of β

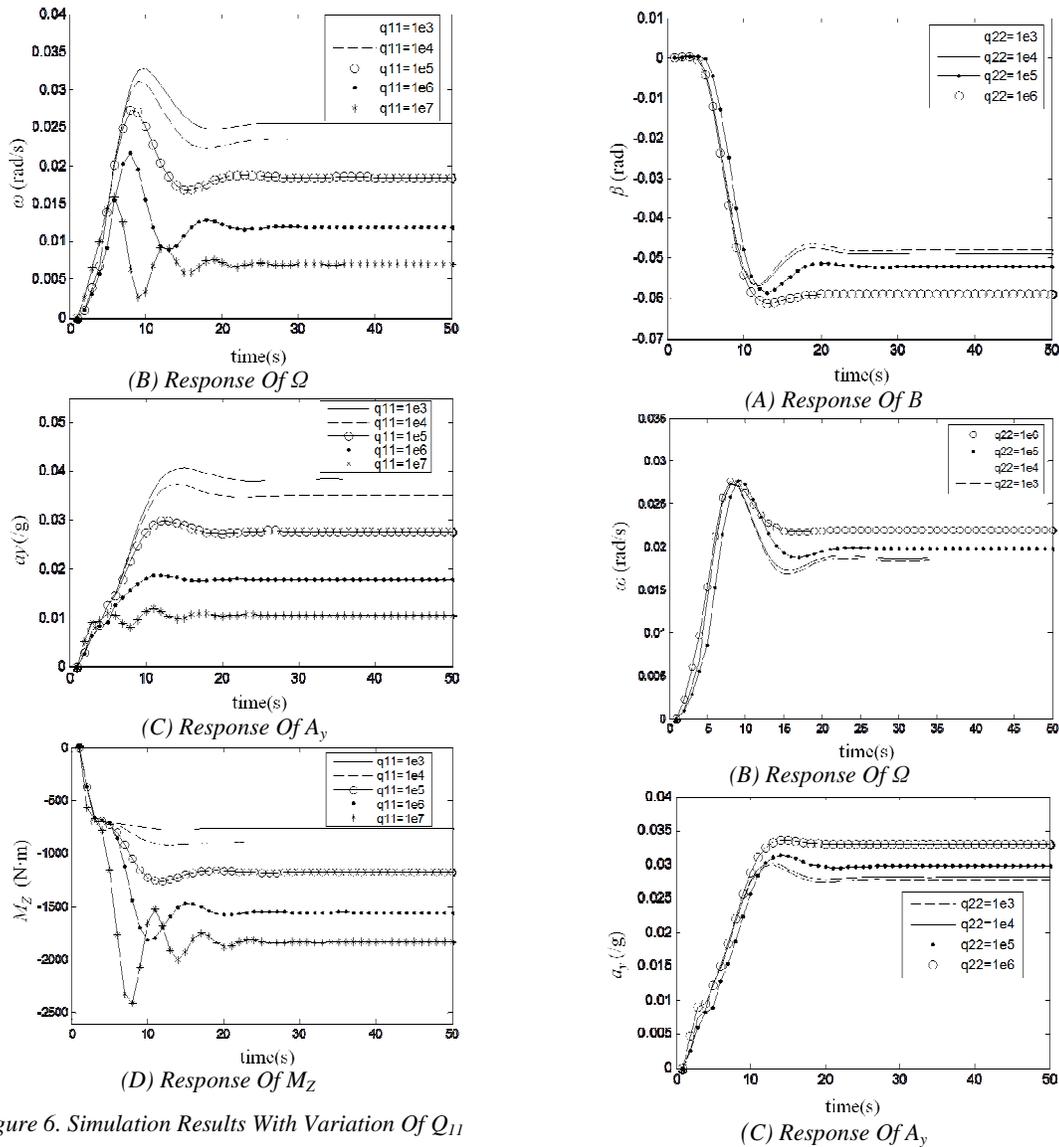


Figure 6. Simulation Results With Variation Of Q_{11}

4.2.2. Influence of q_{22}

The response curves of β , ω , a_y , and M_z when q_{22} varies are shown in Fig.7. When q_{22} rises, ω gets closer to its ideal value ($\omega_d=0.0225\text{rad/s}$), but the values of β and a_y increase. When q_{22} varies from 1000 to 10000, the control effect has little change. Compare with Fig.6, the influence of q_{22} on the vehicle stability is less obvious than that of q_{11} .

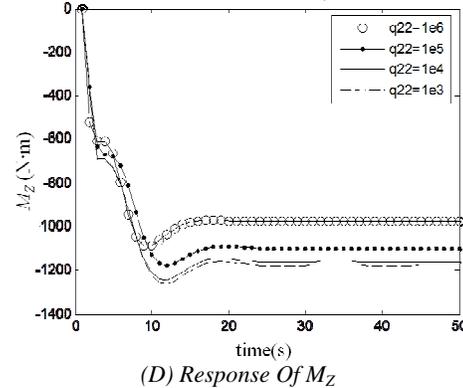


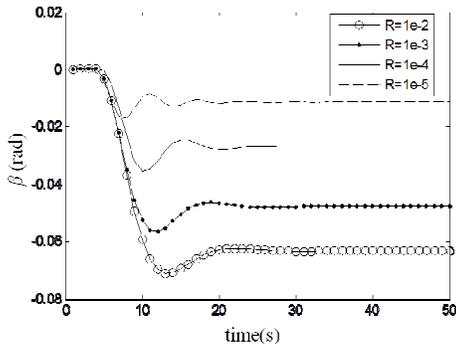
Figure 7. Simulation Results With Variation Of Q_{22}

4.2.3. Influence of R

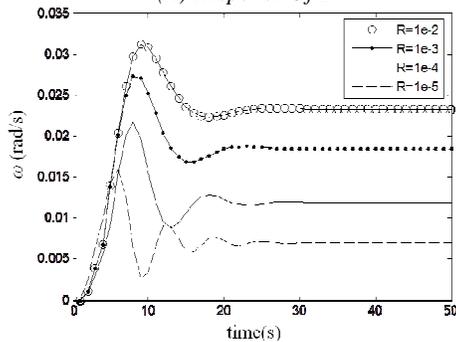
The response curves of β , ω , a_y , and M_z when R varies are shown in Fig.8. β is closer to its ideal value with the decrease of R . But ω approaches to

its ideal value with the rise of R . When R varies from 10^{-2} to 10^{-5} , a_y decreases from 0.035g to 0.01g, while the value of M_Z rises from 900N·m to 1900N·m. A small R also leads to the fluctuation of M_Z . The rise of R increases response time and lowers the vehicle stability.

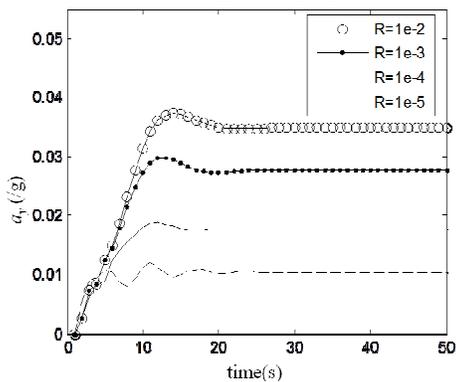
Regulating the weight coefficients have a great impact on the control effect based on the discussion above, so it is necessary to regulate them to reach optimum effect in the simulation.



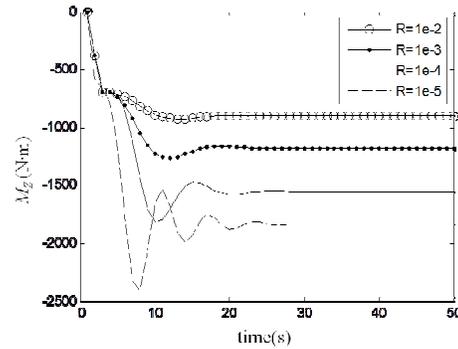
(A) Response Of β



(B) Response Of ω_r



(C) Response Of A_y



(D) Response Of M_Z

Figure 8. Simulation Results With Variation Of R

5. CONCLUSION

To enhance the stability of 6WD articulated dump truck, a feedforward-feedback control system is designed by DYC based on the 3-DOF dynamic model. The feedforward controller is designed to keep the side-slip angle at mass center of front body close to its ideal value, and the feedback compensating controller is built by LQR to make the side-slip angle at mass center and yaw velocity of front body (β and ω) follow their ideal values (β_d and ω_d) effectively.

The simulation result shows that the control system improves stability and raise safety of the vehicle, and the control algorithmic is proved to be effective.

The analysis about the influence of weight coefficients on control effect indicates that β and ω respectively get closer to β_d and ω_d with the increase of q_{11} and q_{22} . But the influence of q_{22} is smaller than that of q_{11} on stability. The decrease of R makes β close to β_d , but makes ω far away from ω_d . And R with a relatively small value will lead to the fluctuation of M_Z .

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