Journal of Theoretical and Applied Information Technology
10th March 2013. Vol. 49 No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

OFDM SYSTEM CHANNEL ESTIMATION ALGORITHM RESEARCH BASED ON KALMAN FILTER COMPRESSED SENSING

[1]JINHE ZHOU, [2]GUOMIN XIA, [3]JURONG WANG

School of Information & Communication Engineering

Beijing Information Science and Technology University, Beijing 100101, China

E-mail: [1]zhoujinhe@bistu.edu.cn, [2]xgmbistu@163.com, [3]wangjurongbistu@163.com

ABSTRACT

Orthogonal frequency division multiplexing (OFDM) has been considered as a kind of effective means to realize wideband data transmission by applying multiple low rates and parallel data transmission sub-carriers. A very important characteristic of wireless channel is multipath propagation, which makes the received signal overlap, and leads to inter symbol interference (ISI). Therefore, OFDM channel estimation is an important part of OFDM system. This paper proposed a channel estimation algorithm based on Kalman filter compression sensing (KF-CS). By the combination of KF and CS algorithm, the KF-CS gets higher estimation precision with fewer pilots compared with classical algorithms. We adopted LS, LMMSE, CS, KF and KF-CS algorithms in pilot insertion method, obtained complete channel spectral response and got effective comparison of mean square error (MSE) of the channel response. Simulation results show that the proposed algorithm of KF-CS has better accuracy than KF algorithm, and can reduce pilots. So the proposed algorithm can get good channel estimation performance with fewer pilots and improved spectrum efficiency.

Keywords: *Channel Estimation (CE), Compressed Sensing (CS), Kalman Filter (KF), Orthogonal Frequency Division Multiplexing (OFDM)*

1. INTRODUCTION

Wireless signal transmission in mobile communication channel is a very complicated process, in order to recover the signal accurately at the receiver, there are some methods which can be used for resisting multipath effect on transmission signal, such as load balancing, multiuser detection, and the space multiplexing technology, all these methods depend on wireless channel state information (CSI), which is the basic information for correctly demodulating out all sorts of aliasing signal, and it's necessary to estimate parameters of the transmission channel at the receiver.

At present, the classic OFDM system channel estimation algorithms are the least square (LS) and linear minimum mean square error (LMMSE) [1]. But due to the initial LS estimation without considering the impact of noise, although the algorithm is simple but the estimation performance is poor. Candes, Tao and Donoho formally put forward the concept of compressed sensing theory[2,3] in 2006, considering the sparse characteristic of wireless channel, and were quickly applied successfully to channel estimation in OFDM system[4-7]. Because CS uses the statistical characteristics of channel model as the priori information, and its reconstruction algorithm can effectively reduce the computational complexity and the requirements of sampling point. In [8], CS is applied to two-way relay system, which indicates CS can play a broader role in channel estimation. For the sake of improving the performance of channel estimation, literature [9,10] introduced several CS reconstruction algorithms. However, the above documents do channel estimation in slow changing system. For time-varying channel, Kalman filter [11,12] is a good choice for channel estimation, but the traditional Kalman filtering method needs to insert pilot signal, witch results in the reduction of transmission efficiency. On the other hand, the combination of KF and CS estimation algorithm can get higher estimation accuracy for time-varying channel.

Kalman filtering, as a typical estimation theory, has a wide range of application in image, radar and navigation fields. Recently, CS theory has become an important method in sparse signal processing. It

Journal of Theoretical and Applied Information Technology
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breaks through the sampling rate limit of traditional Nyquist sampling theorem. When the signal in a certain transform domain has sparse characteristic, the rate of the signal sampling can be lower than the Nyquist sampling rate. Considering the complexity of system channel estimation algorithm, this paper introduced a method combined the Kalman filter and CS technology in OFDM channel. From the known sparse support set point of view, the Kalman filter can obtain channel estimation with minimum mean square error (MMSE), and the compression support set of the channel can be determined by CS technology. So the combination of Kalman filter and CS can get desired effect with lower signal-to-noise ratio (SNR) and the requirements of the observation dimension are reduced. Compared with other traditional channel estimation algorithms, the proposed KF-CS algorithm has better application prospect in sparse signal analysis.

Through the following sections, this article discusses the principle of the proposed KF-CS algorithm. And for the sake of demonstrating the algorithm by simulation compared with traditional, specific content is as follows: the second part, introduced OFDM system sparse channel model and channel estimation technological process; the third part, introduced Kalman filter algorithm and CS technology to estimate channel respectively; the fourth part explained the feasibility and estimation steps for the KF-CS method; the fifth section made some corresponding simulations to compare KF-CS channel estimation algorithm and the traditional estimation algorithm, to verify the new KF-CS algorithm is more effective than other algorithms with lower computational complexity; section six concluded this paper.

2. OFDM SYSTEM MODEL

OFDM is a highly efficient multi-carriers transmission technology, N sub-carriers divide the channel into N sub-channels for parallel transmission, and it has very high spectrum efficiency. The receiver adopts incoherent detection and coherent detection in OFDM systems. For incoherent detection, the transmitter does differential coding with continuous symbols of sub-carriers which are corresponding to OFDM codes. Then uses IFFT processing and adds protect interval. The biggest advantages of incoherent detection are that the channel state information is not necessary and complexity is low for the receivers. However, incoherent detection is only applicable in transmission system with low data rate. Therefore, in order to perform the high-speed data transmission, we generally consider the coherent detection in OFDM system, and the data on the subcarrier are demodulated by PSK or QAM modulation scheme.

The principle diagrams of OFDM system and wireless channel estimation are shown in the following Fig.1 and Fig.2. Firstly, the sender does convolution and interleaving, and then goes to QAM mapping, inserts pilots after de-serialization, again completes sub-carrier modulation through the IFFT transform, finally adds circulation prefix, and sends into the multipath time-varying channel. Similarly, the receiver removes the cycle prefix, and takes out the pilots for channel estimation, then does channel compensation and completes data detection according to the estimation results. Suppose the length of cycle prefix is greater than maximum delay of multipath, and has a very good synchronization, therefore, the ISI and inter carrier interference (ICI) can be ignored.

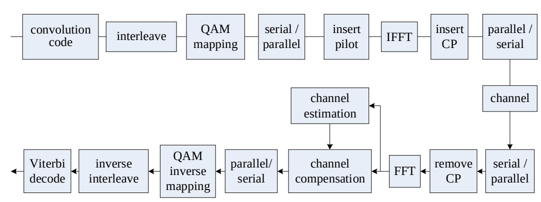

Fig.1 System Structure of OFDM

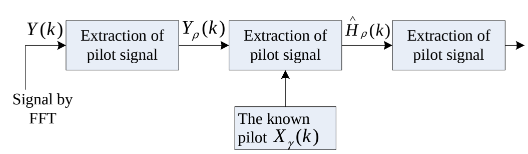

Fig.2 Channel Estimation Diagram

Hypothesis discrete OFDM channel model as follows,

$$h(n)=\sum_{i=0}^{L-1} h_i \delta(n-i) \qquad (2\text{-}1)$$

In (2-1) L is the channel length of OFDM system, and $L=\tau_{\max}/T_s$ ($\tau_{\max}$ is maximum delay, T_s is sampling period).

The vector $h=[h_0,...,h_{L-1}]^T$ only has fewer nonzero value. Then the received signal vector Y can be expressed as,

$$Y = Xg+v = XWh+v \qquad (2\text{-}2)$$

In (2-2) $X=diag(x_1,x_2,...,x_N)$ and g is channel frequency response, v is Gaussian white noise, W can be expressed as,

$$[W]_{a,b}=\frac{1}{\sqrt{N}}e^{-j2\pi ab/N}, a,b=1,2,...,L$$

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Hypothesis P represents pilots of OFDM system, S is the position of pilots of N sub-carriers. Then the pilot signal at receiver can be expressed as,

$$Y_P = X_P W_P h + v_P \qquad (2\text{-}3)$$

In (2-3), $Y_P = SY$, $X_P = SXS^T$, $W_P = SW$, $v_P = Sv$, by Y_P, X_P, W_P, we can estimate vector h, and get channel frequency response by $g = Wh$.

3. KF AND CS PRINCIPLE

3.1 KF Algorithm

Kalman filter is a kind of iterative LMMSE estimation method, each iteration uses the measured value of the current and past. On the other hand, Kalman filter brings in the conception of state, and uses state equation of the system to describe the process of estimating parameters in each moment. So we can apply Kalman filter in non-stationary processing. The optimal criteria of Kalman filtering and the estimation criterion of MMSE are about to ensure that the MSE of every moment estimation is minimum. Considering Gaussian noise interference of linear discrete system as follows,

$$\begin{aligned} X(k) &= F(K,k-1)\cdot X(k-1) \\ &+ T(k,k-1)\cdot U(k-1) \end{aligned} \qquad (3\text{-}1)$$

$$Y(k) = H(k)\cdot X(k) + N(k) \qquad (3\text{-}2)$$

In (3-2), $X(k)$ and $Y(k)$ are state vector and observation vector respectively. $F(k,k-1)$ is state transition matrix, $U(k)$ is noise, $T(k,k-1)$ is system control matrix, $H(k)$ is observation matrix and $N(k)$ is observation noise.

Equation (3-1) is the state equation, and equation (3-2) is the measurement equation. The main principle of KF algorithm is to get state estimation by feedback control. Therefore, KF can mainly be divided into the following two steps: time update and measurement update. Time update is also known as the forecast period, which is mainly based on the current status and error variance estimation, and its objective is to get the time point of a prior estimation. Measurement update is mainly used for feedback, which improves estimation accuracy by introducing new measurements in a prior estimation. The two stages of the specific calculation method apply the following iteration steps,

Predicted value: $\hat{X}(k) = F(k,k-1)\cdot X(k-1)$

1, Calculate pre-estimate covariance matrix,

$$\begin{aligned} \hat{C}(k) &= F(k,k-1)\times C(k)\times F(k,k-1)^t \\ &+ T(k,k-1)\times Q(k)\times T(k,k-1)^t \end{aligned}$$

$$Q(k) = U(k)\times U(k)^t$$

2, Calculate Kalman gain matrix,

$$K(k) = \hat{C}(k)\times H(k)^t \times \hat{C}(k)\times H(k)^t + \hat{R}(k)(-1)$$

$$R(k) = N(k)\times N(k)^t$$

3, Update estimation,

$$\hat{X}(k+1) = \hat{X}(k) + K(k)\times\left[Y(k) - H(k)\times\hat{X}(k)\right]$$

4, Calculate estimation of covariance matrix after Update,

$$\begin{aligned} \hat{C}(k+1) &= [I - K(k)\times H(k)]\times\hat{C}(k)\times \\ &[I - K(k)\times H(k)]' + K(k)\times R(k)K(k)' \end{aligned}$$

3.2 CS Theory

CS is mainly aimed at sparse signal or compressible signal, which plays sampling and compression simultaneously.

Assuming that a signal is $f\,(f \in R^N)$, its length is N. Transform base vector is $\Psi_i (i = 1,2,...,N)$, to transform the signal as follows,

$$f = \sum_{i=1}^{N} a_i \psi_i = \psi\alpha \qquad (3\text{-}3)$$

Suppose α only has $K\,(N >> K)$ nonzero value or close to zero by exponential attenuation after transformation in formula (3-3), so the signal is sparse. In the premise of known compressed signal, CS process can be divided into two steps,

Firstly, design a $M\times N(M << N)$ dimension observation matrix to measure signal, the matrix is not relevant to the transformed base, get $M\times 1$ order measurement vector.

Secondly, reconstruct according to the measurement matrix. Do linear project for signal, and obtain measured value named y that can be expressed as,

$$y = \varphi f \qquad (3\text{-}4)$$

In (3-4), φ is a $M\times N$ measure matrix, which makes measured content reduced to M dimension. So that,

$$y = \varphi f = \varphi\psi\alpha = T^{CS}\alpha \qquad (3\text{-}5)$$

Due to that M is much less than N , for formula (3-5) the number of equations is less than that of unknowns, by mathematical theory, this equation

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

has more than one solution. Therefore, CS uses f vector, which is K sparse, and measurement matrix T^{CS} needs to meet restricted isometry property (RIP). When the specific position of nonzero value in f vector is unknown, through optimization theory, it can recover the initial signal from the observation vector.

Signal reconstruction algorithm is the core of CS theory, it is to accurately recover the original high dimensional data from the low dimensional data, and the reconstruction accuracy is very important. The typical algorithms include basis pursuit (BP) which based on the norm minimum greed tracking series algorithm, and other algorithms such as the matching pursuit (MP), orthogonal matching pursuit (OMP) and so on. This paper mainly introduced the general OMP algorithm.

The basic idea of OMP algorithm is to select column in φ with greed iterative method, the selected column must be related greatest to the current redundancy vector in each iteration, then subtract the relevant parts from measurement vector and repeat iteration, until iterations reach the sparse degree K.

The core algorithm steps are shown as follows,

Input, the sensing matrix is T^{CS}, sampling vector is y, and the sparse degree is K;

Output, the closed estimation is $\hat{f}$;

Initial, the residual $r_0 = y$,

index set $\wedge_0 = \Phi$, $t = 1$;

Cycle steps are as follows,

Step one, find residual r and the corresponding subscript λ of max column T_j product of sensing matrix. Namely,

$$\lambda_j = \arg\max_{j=1,\dots,N} \left|\left\langle r_{t-1}, T_j \right\rangle\right|$$

Step two, update the index set $\wedge_t = \wedge_{t-1} \cup \{\lambda_t\}$, record reconstruction atomic set in the found sensing matrix $\varphi_t = [\varphi_{t-1}, T_{\lambda_t}]$;

Step three, through the LS, deduce the following formula,

$$\hat{f}_t = \arg\max \left\| y - \varphi_t \hat{f} \right\|_2$$

Step four, update residual,

$$r_t = y - \varphi_t \hat{f}_t,\ t = t + 1$$

Step Five, determine whether it meets $t \succ k$, if it meets, then stop the iteration, otherwise executes step one.

In each iteration process, OMP algorithm selects the best matching atom to reconstruct sparse approximation from over-complete atom library (namely $T^{cs} = \varphi\psi$), and finds the residual of signal, then continues to select the most matching of the atoms with signal residual. After a certain number of iteration, the signal can be made linear expression by some atoms. Then makes the chosen atomic group orthogonalization recursively for ensuring that the iterative optimality. So OMP algorithm is very suitable for sparse multipath channel estimation. Compared with LS, LMMSE, OMP needs fewer pilots

4. CHANNEL ESTIAMTION ALGORITHM BASED ON KF-CS

According to the above content, we combined adaptive Kalman filter with CS to estimate OFDM channel parameters. According to OFDM system of basic model, firstly, using CS technology for the init channel estimation, and then process the estimation based on the h, using the adaptive Kalman filter to predict a group of new h value, and finally we can get the whole channel frequency response value.

The main implementation steps of the proposed algorithm are as follows:

1) Initialization parameters: non zero channel estimation of the response set $\Gamma = \Phi$ (empty set), $P_0 = 0$.

2) In the first OFDM symbols, using the received signals and the known pilot sequence to estimate pilot frequency positions of the initial value:

$$\tilde{H}(1) = \frac{Y(1)}{X(1)} \tag{4-1}$$

3) OMP algorithm is adopted to estimate the initial CS, through which to get nonzero response sets Γ and Kalman iterative initial value $\hat{h}_1$, the detailed steps are as follows,

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

The first step: initialize the iteration index $l = 1$, setting the initial residual value $R_0 = \tilde{H}(1)$;

The second step: look for a position index $\lambda_t = \arg\max_{j=1,2,\cdots,N} \left|\left\langle R_{t-1}, F_j \right\rangle\right|$, in which F_j represents FFT matrix section j column;

The third step: update $\Lambda_t = \Lambda_{t-1} \cup \lambda_t$ according to Λ_t calculate FFT submatrix $\mathbf{F}_{sub}$;

The forth step: use the least square algorithm to solve the following problems:

$$\mathbf{Z}_t = \arg\min \left\|\mathbf{R} - \mathbf{F}_{sub} \cdot \mathbf{Z}\right\|_2 \text{ get } \mathbf{Z}_t$$

The fifth step: calculate the residual error $\mathbf{R}_t = \mathbf{R}_{t-1} - \mathbf{F}_{sub} \cdot \mathbf{Z}_t$, when $t < k$, return to second step, while $t = k$, stop the iteration;

The sixth step: restore the original channel using position set Λ_t.

4) For number j $(j \geq 2)$ OFDM symbol, calculate parameters $r_j = \left\|\hat{h}_j - \hat{h}_{j-1}\right\|_2^2$ before channel estimation, and compare with the preset threshold $\in_j$. If $r_j > \in_j$ which indicates that the time-varying channel response position has changed, on this condition, we can estimate the channel coefficient values by KF algorithm, but because of multipath time delay which causesthe position change, this will lead to the failure of demodulation at receiver. Thus it still need to use CS technology to estimate delay parameter of the non-zero coefficient path and non zero set Γ.

5) If $r_j < \in_j$ in above 4), which demonstrates that position has not changed, then we can track the channel impulse response $\hat{h}_j$ directly according to the last non-zero set Γ.

6) Update the system noise covariance matrix Q and the measurement noise covariance matrix R.

7) $j = j + 1$, return to the step 3) for the next iteration.

According to the filtering cycle formula we can predict channel estimation value. Now we consider to compare the channel MSE of different channel estimation algorithms, and the MSE is defined as follows,

$$\frac{E\left[\sum_i \left|h_i - \hat{h}_i\right|^2\right]}{E\left[\sum_i \left|h_i\right|^2\right]} \qquad (4\text{-}2)$$

In (4-2), h_i is vector elements of h.

5. SIMULATION RESULTS AND ANALYSIS

According to the algorithms described above sections, we apply 16QAM as modulation method, using Rayleigh multipath fading channel as the transmission channel, using MSE to measure the kind of channel estimation algorithms, and compare with traditional OFDM channel estimation algorithm, such as LS, LMMSE and KF algorithms. The MSE of channel and frequency response curve are as follows,

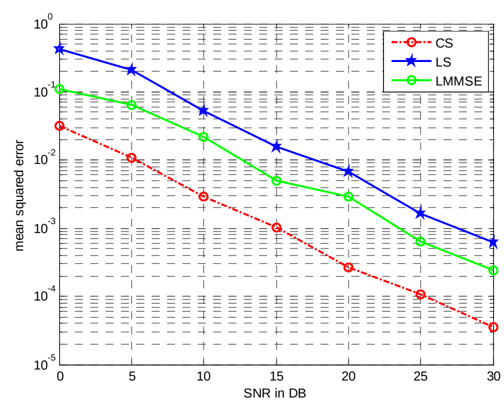

Fig.3 Contrast Figure of LS, LMMSE and CS Algorithm

From the Fig.3 we can see the performance of the CS algorithm is better than LS and LMMSE algorithm obviously. This is because the LS algorithm doesn't consider the influence of channel noise, and CS uses the OMP algorithm to estimate, which can better reconstruct channel frequency response. In addition, LMMSE algorithm doesn't consider the sparse of channel, which is very important for the estimation result, so to reach the same estimated MSE of the channel, the reconstructed algorithm based on CS is more efficient under the same SNR.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

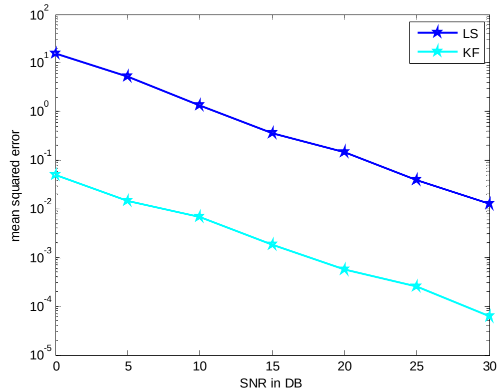

Fig.4 SNR-MSE Figure of LS and KF Algorithm

As the Fig.4 shows that Kalman filter can significantly lower MSE of the channel estimation compared to the LS algorithm, and can make a very good approximation of the true channel. This effect is brought by the algorithm itself characteristics of Kalman filter. In each iteration, the Kalman filter updates channel estimation error, and adjusts estimated value for the next iteration, especially in Gaussian noise environment it can accurately track time-varying channel.

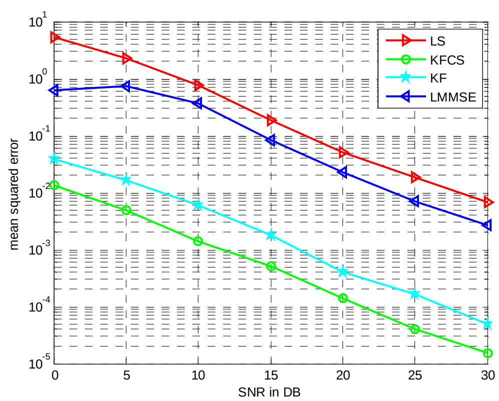

Fig.5 SNR-MSE Figure of LS , KF and KF-CS Algorithm

As the above Fig.5 and Fig.6 shown, after improving Kalman algorithms by CS technology, KF-CS performs significantly better than traditional KF algorithm and the classic LS, LMMSE algorithm, this is because the KF-CS method firstly estimates the nonzero response location sets through the OMP algorithm, then estimates the initial value by Kalman filter. But the traditional Kalman filter initial value requires assumptions, so the proposed KF-CS can reduce the MSE of the channel estimation.

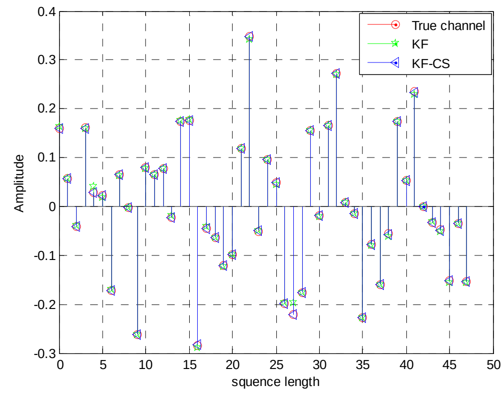

Fig.6 Contrast Figure of KF, KF-CS and True Channel

6. CONCLUSIONS

To play channel estimation in OFDM system, this paper proposed a kind of channel estimation algorithms, which is combined Kalman filter and CS. We applied under-sampling to direct sequence modulation signal, it not only reduces the sampling node, but also builds random observation process model of CS, then at the receiver, we used KF-CS reconstruction algorithm to estimate the channel time domain response. Simulation results show that the proposed KF-CS has lower channel estimation MSE, which can reduce the pilot quantity and improve the spectrum efficiency. For channel estimation, by CS, we can get the pilot signal location by random selection to achieve optimal performance, i.e. randomized pilot pattern that can approximately satisfy the sensing matrix constraint isometric condition. However in this case the performance of channel estimation is random, not whether can we design the best fixed pilot pattern, to make the performance of channel estimation with higher accuracy and stability, but how to design such a pilot pattern, which is worthy of taking the next step.

ACKNOWLEDGMENTS

This work was supported by National Natural Science Foundation of China (Grant No.61271198) and Key Program of Beijing Municipal Natural Science Foundation (Grant No. 4131003).

REFERENCES:

[1] Martin Henkel, Christoph Schiling and Wolfgang Schroer, "Comparison of Channel Estimation methods for pilot aided OFDM systems", *IEEE Transactions on communication*, 2007, pp.1435-1439.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

[2] D. L. Donoho, "Compressed sensing", *IEEE Trans. on information Theory*, Vol.52, No.4, 2006, pp.1289-1306.

[3] Canddes E and Tao .T, "Near optimal signal recovery from random projections: universal encoding strategies", *IEEE Trans on Information Theory*, Vol. 52, No.12, 2006, pp.5406-5425.

[4] Ali Hormati and Martin Vetterli, "Compressive Sampling of Multiple Sparse Signals Having Common Support Using Finite Rate of Innovation Principles", *IEEE Signal processing letters*, Vol.18, No.5, 2011, pp. 331-334.

[5] C. R. Berger, S. Zhou, W. Chen and P.Willett, "Sparse channel estimation for OFDM: Over-complete dictionaries and super-resolution", *IEEE Workshop on Signal Processing Advanced in Wireless Communications*, 2009, pp.196-200.

[6] Matthew Sharp and Anna Scaglione, "A Useful Performance Metric for Compressed Channel Sensing", *IEEE Trans on signal processing*, Vol.59, No.6, 2011, pp. 2982-2988.

[7] Patrick Kuppinger, Giuseppe Durisi and Helmut Bolcskei, "Uncertainy Relations and Sparse Signal Recovery for Pairs of General Signal Sets", *IEEE Trans. on information theory*, Vol.58, No.1, 2012, pp.263-277.

[8] Peng Cheng, Lin Gui,Yun Rui,Y. Jay Guo, Xiaojing Huang and Wenjun Zhang, "Compressed Sensing Based Channel Estimation for Two-Way Relay Networks", *IEEE Wireless communications letters*, Vol.1, No.3, 2012, pp.201-204.

[9] Qi, Chenhao and Wu, Lenan, "Sparse recovery algorithms for pilot assisted MIMO OFDM channel estimation" , *IEICE Transactions on Communications*, Vol. E93-B, No. 5, 2010, pp.1313-1316.

[10] Sun Teng, Song Zhiqun and Zhang Yongjie, " Matching pursuit based sparse channel estimation using pseudorandom sequences", *Proceedings of 2012 5th Global Symposium on Millimeter-Waves (GSMW)*, 2012, pp. 33-37.

[11] Gupta, Prerana and Mehra D.K, "Kalman filter-based channel estimation and ICI suppression for high-mobility OFDM systems", *International Journal of Communication Systems*, October, Vol.21, No.10, 2008, pp.1075-1090.

[12] Schmidt, J.F,Wichman.R and Gregorio F.H, "Fast-fading channel estimator using DCT and simplified Kalman filter", *IEEE Workshop on Signal Processing Advances in Wireless Communications(*SPAWC), Vol.2, No.6, 2007, pp.386-398.