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RESEARCH ON THE KINEMICS OF 3/3-RRRS PARALLEL MANIPULATOR

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ABSTRACT

It is known that a serious disadvantage of parallel manipulators is the complexity of calculating forward kinematics. Briefly introducing the mathematical model and the corresponding coordinate system, this paper mainly addressed the forward and inverse solution analyses and examples of the 3/3-RRRS parallel manipulator in detail. Especially, a novel geometrical method, i.e. equivalent mechanism is proposed for the forward solution analysis of the manipulator.

Keywords: *Kinematics Analysis; Workspace Analysis; Equivalent Mechanism.*

1. INTRODUCTION

The parallel kinematics analysis is the base of Kinematics speed, acceleration, workspace, singular configuration, and dynamics analysis [2] and determines the success or failure of the mechanism late applications [1]. The parallel kinematics study is aimed at Forward and Inverse solution. The given parallel robot platform position and orientation, solving each driver input joint position is the inverse kinematics problem; and the given parallel robot each input joint position to solve the position and orientation of the platform is the Forward Kinematics problem [3]. The one-dimensional search method had already been put forward, which can make parallel manipulator with triangular platform simplify to contain only one variable nonlinear equation, significantly to improve forward solutions' speed [4]. Because of the influence of the initial value and the shortcoming of leakage solution to the selected operation, a homotopy iteration method based on homotopy function had been come forward, and there is no need to select initial values, and all values can be calculated [5]. Meanwhile, the method to solve the positive position of the general 6-SPS manipulator had been applied, very easy to find the all 40 groups of solutions to improve the computational efficiency and reliability [6]. A kind of symmetry parallel manipulator 6-6 the SPS Stewart position has been analyzed based on homotopy method [7]. The vector analytic principle has been adopted successfully to 3-RPS parallel manipulator to solve the location positive solutions of the parallel manipulator, and verify the method by geometric analysis [8] et al. [9-12].

2. THE MECHANISM ANALYSIS OF THE 3-RRRS PARALLEL MANIPULATOR

The architecture of the 3/3-RRRS parallel manipulator is composed of a moving platform, a fixed platform and three connecting legs with identical kinematic chain of three one-DOF joints (A_i, B_i, C_i) and a three-DOF spherical joint, as shown in Fig.1.

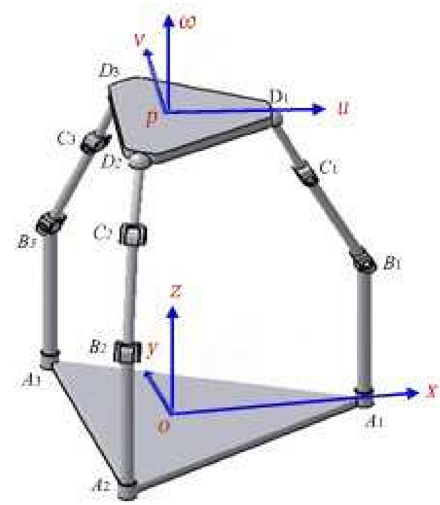

Figure 1: Three-Dimension Diagram of the 3-RRRS Parallel Manipulator

Among these three one-DOF joints, two are active and one is passive. The fixed reference frame *O-xyz* is attached to the fixed platform with the

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origin *O* located at the centroid of the equilateral triangle *A*1*A*2*A*3. Similarly, the moving coordinate frame *P-μνω* is attached to the moving platform with the origin P located at the centroid of the equilateral triangle D1D2D3. The axis of rotation at the connection of rotation pair are depicted as Si1, Si2 and Si3 and these at the connection of spherical pair are described as Si4, Si5, Si6, as shown in Fig. 2.

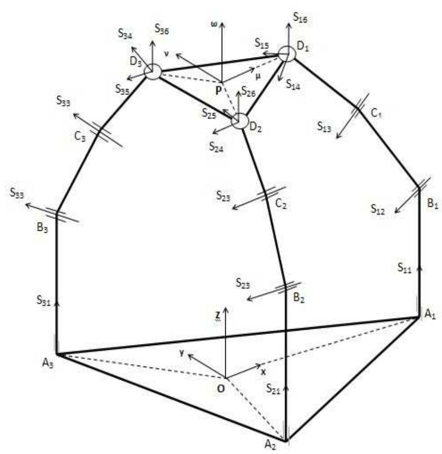

Figure 2: Structure Diagram of the 3-RRRS Parallel Manipulator

3. INVERSE AND FORWARD SOLUTION ANALYSES OF THE 3-RRRS PARALLEL MANIPULATOR

For the convenience of the study, essential parameters are introduced. r and R are assigned as the corresponding circumradius of the moving and fixed platforms. Constant parameters hi, L1i, L2i (i=1, 2, 3) are respectively defined as the length between Ai and Bi, Bi and Ci, Ci and Di. Meanwhile, γi is expressed as the angle from the positive direction of x-axis to cross-products vector between the vector $\overrightarrow{B_iD_i}$ and the vertical vector (0, 0, 1), similarly, αi is defined as the angle between the vector $\overrightarrow{B_iC_i}$ and the vertical vector (0, 0, 1). The position of the centroid P corresponding to the fixed reference frame is denoted as a position vector rp=(a, b, c)T; the orientation is defined by three Euler angles (η, φ, δ) rotating about z-, y- and x-axes of the fixed reference frame orderly. The the moving platform orientation direction cosine transformation matrix can be described by Rzyx(η, φ, δ). Drawing the structure diagram of the first branched chain, the representation of the various parameters on the other branched is similar because of the identical three branched-chains completely, showing as follows Fig. 3.

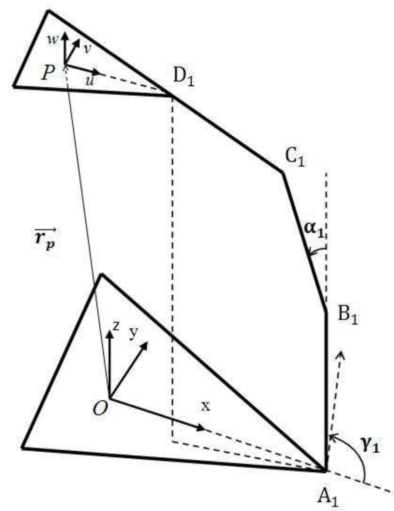

Figure 3: Branched Chain 1 Kinematics Parameter Structure Diagram

3.1 Forward Solution Analysis

to The location forward solution is to determine the location (position and orientation) of the output platform according to the drive angles αi γi (i=1, 2, 3) given. Though the traditional numerical method has a computational shortcoming of complexity, the simple analysis method is still widely used. The coordinates of point Di with respect to the fixed reference O-xyz can be expressed by parameters (a, b, c; η, φ, δ) by equation Di=Rzyx(η, φ, δ)DipT+rp. Coordinates of Ci are obtained on the base of the geometrical relationship and any given values of drive angles αi, γi , six simultaneous equations can be derived as well:

$$\left|\overrightarrow{D_iC_i}\right| = L_{21},\ \text{(i=1, 2, 3)} \qquad (1)$$

$$(\overrightarrow{B_iC_i} \times \overrightarrow{C_iD_i}).\overrightarrow{A_iB_i} = 0,\ \text{(i=1, 2, 3)} \qquad (2)$$

The conventional numerical method to the forward solutions is relatively complicated, which needs to solve a group of higher order nonlinear multivariable equations with strong coupling, leading to determine the workspace boundary, is not practical.

For predigesting the problem, we can adopt a novel solving method of position forward solution - equivalent mechanism method.

Under the condition coordinates value of Ci can be expressed as the function of (γi, αi) (i=1, 2, 3). We can introduce a new parameter 'β', which is the angle between vector $\overrightarrow{C_iD_i}$ and vertical vector (0,

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0, 1). Accordingly the equivalent mechanism forms can be achieved, as shown in Fig.4

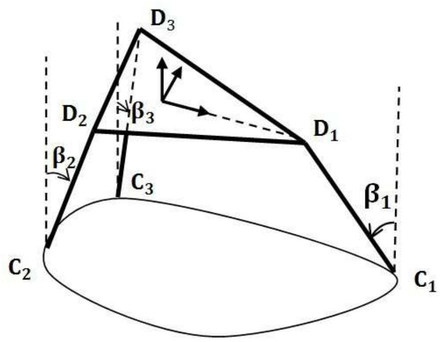


Figure 4: Equivalent Mechanism Method Parameter Structure Diagram

According to the coordinates Ci in the fixed coordinate system and mechanism geometrical relationship, point Di relative to the fixed coordinate system can be expressed by the coordinate Ci and angle variable coordinate βi as three simultaneous equations below:

$$\left|\overrightarrow{D_1D_2}\right| = \sqrt{3}r \quad (3)$$

$$\left|\overrightarrow{D_2D_3}\right| = \sqrt{3}r \quad (4)$$

$$\left|\overrightarrow{D_1D_3}\right| = \sqrt{3}r \quad (5)$$

Then the position vector of the moving platform is obtained as:

$$r_p = (a, b, c)^T = (D_1 + D_2 + D_3)/3 \quad (6)$$

Meanwhile, the rotation vectors may be obtained as:

$$\vec{u} = (D_1 - r_p)/\left|D_1 - r_p\right| \quad (7)$$

$$\vec{\upsilon} = (D_1/3+2D_3/3-r_p)/\left|D_1/3+2D_3/3-r_p\right| \quad (8)$$

$$\vec{\omega} = \vec{u}\times\vec{\upsilon} \quad (9)$$

The Euler angles (η, φ, δ) can be solved by using the following Matlab function:

$$\eta = A\tan 2\,(u(2), u(1)) \quad (10)$$

$$\varphi = A\tan 2\,(-u(3), sqrt(u(1)\wedge 2 + u(2)\wedge 2)) \quad (11)$$

$$\delta = A\tan 2\,(\upsilon(3), \omega(3)) \quad (12)$$

So far, equivalent mechanism method is achieved successfully.

3.2 Numerical Examples of Location Forward Kinematics

On the basis of the above theoretical analysis, defining the parallel manipulator structure parameters and taking a group of the input driving angle valueαi, γi， , using the two methods to work out forward kinematics solution respectively, and all results can be drawn corresponding to the three-dimensional model to further verify the correctness of the results.3/3-RRRS parallel manipulator structure parameters are given as shown in Table1

Table1: 3/3-Rrrs Parallel Manipulator Structure Parameters

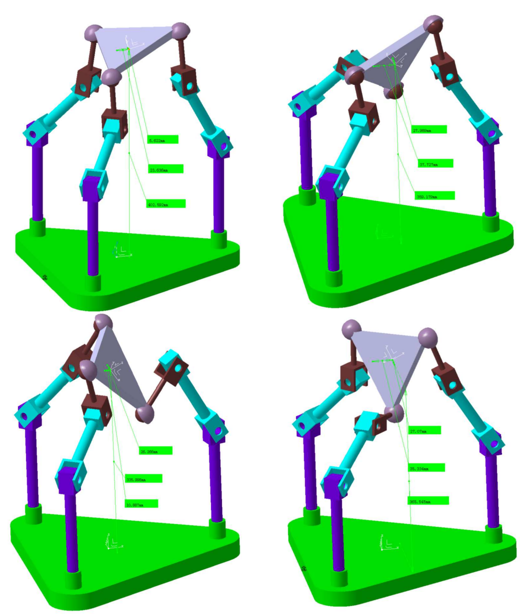
Structure Parameters	R (mm)	R (mm)	A_iB_i (mm)	B_iC_i (mm)	C_iD_i (mm)
Parameter Value	100	200	200	150	100

Taking the group of the moving platform motion input, as shown in Table2

Table 2 : 3/3-Rrrs Parallel Manipulator Motion Input

Drive input	γ1 (rad)	γ2 (rad)	γ3 (rad)	α1 (rad)	α2 (rad)	α3 (rad)
Parameter value	π/12	-π/5	-3π/5	π/6	π/4	π/4

Writing the computing program in MATLAB software environment based on the equivalent mechanism method of the forward Kinematics solution, the moving platform location parameters can be obtained by calculating. Eight solutions gotten can be expressed in the CATIA model of 3/3-RRRS parallel manipulator and the corresponding three-dimensional location model are showing in Fig.5



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(e) location forward solutions model 5.(f) location forward solutions model 6.(g) location forward solutions model 7.(h) location forward solutions model 8.

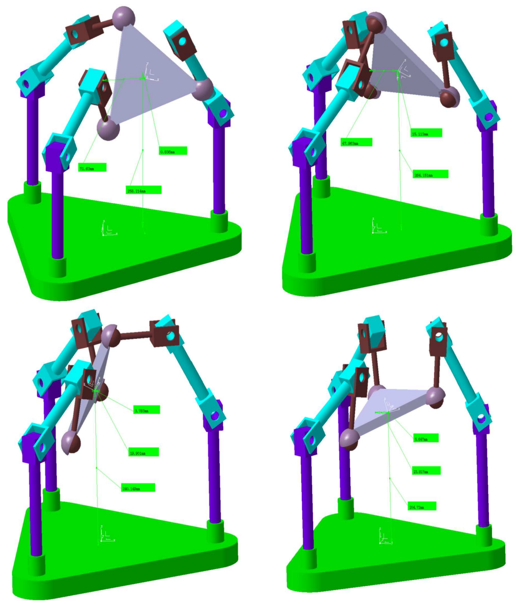


Figure 5: The Group Kinematics Model

(a) location forward solution model 1. (b) location forward solution model 2.(c) location forward solutions model 3. (d) location forward solutions model 4.

3.3 Inverse Solution of Location Numerical Method

Inverse solution is to determine the six drive angles (αi, γi) according to given moving platform location parameter rp and three Euler angles η, φ, δ.

Rod length equation can be listed according to the length of each rod as well:

$$[\overrightarrow{B_iC_i}\times(0,0,1)]\cdot\overrightarrow{C_iD_i}=0 \tag{13}$$

$$\left|\overrightarrow{B_iC_i}\right|=L_{1i} \tag{14}$$

$$\left|\overrightarrow{C_iD_i}\right|=L_{2i} \tag{15}$$

From the three simultaneous equations before, the coordinates of points Ci can be obtained. According to the geometrical relationship, writing the corresponding computing program in MATLAB environment, αi, γi can be obtained as following.

$$\kappa_i=\overrightarrow{B_iD_i}\times(0,0,1) \tag{16}$$

$$\gamma_i=A\tan 2\,(\kappa_i(2),\ \kappa_i(1)) \tag{17}$$

$$\alpha_i=A\tan 2\,((R-X_i)/\sin\gamma_i,\ Z_i-\left|\overrightarrow{A_iB_i}\right|) \tag{18}$$

In the formula, k(i) denotes a vector value of the column i.

3.4 Numerical Examples of the Location Inverse Solution

On the basis of the above analysis, we make the results of location forward solutions as the input of the inverse solution, in order to achieve the location forward and inverse mutual validation, and draw all the results of three-dimensional model.

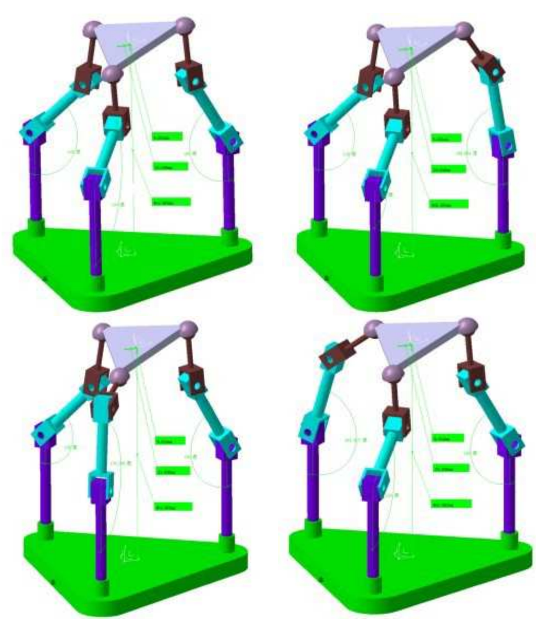
Given 3/3-RRRS parallel manipulator structure parameters as shown in Table 3.

Making location forward solutions 1 in the aforesaid group as the input of the inverse solution, the moving platform location parameters can be shown as the following table:

Table 3: Location Parameter Values of 3/3-RRRS Parallel Manipulator

Location	*a* mm	*b* mm	*c* mm	*η* rad	*φ* rad	*δ* rad
Parameter Value	23.6	-5.6	402.5	-0.1	-0.2	0.1

Then writing the corresponding MATLAB program, through the calculation, the results of drive angles αi, γi can be got The corresponding 3 d location model of the eight group obtained solutions can be got correspondingly as shown in Fig.6:



(a) location inverse solution model 1. (b) location inverse solution model 2.
(c) location inverse solution model 3. (d) location inverse solution model4.

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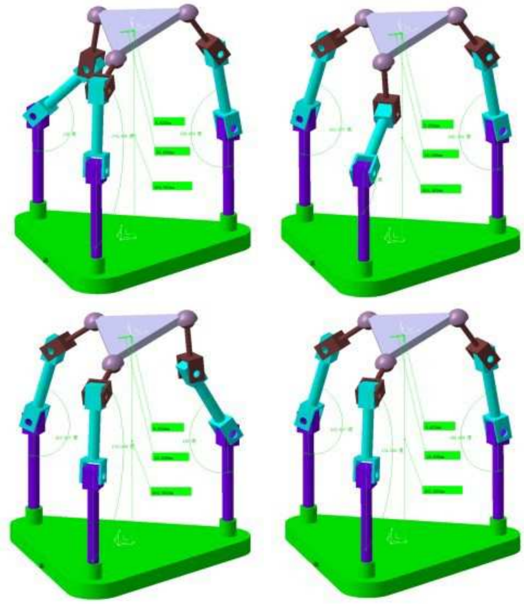

(e) location inverse solution model 5. (f) location inverse solution model 6.
(g) location inverse solution model 7. (h) location inverse solution model 8.

Figure 6: The First Group Kinematics Inverse Solution Results Model

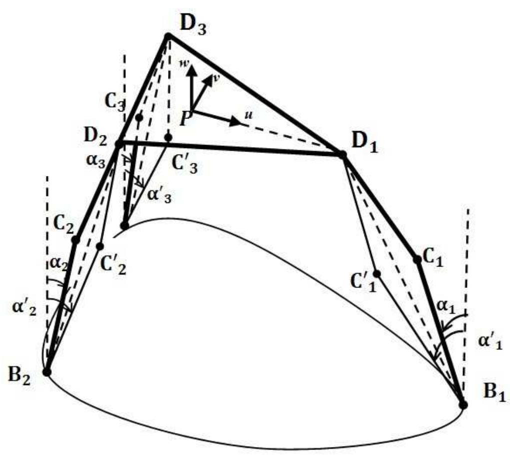

Figure 7: The Eight Groups' Solution of Location Inverse Solution

From the above the group of numerical inverse solution applied examples, we can know, corresponding to the location in a moving platform, γi only has a set of solution, αi have two groups of solution, so reverse solution have 2×2×2=8 groups in total, shown in Fig.7.

4. SUMMARY

This paper mainly addressed the kinematics analyses of a 3/3-RRRS parallel manipulator. Analyzing of the conventional solution of forward Kinematics on the basis of the definition of 3/3-RRRS parallel kinematical parameters, proposing a novel solution, the equivalent mechanism, the numerical solution needs solving three equations transformed by nine equations successfully, to simplify the kinematics forward Solutions. At the same time, the paper also completed a set of applied example of the forward and inverse solutions. All kinematics results of the forward and inverse in 3D institutional model is a more intuitive, and achieve the purpose of geometrical verification.

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