Journal of Theoretical and Applied Information Technology
10th March 2013. Vol. 49 No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

MR IMAGE RECONSTRUCTION BY PATCH-BASED SPARSE REPRESENTATION

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ABSTRACT

Compressed sensing has been widely used to reconstruct magnetic resonance(MR) images from highly under-sampled k-space data. Generally, there are two main types of dictionary: analytic dictionary and trained dictionary. In this paper, we propose a novel framework to adaptively learn the dictionary and reconstruct MR images from highly under-sampled k-space data. We use the beta-Bernoulli process as a Bayesian nonparametric prior, which can learn dictionary adaptively. This patch-based dictionary learning process can also infers the sparsity of each patch and the noise variance. Our numerical experiments demonstrate that our reconstruction is more accurate compared to existing algorithms..

Keywords: *Magnetic Resonance Imaging(MRI), Compressed Sensing(CS), Dictionary Learning, Bayesian Nonparametric*

1. INTRODUCTION

Magnetic Resonance imaging (MRI) has been widely used in medical diagnosis, but its application is limited by the slowly imaging process. This is mainly because of the K-space data is acquired sequentially in time [1]. Recent years, sparse representation theory shows that high quality of MR images reconstruction from highly under-sampled k-space data is possible [2]. This enables fewer sampling data than traditional Nyquist Sampling, and still recovers images accurately. Different algorithms can be classified by the choice of dictionary. Generally, there are two main ways to build the dictionary [3]: (i) use mathematical model to build an analytic dictionary, such as wavelets, curvelets and contourlets; (ii) trained a dictionary based on a set of samples data, like MOD [4], KSVD [5].

Analytic dictionary employs fast mathematical transformation to shift the images to transform domain, like wavelets, contourlets. M. Lustig et al. use wavelets to build the dictionary [2], and can effectively reconstruct MR images from highly under-sampled k-space data. Junzhou Huang et al. use wavelets dictionary and TV norm regularization sub problems to obtain the reconstruction images [6]. Recently years, patch-based MR images denoising studies show that patch-based dictionaries can potential remove noise. A patch-based directional wavelets dictionary is proposed by Xiaobo Qu et al. to reconstruct MR images from highly under-sampled k-space data [7]. However analytic dictionaries are built by some general mathematical transformations, they cannot describe the local image features suitable. So dictionary learning methods are developed to build better dictionary.

Trained dictionaries, such as MOD [4], K-SVD [5], are learned from a set of samples, so the trained atoms are more suitable to the specific images. They use patch-based dictionary learning method to effectively capture local image features. The trained atoms can be regarded as elementary structures of the samples images. Therefore, we are able to reconstruct MR images with sufficient details information and obtain a better reconstruction result. But dictionary learning with these models needs to set an error threshold or sparsity level. In practice, we do not know them exactly. In year 2001, Mingyuan Zhou et al. [8] uses beta process factor analysis method (BPFA) to learn dictionary automatically and adaptively estimate the number of coefficients are used for representation of each patch. This gives an adaptive way to learn dictionary.

In this paper, we proposed a novel Bayesian framework for learning the dictionary and used the trained dictionary to reconstruct the MR images from highly under-sampled k-space data. We use

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

nonparametric factor analysis with beta process (BPFA) to learn the dictionary automatically and calculate the number of coefficients which are used for representation of each patch (or sparsity level). This avoids setting the sparsity and noise levels by hand. Meanwhile we can also obtain the noise variance of the MR image.

The rest of this paper is organized as follows. Section 2 talks about related work in CSMRI and dictionary learning with BPFA. In section 3, we introduce our problem formulation for patch-based MR image reconstruction. Experiments are conducted in section 4. Section 5 gives a conclusion to our work.

2. BACKGROUND AND RELATED WORK

We use $x \in C^N$ to represent as a vector, which corresponding $\sqrt{N} \times \sqrt{N}$ image which we want to reconstruct, and $y \in C^M$ represents the k-space measurements. The two are related as $y = F_u * x$, where $F_u \in C^{M \times N}$, $M < N$ is the under-sampled Fourier encoding matrix. Let R_i be the ith patch extraction operator, $R_i x = x_i \in C^P, i = 1, \cdots, I$.

2.1 Csmri with Analytic Dictionary

Compressed Sensing reconstructs the unknown x from the measurements y, or equivalently solves an underdetermined system of linear equations $y = F_u * x$ by minimizing the l_0 quasi norm of the sparsified image Ψx, where Ψ represents a global, typically orthogonal sparsifying transform for the image. For example, Ψ may be the wavelets transform, so that Ψx corresponds to the wavelets coefficients of x and is assumed to be sparse. The corresponding optimization problem is

$$\min_x \|\Psi x\|_0 \text{ s.t. } y = F_u x \tag{1}$$

The typical formulation of the CS reconstruction problem uses l_1 relaxation of the l_0 quasi norm, and accounts for the noise in the k-space measurements in the following Lagrangian setup

$$\min_x \|F_u x - y\|_2^2 + \lambda \|\Psi x\|_1 \tag{2}$$

This problem formulation involves a global sparsity measure and an analytical, fast sparsifying transform Ψ. However, common transforms such as wavelets result in artifacts such as Gibbs ringing in the reconstruction images.

2.2 Trained Dictionary

The MR images reconstruction techniques with analytic dictionary are limited by the degree of under-sampling. Trained dictionary makes the higher under-sampling degree available and obtains better image reconstructions.

K-SVD algorithm can learn a dictionary from a set of training data [1]. It performs the dictionary update step with singular value decomposition (SVD), where each atom of dictionary and corresponding coefficients are updated jointly for the image patches used currently. Saiprasad Ravishankar et al. use this algorithm to reconstruct MR images from highly under-sampled k-space data, and obtain a well result.

Mingyuan Zhou et al. uses beta process factor analysis method (BPFA) to learn the dictionary automatically and adaptively estimate the number of coefficients is used for representation of each patch [8]. We use BPFA to train the dictionary and reconstruct MR images with the trained dictionary.

Let the image patch as $x_i = D\alpha_i + \varepsilon_i$, ε_i is additive noise of input image patch, $D\alpha_i$ is ideal image patch. We also aim to solve a patch-based optimization problem:

$$\min_{\alpha, x, D} \sum_i \|R_i x - D\alpha_i\|_2^2, s.t. \|\alpha_i\|_0 < T_0 \tag{3}$$

In order to estimate each patch's sparsity level, we let $\alpha_i = z_i \circ s_i$, where $\circ$ represents the Hadamard multiplication. The noise variance is unknown and we can estimate it. To train a dictionary with BPFA, we use the following steps:

(1) construct a dictionary $D = [d_1, \cdots, d_k]$:

$$d_k \quad N(0, P^{-1} I_P),\ k = 1, \cdots, K.$$

(2) draw a probability π_k to suit each d_k

$$\pi_k \quad Beta(a_0, b_0),\ k = 1, \cdots, K.$$

(3) construct precision value

$$\gamma_s \quad Gamma(c_0, d_0), \gamma_\varepsilon \quad Gamma(e_0, f_0)$$

(4) for every patch

(a) Construct $s_i \quad N(0, \gamma_s^{-1} I_K)$.

Journal of Theoretical and Applied Information Technology
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ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

(b) Draw a binary vector z_i with

$$z_{ik} \quad Bernoulli(\pi_k)$$

(c) Construct the image patches

$$R_i x = D\alpha_i + \varepsilon_i \ ,\varepsilon_i \quad N(0,\gamma_\varepsilon^{-1} I_P)$$

(5) Obtain the dictionary and noise variance after iteration.

With this approach, the model constructs a dictionary matrix D, and assigns probability π_k to vector d_k.

3. ALGORITHM

We next present our problem formulation for reconstructing MR images from highly under-sampled k-space data. As discussed in section II, we propose a formulation as follow:

$$\arg\min_x \lambda\rho(x) + v\left\|F_u x - y\right\|_2^2. \qquad (4)$$

In [2], $\rho(x) := Wavelet(x)$, it uses wavelets transform to build an analytic dictionary. We use patch-based image dictionary learning algorithm, so we have

$$\rho(x) := \sum_i \left\|R_i x - D\alpha_i\right\|_2^2 + f(\alpha_i, D) \qquad (5)$$

The additional function $f(\alpha_i, D)$ is sparsity constraint. We use BPFA to learn the dictionary, so the function $f(\ )$ enforce sparsity and learning a D matrix for which α_i is sparse. This problem can be solved by using an alternating minimization procedure, first fix x to update dictionary and sparse coefficients, then fix dictionary and coefficients to obtain reconstruction image. The problem is show as follow, where j is the iteration .

$$\begin{cases} (D^{j+1}, \alpha^{j+1}) = \arg\min \rho(x^j) \\ x^{j+1} = \arg\min \sum_i \left\|R_i x - D^{j+1}\alpha_i^{j+1}\right\|_2^2 + v\left\|F_u x - y\right\|_2^2 \end{cases} \qquad (6)$$

3.1 Dictionary Learning

In first step, we fix x. Then we can rewrite the problem as

$$\min_{\alpha,x,D} \sum_i \left\|R_i x - D\alpha_i\right\|_2^2, s.t.\left\|\alpha_i\right\|_0 < T_0 \qquad (7)$$

We can use Gibbs sampling to obtain variables posterior density for dictionary and coefficients, as show in the following

$$p(d_k|-) \propto \prod_{i=1}^N N\left(x_i \middle| D(s_i \circ z_i), \gamma_\varepsilon^{-1} I\right) N\left(d_k \middle| 0, P^{-1} I_P\right) \quad (8)$$

$$p(s_{ik}|\sim) \propto N\left(x_i \middle| D(s_i \circ z_i), \gamma_\varepsilon^{-1} I_p\right) N(s_i | 0, \gamma_s^{-1} I_K) \quad (9)$$

$$p(z_{ik}|\sim) \propto N\left(x_i \middle| D(s_i \circ z_i), \gamma_\varepsilon^{-1} I_p\right) Bernoulli(z_{ik} | \pi_k) \quad (10)$$

With these variables posterior density, we can obtain dictionary and coefficients.

3.2 Update Image:

In this step, we fix dictionary and coefficients. We can also rewrite the problem as

$$\min_x \sum_i \left\|R_i x - D\alpha_i\right\|_2^2 + v\left\|F_u x - y\right\|_2^2. \qquad (11)$$

This is a simple least squares problem, the solution can rewrite as follow

$$(\sum_i R_i^T R_i + vF_u^H F_u)x = \sum_i R_i^T D\alpha_i + vF_u^H y \qquad (12)$$

Solving (12) directly may take a long time, as it requires to invert the $N \times N$ matrix premultiplying x. So the solution must be simplified. The term $\sum_i R_i^T R_i$ is a diagonal matrix, their values are the same as the number of overlapping patches contributing at those pixel locations. The diagonal entries become all equal if we wrap around the image at boundaries. Then we have the term $\sum_i R_i^T R_i = PI_N$. Another simplification is obtained by the quantity of transforming matrix F. Let $F \in C^{N\times N}$ denote the full Fourier encoding matrix normalized such that $F^H F = I_N$. Then we can get

$$F(PI_N + vF_u^H F_u)F^H Fx = F\sum_i R_i^T D\alpha_i + vFF_u^H y \qquad (13)$$

and rewrite as

$$(I_N + v' FF_u^H F_u F^H)Fx = \frac{1}{P} F\sum_i R_i^T D\alpha_i + v' FF_u^H y \qquad (14)$$

The matrix $FF_u^H F_u F^H$ is a diagonal matrix with ones and zeros entries which correspond to sampled and unsampled data in k-space. $FF_u^H y$ represents the zero-filled in unsampled k-space data.

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10th March 2013. Vol. 49 No.1

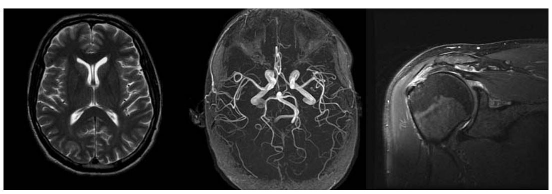
ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

Let $S = \frac{F\sum_i R_i^T D\omega_i}{P}, S_0 = FF_u^H y$, the solution is then

$$Fx(i,j) = \begin{cases} S & (i,j)\notin \Omega \\ \frac{S}{1+v^{'}} + \frac{v^{'} S_0}{1+v^{'}} & (i,j)\in \Omega \end{cases} \tag{15}$$

$Fx(i,j)$ represents the updated k-space value at location (i, j), S_0 represents the zero-filled k-space data, and Ω represents the subset of k-space data that has been sampled. We can get the reconstruction image by IFFT.

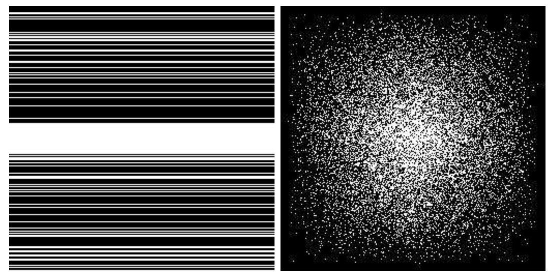
The proposed algorithm uses zero-filled k-space data to initialize input x, and S to fill the unsampled location.

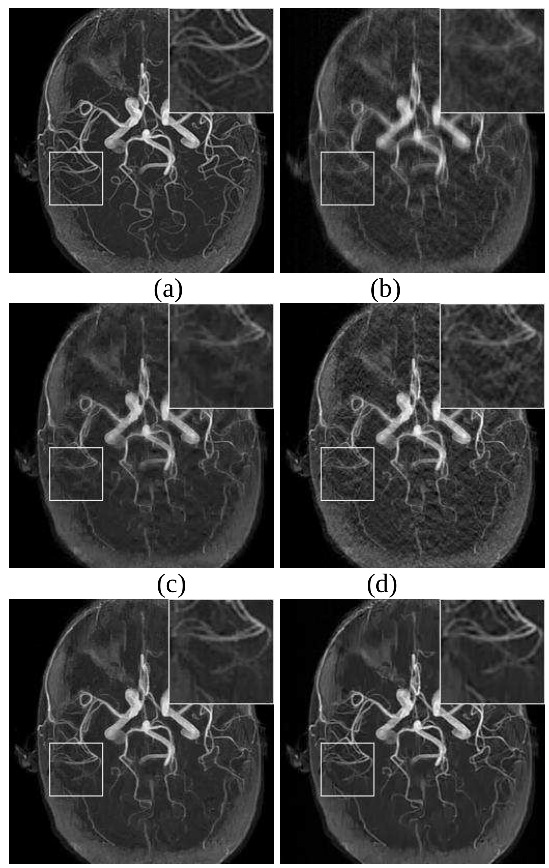

(a) (b) (c)
Fig.1 The images we used in the test. (a) Circle (b) brain (c) shoulder.

4. EXPERIMENTS

In this section, the experiments are performed at a variety of highly under-sampled ratio. The images we used in the test are in vivo MR data (see Fig. 1). The sampling schemes we used in k-space data include random phase encoding in Cartesian sampling and pseudo random sampling (see Fig. 2). We use Cartesian sampling mask as its wide use for k-space data acquisition. However our method works well for non-Cartesian sampling schemes. The k-space data acquisition was simulated by 2D discrete Fourier transform of the MR images. We obtain more data in low frequencies, and less data in high frequencies. Our experiments are compared with SparseMRI, FCSA and PBDW (as DLMRI do not provide experiment code, we can't compare with it).

In our experiments, we conduct k-space data both with noise and without noise. The parameters were set as patch size N=36, dictionary size 36*72, v=100. All the reference MR images for the experiments were normalized such that the intensities of them have range [0, 1]. All the experiments are implemented with Matlab 2008 with Intel Celeron CPU E3200 at 2.40GHz and 1.96G memory. We under-sample k-space data, and fill the unsampled location with zero, then use different algorithms to reconstruct the MR images from the under-sampled k-space data and compare their results.


(a) (b)
Fig.2 sample mask (a) Cartesian sample mask. (b) Random sample mask.


(a) (b) (c) (d) (d) (e)
Fig.3 The reconstruction images. (a) orignal image. (b) Reconstruction image with zero filled in unsampled pixes. (c) - (d) reconstruction images by SparseMRI, FCSA, PBDW, and BPFA.

Journal of Theoretical and Applied Information Technology
10th March 2013. Vol. 49 No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

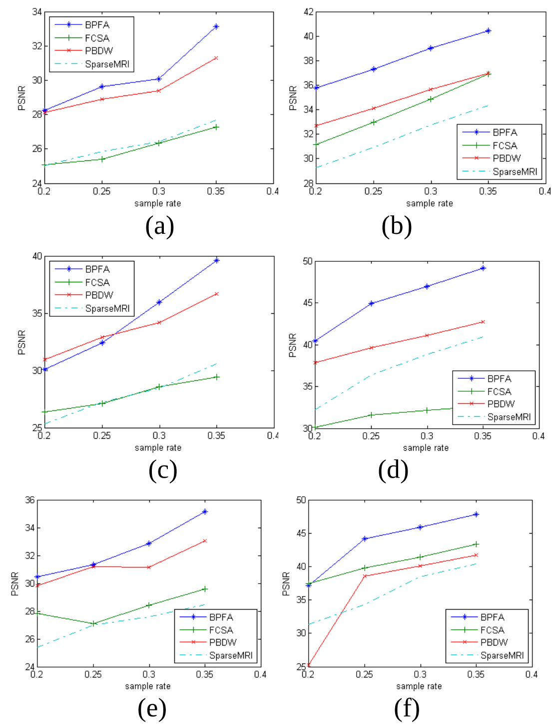


Fig. 4 PSNR of the different images. (a)-(b) circle image with Cartesian and Random sampling. (c)-(d) shoulder image with Cartesian and Random sampling. (e)-(f) brain image with Cartesian and Random sampling.

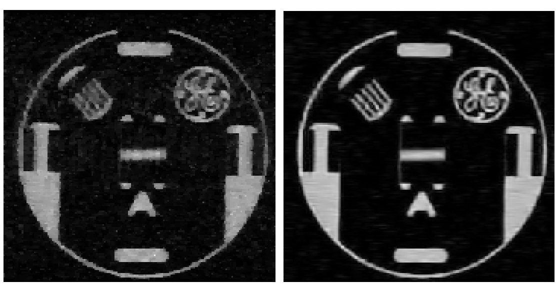


Fig. 5 the reconstruction images with noise data in k-space. (a) SparseMRI result. (b) BPFA result.

Fig. 3 shows the reconstruction images of noiseless k-space data with the Cartesian sampling at 35% sampling rate. The SparseMRI and FCSA algorithms are unable to remove the aliasing very well, and the reconstruction has significant error in image texture. The PBDW results can get a better reconstruction than SparseMRI and FCSA in image texture, but from the zoom in patches we can find that our reconstruction image can obtain a clear edge feature. The reconstruction of sparseMRI has a PSNR of 27.67 dB, FCSA with 27.29 dB, PBDW with 31.29 dB and BPFA with 33.11 dB. The PSNR and reconstruction images both show that BPFA performs better than other algorithms we compared.

Fig. 4 is the PSNR of different sampling rates with two sampling mask.

Fig. 5 shows the reconstruction images with noise data in k-space (the data is get from Lustig's website). We use 25% Cartesian sampling mask to under-sample the k-space data. The result shows that the proposed algorithm can effectively remove aliasing and preserve better detail.

5. CONCLUSION

In this paper, a novel framework is proposed to learn a dictionary for a specific image and reconstruct MR images from highly undersampled k-space data. Beta-Bernoulli process was used as a Bayesian nonparametric prior to learn a dictionary adaptively and determine the sparsity level. Our experiments show that our patch-based dictionary learning method can effectively remove aliasing and noise in reconstructing images from highly undersampled k-space data.

ACKNOWLEDGE

The work is supported by National Natural Science Funding of China, NO. 61172179, 30900328 and Fundamental Research Funds for the Central Universities 2011121051, the Natural Science Foundation of Fujian Province of China 2012J05160.

REFRENCES:

[1] S. Ravishankar and Y. Bresler, “MR Image Reconstruction from Highly Undersampled k-space Data by Dictionary Learning,” IEEE Trans. on Medical Imaging, vol. 30, no. 5, pp. 1028-1041, 2011.

[2] M. Lustig, D. Donoho, and J. M. Pauly, “Sparse MRI: The Application of Compressed Sensing for Rapid MR Imaging,”Magnetic Resonance in Medicine, vol. 58, no. 6, pp. 1182-1195, 2007.

[3] R. Rubinstein, A. M. Bruckstein, and M. Elad, “Dictionaries for Sparse Representation Modeling,” Proc. of IEEE, vol. 98, no. 6, pp. 1045-1057, 2010.

[4] K. Engan, S. O. Aase, and J. H. Husoy, “Method of Optimal Directions for Frame Design,” inProc. IEEE Int. Conf. Acoustics, Speech, Signal Processing, pp. 2443-2446, 1999.

[5] M. Aharon, M. Elad, A. Bruckstein, and Y. Katz, “K-SVD: An Algorithm for Designing of Overcomplete Dictionaries for Sparse

Representation," IEEE Trans. on Signal Processing, vol. 54, pp. 4311-4322, 2006.

[6] J. Huang, S. Zhang, and D. Metaxas, "Efficient MR Image Reconstruction for Compressed MR Imaging," Medical Image Analysis, vol. 15, no. 5, pp. 670-679, 2011.

[7] X. Qu, D. Guo, B. Ning, Y. Hou, Y. Lin, S. Cai, and Z. Chen, "Undersampled MRI Reconstruction with the Patch-Based Directional Wavelets," Magnetic Resonance in Imaging, doi:10.1016/j.mri.2012.02.019, Feb. 2012.

[8] M. Zhou, H. Chen, J. Paisley, L. Ren, L. Li, Z. Xing, D. Dunson, G. Sapiro, and L. Carin, " Nonparametric Bayesian Dictionary Learning for Analysis of Noisy and Incomplete Images," IEEE Trans. Image Process., vol. 21, no. 1, pp.130-144, Jun. 2011.

[9] K. Dabov, A. Foi, V. Katkovnik, and K. Egiazarian, "Image Denoising by Sparse 3D Transform-Domain Collaborative Filtering," IEEE Trans. on Image Process, vol. 16, no. 8, pp. 2080-2095, 2007.

[10] M. Akcakaya, T. A. Basha, B. Goddu, L. A. Goepfert, K. V. Kissinger, V. Tarokh, W. J. Manning, and R. Nezafat, "Low-Dimensional-Structure Self-Learning and Thresholding: Regularization beyond Compressed Sensing for MRI Reconstruction," Magnetic Resonance in Medicine, vol. 66, pp. 756-767, 2011.

[11] Shihao Ji, Ya Xue, and Lawrence Carin, IEEE Trans. Signal Processing, vol. 56, no. 6, pp. 2346-2356, June 2008.

[12] S. Ma, W. Yin, Y. Zhang, and A. Chakraborty, "An Efficient Algorithm for Compressed MR Imaging using Total Variation and Wavelets," inProc. of CVPR, 2008.