Journal of Theoretical and Applied Information Technology
10th March 2013. Vol. 49 No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

A HYBRID ARTIFICIAL BEE COLONY ALGORITHM FOR PORTFOLIO OPTIMIZATION PROBLEM

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ABSTRACT

In this paper, a constrained mean-variance model is constructed for the portfolio optimization problems. The model is a mixed quadratic integer programming problem, and it is too hard to solve by using the traditional optimal algorithms. The purpose of this paper is to use a heuristic algorithm to solve this problem. Combined with the differential evolution strategy, a new hybrid artificial bee colony algorithm is proposed for solving the constrained mean-variance model. In the new hybrid artificial bee colony algorithm, a new search scheme and obsolete rules are presented to improve convergent speed of the algorithm. Computational results show that the algorithm has great capability to solve the portfolio optimization problem. It supplies a new method for solving the portfolio optimization problem.

Keywords: *Artificial Bee Colony; Differential Evolution; Portfolio Optimization; Efficient Frontier*

1. INTRODUCTION

Portfolio optimization is one of the most important problems in modern financial management, which addresses the problem of allocating a particular amount of wealth among a given set of assets in financial market properly, such that the investor can obtain the highest returns along with the lowest risk. When investors seek the high returns, it always accompanies with the high risks. In the 1950's, a fundamental research on this problem was given by Markowitz [1][2], who proposed the mean-variance model (MV) in his researches. The key point of the MV model is to use the expected returns of a portfolio as the investment return and the variance of returns of the portfolio as the investment risk. The mean-variance model proposed by Markowitz plays an important role in the modern portfolio selection theory. However, the MV model was constructed under strictly assumptions, such that it is not always suitable for real applications. Therefore, in this paper, based on the MV model, a constrained mean-variance model is proposed.

Recently, there are some researches using heuristic methods and artificial intelligent algorithms to solve the portfolio optimization problem (see for example, [3-10]). Arnone et al. were the ones who first applied genetic algorithm to portfolio optimization problem [3]. Fieldsend et al. and Jobst et al. discussed the computational aspects in presence of the discrete asset choice constraints [4][5]. In [6], genetic algorithm, tabu search and simulated annealing were applied to solve the cardinality constrained portfolio optimization problem, and outstanding results were obtained. Fernandez [7] used neural network algorithm to solve the portfolio optimization problem and compared the results with that in [6]. The results indicated that neural network algorithm outperforms the other algorithms in [6]. In [8], the particle swarm optimization was used to portfolio optimization problem, and part of the results of test problems also performed well.

The entire algorithms mentioned above have their own advantages for solving portfolio optimization problem. However, there are still some shortages, such as slower convergent speed and easily trapped in the local optima. To overcome these issues, based on the artificial bee colony algorithm, a hybrid artificial bee colony algorithm (HABC) is proposed for constrained portfolio optimization problem in this paper. The artificial bee colony algorithm (ABC), proposed by Karaboga, is recently introduced optimization algorithm which simulates the foraging behavior of a honey bee colony (see for example, [11-15]). In the HABC algorithm, a new search scheme and the obsolete

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

rules are presented to improve convergent speed, while differential evolution is introduced to maintain the population diversity. Furthermore, the Deb's selection rule is employed to make sure that the candidate solutions are feasible in the search area. Experimental results and comparisons demonstrated the effectiveness and efficiency of the proposed HABC.

The rest of the paper is organized as follows. In Section 2, the portfolio optimization problem is described and the constrained mean-variance model is presented. In Section 3, the hybrid artificial bee colony algorithm for constrained mean-variance model is proposed. In Section 4, some supporting numerical experiments are presented. Finally, a conclusion is provided in Section 5.

2. PORTFOLIO OPTIMIZATION PROBLEM FORMULATION

Consider a financial market where an investor can trade N risky assets. Assume that μ_i represents the mean returns of the i th asset, σ_{ij} is the covariance of the i th asset and the j th asset returns, $\lambda \in [0,1]$ is the risk aversion parameter, and decision variable x_i represent the proportion an investor invest on the i th asset. Therefore, the mean-variance model can be described as follows:

$$\begin{aligned} \min \quad & \lambda\left[\sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij}\right] - (1-\lambda)\left[\sum_{i=1}^{N} x_i \mu_i\right] \\ s.t. \quad & \sum_{i=1}^{N} x_i = 1, \\ & 0 \le x_i \le 1, \qquad i = 1,\dots,N. \end{aligned} \tag{1}$$

For every $\lambda \in [0,1]$, by solving problem (1), a pair of mean returns and risk values can be obtained. All these pairs consist of the efficient frontier, which is called the standard efficient frontier.

Assume that K represents the number of assets needed to be selected in portfolio, ε_i and δ_i represent the lower bound and upper bound of the i th asset, respectively. z_i is a 0-1 variable, when $z_i = 1$ the i th asset is selected in the portfolio; when $z_i = 0$ the i th asset is not considered in the portfolio. Thus, the constrained mean-variance model can be constructed as:

$$\begin{aligned} \min \quad & \lambda\left[\sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij}\right] - (1-\lambda)\left[\sum_{i=1}^{N} x_i \mu_i\right] \\ s.t. \quad & \sum_{i=1}^{N} x_i = 1, \\ & \sum_{i=1}^{N} z_i = K, \\ & \varepsilon_i z_i \le x_i \le \delta_i z_i, \quad i = 1,\dots,N, \\ & z_i \in \{0,1\}, i = 1,\dots,N. \end{aligned} \tag{2}$$

3. HYBRID ARTIFICIAL BEE COLONY ALGORITHM

The hybrid artificial bee colony (HABC) algorithm proposed in this paper is a new algorithm, which combines an improved artificial bee colony (IABC) algorithm and differential evolution (DE) [16]. The main idea of HABC algorithm is as follows:

First, a new search strategy is constructed to improve the convergence speed;

Secondly, the obsolete rules are introduced to further improve the convergence speed;

Finally, execute the differential evolution to the individuals in the population to protect the maintain the population diversity.

3.1 Differential evolution

The main idea of DE is that starting from the initial random population repeat the three operations: mutation operation, crossover operation and selection operation, until the optimal solution is obtained. In the selection operation, the idea of survival of the fittest is employed to select the optimal individual by its fitness function value.

Here, the mutation operation can be described as follows:

$$U = X_{r1} + F(X_{r2} - X_{r3}), \tag{3}$$

where X_{r1}, X_{r2}, X_{r3} represent three different random selected individuals in the population, F is the amplification factor of the difference vector.

The crossover operation is:

$$v(i) = \begin{cases} u(i), & p \le CR, \\ x(i), & p > CR, \end{cases} \quad i = 1,\dots,N, \tag{4}$$

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

where $CR \in [0,1]$ is the crossover control parameter, p is the random number in $[0,1]$.

3.2 Artificial Bee Colony Algorithm

The artificial bee colony algorithm (ABC), proposed by Karaboga, is recently introduced optimization algorithm which simulates the foraging behavior of a honey bee colony. In ABC algorithm, the swarm population consists of employed bees, onlooker bees and scout bees. Employed bees exploit the food source and share the information to the onlooker bees. Onlooker bees further exploit the food source based on the information shared by employed bees. The employed bee whose food source has been abandoned becomes a scout bee to search a new food source.

The basic steps of the ABC algorithm are given below.

Initialize.

Repeat:

Place the employed bees on the food source on the memory;

Place the onlooker bees on the food source on the memory;

Send the scouts to the search area for discovering new food source.

until requirements are satisfied.

The search strategy for both employed bees and onlooker bees is:

$$v_{ij} = x_{ij} + \phi_{ij}(x_{ij} - x_{kj}) \qquad (5)$$

where k, j are the randomly selected indices, $k \in \{1,2,...,SN\}$, $k \neq i$, $j \in \{1,2,...,N\}$, and $\phi_{ij} \in [-1,1]$ is a random number. Thus, a greedy selection is applied between v_{ij} and x_{ij} for bees.

An onlooker bee chooses a food source depending on the probability value associated with the quality of food source, which is given by

$$p_i = \frac{fitness_i}{\sum_{i=n}^{SN} fitness_i}, \qquad (6)$$

where $fitness_i$ is the fitness value of the i th food source, SN is the food source number.

If the abandoned food source is x_i, the scout bee discovers a new food source according to

$$x_{ij} = x_j^{\min} + rand(0,1)(x_j^{\max} - x_j^{\min}), \qquad (7)$$

where $x_j^{\max}$ and $x_j^{\min}$ are the upper and lower bounds of the j th dimension of the problem space.

Although ABC algorithm is a well-performed algorithm for many optimization problems, there still have several shortages. To overcome these issues, an improved artificial bee colony is proposed below.

3.3 Improved artificial bee colony algorithm

For the constrained mean-variance model, the search strategy of the IABC algorithm is as follows:

$$\tilde{z}_{ij_k} = round\left(\frac{1}{1+e^{-[z_{ij_k}+\phi_{ij_k}(z_{ij_k}-z_{lj_k})]}} - \alpha\right), \qquad (8)$$

$$\tilde{x}_{ij_k} = \begin{cases} x_{ij_k} + \phi_{ij_k}(x_{ij_k} - x_{lj_k}), & \tilde{z}_{ij_k} = 1, \\ x_{ij_k}, & \tilde{z}_{ij_k} = 0, \end{cases} \qquad (9)$$

where $\alpha = 0.06$; $1 \le m \le N$; $k = 1, 2, \cdots, m$; l, j are randomly selected index, and $l \in \{1,2,...,SN\}$, $j \in \{1,2,...,N\}$, $l \neq i$; ϕ_{ij_k} is the random number in $[-1,1]$.

Furthermore, when the decrement of optimal fitness value less than τ after limited cycle L, the obsolete rules execute, which is the hybridization and mutation operation:

$$\bar{z}_s = round\left(\frac{1}{1+e^{-[\lambda_s z_{is} + (1-\lambda_s) z_{best,s}]}} - \alpha\right), \qquad (10)$$

$$\bar{x}_s = \begin{cases} \lambda_s x_{is} + (1-\lambda_s) x_{best,s}, & \bar{z}_s = 1, \\ x_{is}, & \bar{z}_s = 0, \end{cases} \qquad (11)$$

$$Z_z = round(Z_{best} + N(0,1)) \qquad (12)$$

$$Z_x = X_{best} + N(0,1) \qquad (13)$$

where $s = 1, 2, ..., N$, $\lambda_s \in [-1,1]$ is a uniformly distributed random number, Z_{best} and X_{best} are the present best solution, Z_i and X_i represent the randomly selected individuals different form Z_{best} and X_{best} respectively. Then, use the best one between Z_z and Z_x to replace Z_i and X_i.

Journal of Theoretical and Applied Information Technology
10th March 2013. Vol. 49 No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

3.4 Selection Mechanism

Because the constrained mean-variance model is a constraint optimization problem, that it is not suitable to use the greedy selection strategy in the new algorithm. Here, Deb's constrained handling method is adopted instead of the greedy selection. Deb's method uses a tournament selection operator, where two solutions are compared at the same time, and the following criteria are applied:

1) Any feasible solution is preferred to any infeasible solution;
2) Between two feasible solutions, the one having the better objective function value is preferred;
3) Between two infeasible solutions, the one having smaller constraint violation is preferred.

3.5 The main step of HABC algorithm

The solution procedure of the overall IABC is as follows.

Algorithm: Hybrid artificial bee colony algorithm

Step 1: Initialize the food source and set all the parameters SN, τ, L and m in the algorithm.

Step 2: The employed bees search the new food source according to (8) and (9). And evaluate the objective function value, fitness value and violation value of this food source. Then, use Deb's selection scheme to make a choice between the new food source and the old one. If the new food source is preferred to the old one, the new one replaces the old one.

Step 3: Calculate the selective probability and select a food source for onlooker bees. Then, the onlooker bees search the new food source according to (8) and (9). And evaluate the objective function value, fitness value and violation value of this food source. Then, use Deb's selection scheme to make a choice between the new food source and the old one. If the new food source is preferred to the old one, the new one replaces the old one.

Step 4: If there is an abandoned food source, replace it with a new food source discovered by the scout bee.

Step 5: Check whether the obsolete rules are satisfied. If it is, randomly selected an individual and produce two new food sources by (10) - (13). Then, use Deb's selection scheme to make a choice between the new food source and the old one. If the new food source is preferred to the old one, the new one replaces the old one.

Step 6: Execute the differential mutation operation, crossover operation and Deb's selection scheme to all the individuals in the population. Record the present optimal food source and fitness function value.

Step 7: Check whether the termination criteria are satisfied. If they are, stop searching and output the final food position; otherwise, return to Step 2.

4. EXPERIMENTS

To verify the performance of the HABC, it is applied to the test problems proposed in [8]. The test data are obtained from http://people.brunel.ac.uk/astjjb/jeb/orlib/portinfo.html. The data correspond to different stock market indices: the Hong Kong HangSeng index, the German DAX100 index, the British FTSE100 index, the US S&P100 index and the Japanese Nikkei index. The dimensions of the five indices are presented in Table 1.

The parameters in the experiments are given as follows: $K=10$, $\varepsilon_j=0.01$, $\delta_j=1$, $(j=1,\ldots,31)$, $\Delta\lambda=0.02$, $SN=50$, $m=4$, $L=15$, $\tau=0.01$, $CR=0.9$, $F=0.5$.

Table 1. The Dimension Of Test Problems

Index	Dimension
HangSeng	31
DAX100	85
FTSE100	89
S&P100	98
Nikkei	225

To verify the applicable efficiency of IABC algorithm, the three indices: mean Euclidian distance (MED), variance of return error (VRE) and mean return error (MRE), presented in [8] for results evaluation is employed. The results obtained by HABC are compared with those obtained by other algorithms in [8], shown in Table 2.

Journal of Theoretical and Applied Information Technology
10th March 2013. Vol. 49 No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

Table 2. The Results Obtained By HABC And In [8]

Index		GA	TS	SA	PSO	HABC
HangSeng	MED	0.0040	0.0040	0.0040	0.0049	5. 36e-005
	VRE(%)	1.6441	1.6578	1.6628	2.2421	0. 8017
	MRE(%)	0.6072	0.6107	0.6238	0.7427	0. 3427
DAX100	MED	0.0076	0.0082	0.0078	0.0090	1.73e-005
	VRE(%)	1.6441	1.6578	1.6628	2.2421	1.5083
	MRE(%)	0.6072	0.6107	0.6238	0.7427	0. 2967
FTSE100	MED	0.0020	0.0021	0.0021	0.0022	1. 95e-004
	VRE(%)	2.8660	4.0123	3.8205	3.0596	2. 2794
	MRE(%)	0.3277	0.3298	0.3304	0.3640	0.3043
S&P100	MED	0.0041	0.0041	0.0041	0.0052	9.72e-005
	VRE(%)	3.4802	5.7139	5.4247	3.9136	2. 9023
	MRE(%)	1.2258	0.7125	0.8416	1.4040	0. 4436
Nikkei	MED	0.0093	0.0010	0.0010	0.0019	0.53e-005
	VRE(%)	1.2056	1.2431	1.2017	2.4274	1.2635
	MRE(%)	5.3266	0.4207	0.4126	0.7997	0. 5307

Meanwhile, the comparison of standard efficient frontiers and HABC efficient frontiers are showed in Figure 1-5.

The experiment results in Table 2 indicate that none of the other four algorithms has clearly advantages in all kinds of investment policies; however, the results obtained by HABC are better than the other four algorithms.

Figure 1-5 show that the efficient frontiers obtained by HABC are more close to the standard efficient frontiers. Under the same risk values, the portfolio return obtained by HABC are higher than the other algorithms.

From the above analysis, compared with other four algorithms we can see that, the results calculated by HABC have the higher accuracy, and the obtained investment strategy can efficiently diversified the risk of the portfolio, which is more valuable for investors.

5. CONCLUSION

HABC algorithm proposed in this paper is a new population based intelligent algorithm. It has the advantages of easy operation, simple achieve and less control parameters. Combine the characteristics of constrained mean-variance model, an optimal algorithm based on the HABC is proposed to solve portfolio optimization problem. The experiment results show that the results obtained by the new algorithm are superior to the results obtained by other intelligent algorithm. It verifies the HABC algorithm is efficiency and applicability for the portfolio optimization problems.

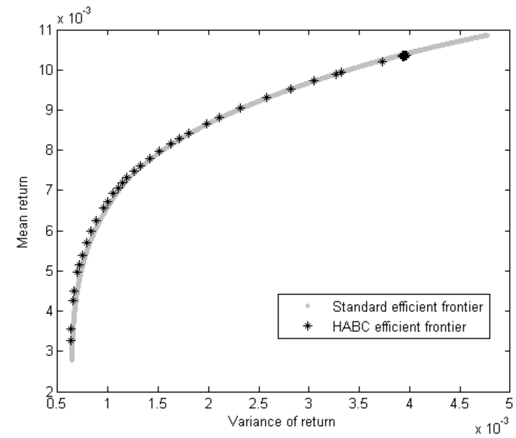

Figure 1. Efficient frontier for HengSeng index

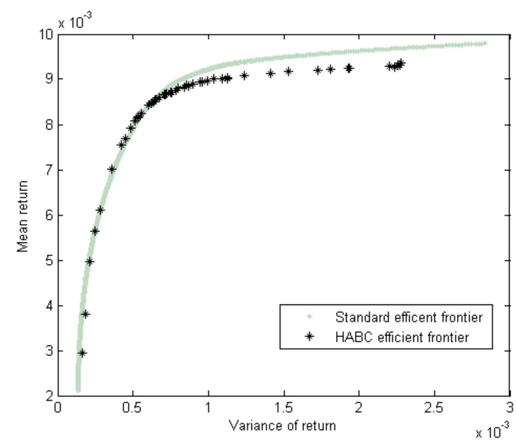

Figure 2. Efficient frontier for DAX100 index

Journal of Theoretical and Applied Information Technology
10th March 2013. Vol. 49 No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

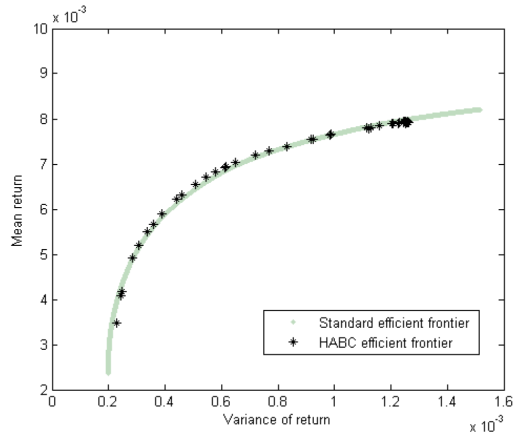

Figure 3. Efficient frontier for FEST100 index

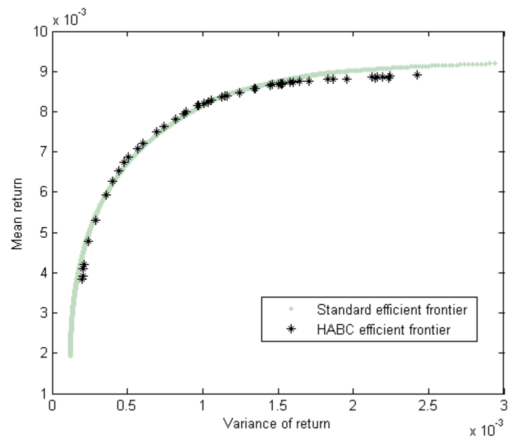

Figure 4. Efficient frontier for S&P100 index

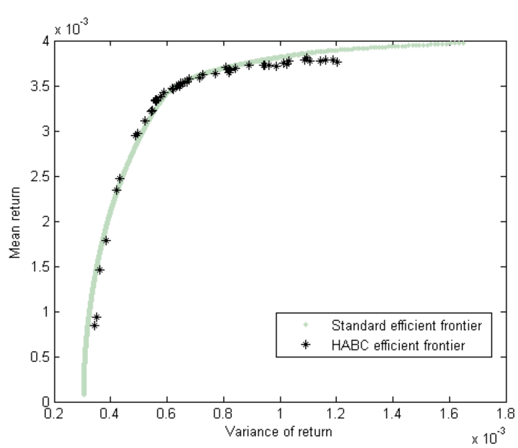

Figure 5. Efficient frontier for Nekkei index

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