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D-STABLE ROBUST RELIABLE CONTROL FOR UNCERTAIN DELTA OPERATOR SYSTEMS

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ABSTRACT

The D-stable robust reliable control for uncertain delta operator systems is mainly studied by this article. It proposes a sufficient condition of placing poles of the closed-loop systems in a specified circular disc, in terms of linear matrix inequalities by using state feedback. It also gives a design procedure of such controllers. The proposed results can also unify related results of continuous and discrete systems. Furthermore, a numerical example is provided to demonstrate the feasibility and the effectiveness of the design method.

Keywords: *Delta Operator, Reliable Control, Linear Matrix Inequality*

1. INTRODUCTION

The stability of linear systems has direct connection with the location of its poles. Therefore, pole assignment is one of the most significant research subjects on linear systems' analysis and synthesis. By using the delta operator to describe discrete system can avoid the trouble when we handle problem of abnormal condition, and overcome the instability of sampling with high speed at the same time. The model of discretization of delta operator tends to continuous system model, and this would enable the controlled studies of continuous system and the discrete systems on the issue come down to study for delta operator systems. Studying on delta operator in China began in the 1990 of the 20th century. Zhang and Yang [1] published the first summary paper of delta operator method. In recent years, the study of delta operator systems control has got a large number of results [2]~[5]. In addition, a lot of achievements have been made in neural network, parameter estimation, and signal processing, and so on.

Reliable control means for any fault of system components (actuator and sensor) that may appear, designs the appropriate controllers, ensures that the system can still operate correctly in the event of fault. D-stability control means placing the poles of the closed-loop system in the specified disk in the complex plane. Both researches possess practical meaning. Since Siljak made reliable control for the first time in the 1970 of the 20th century, the design method of robust controller has moved [6]~[8]

In fact, as long as we assign the poles of the closed-loop systems in the complex plane with an appropriate area, the system will has some dynamic and stabilization characteristics. For instance, step response of second order systems with the poles: $\lambda = \zeta\omega_n \pm j\omega_d$ can be completely specified by natural frequency $\omega_n = |\lambda|$, damping ratio ζ and damped natural frequency ω_d. The ζ, ω_n and ω_d can satisfy some given bounds by limiting λ in the complex plane with an appropriate area. Consequently, we can make the systems have prescribed characteristics.

This article mainly studies the delta operator systems with continuous fault models, and gives the existence condition and design method of the D-stable robust reliable controller of the linear system with actuator continuous fault.

2. PROBLEM FORMULATION

Consider the following uncertain Delta operator system:

$$\delta x(t) = (A + \Delta A)x(t) + (B + \Delta B)u(t) \quad (1)$$

where δ is symbol of Delta, and

$$\delta x(t) = \begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t}, & T = 0 \\ \dfrac{x(t+T) - x(t)}{T}, & T \neq 0 \end{cases}$$

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$x(t) \in R^n$ is the state, $u(t) \in R^p$ is the control input; $A \in R^{n\times n}, B \in R^{n\times p}$ are known constant matrices; ΔA and ΔB are uncertainty of norm bounded type written as:

$$[\Delta A \quad \Delta B] = EF[G_1 \quad G_2] \qquad (2)$$

where E , G_1 , G_2 are known constant matrices with appropriate dimension, and define the structure of the uncertainty. And the parameter uncertainty F satisfy $F^{\mathrm{T}}F \le I$, where I is identity matrix with appropriate dimension.

Using the state feedback, the form of the controller and the continuous fault model respectively are:

$$u(t) = Kx(t) \qquad (3)$$

$$u^f(t) = MKx(t) \qquad (4)$$

Where $M = \mathrm{diag}(m_1, m_2, \cdots, m_n)$ is continuous fault matrix, while $m_i = 0$, means outage of the ith actuator control signal. $m_i = 1$ means the normal operation of ith actuator control signal. When $m_{di} \le m_i \le m_{ui}$, $m_i \ne 1$, partial fault of the ith actuator control signal occurs, where $0 \le m_{di} \le 1 \le m_{ui}$, $i = 1, 2, \cdots n$.

Let

$$M_d = \mathrm{diag}(m_{d,1}, m_{d,2}, \cdots, m_{d,n})$$

$$M_u = \mathrm{diag}(m_{u,1}, m_{u,2}, \cdots, m_{u,n})$$

$$M_0 = \frac{1}{2}(M_u + M_d),\ M_1 = \frac{1}{2}(M_u - M_d)$$

then

$$M = M_0 + M_1^{\frac{1}{2}} \Sigma M_1^{\frac{1}{2}}$$

where

$$\Sigma = \mathrm{diag}(\sigma_1, \sigma_2, \cdots, \sigma_n),\ |\sigma_i| \le 1, i = 1, 2, \cdots, n$$

The system which is composed of (2) and (3) can be described as:

$$\delta x(t) = (A + \Delta A + BK + \Delta BK)x(t) = \bar{A}x(t) \quad (5)$$

where $\bar{A} = A + \Delta A + BK + \Delta BK$.

The system which is composed of (2), (3) and (4) can be described as:

$$\delta x(t) = (A + \Delta A + BMK + \Delta BMK)x(t) = \tilde{A}x(t) \quad (6)$$

where $\tilde{A} = A + \Delta A + BMK + \Delta BMK$.

The purpose of this paper is to determine the state feedback $u(t) = Kx(t)$ such that the poles of the closed loop system (5) and (6) lie in the specified disc.

3. MAIN RESULTS

Lemma 1[9] Let D and E be given real matrices with appropriate dimension. Y is a negative definite symmetric matrix; F is a time varying diagonal matrix with m dimension satisfying $F^{\mathrm{T}}F \le I$, then

$$Y + DFE + E^{\mathrm{T}}F^{\mathrm{T}}D^{\mathrm{T}} < 0$$

for all F, if and only if there exists a positive definite symmetric matrix $U \in R^{m\times m}$, such that

$$Y + DUD^{\mathrm{T}} + E^{\mathrm{T}}U^{-1}E < 0$$

Lemma 2[10] Let X and Y be matrices with appropriate dimension, F is a time varying matrix with appropriate dimension satisfying $F^{\mathrm{T}}F \le I$, then, for any scalar quantity $\varepsilon > 0$, we have

$$XFY + Y^{\mathrm{T}}F^{\mathrm{T}}X^{\mathrm{T}} \le \varepsilon XX^{\mathrm{T}} + \varepsilon^{-1}Y^{\mathrm{T}}Y$$

Lemma 3[11] $\lambda(A) \subset D(a, r)$ if and only if there exists a positive definite matrix X satisfying

$$\begin{bmatrix} -rX & -aX + AX \\ -aX + XA^{\mathrm{T}} & -rX \end{bmatrix} < 0$$

Theorem 1 Let $A \in R^{n\times n}$, then $\lambda(A) \subset D(a, r)$ if and only if there exists a positive definite symmetric matrix $P \in R^{n\times n}$ such that

$$(I + TA_a)^{\mathrm{T}} \frac{P}{T}(I + TA_a) - \frac{P}{T} < 0$$

where $A_a = \dfrac{A - aI}{rT} - \dfrac{1}{T}I = \dfrac{A - (a + r)I}{rT}$

Proof

$$\lambda(A) \subset D(a, r) \Leftrightarrow \lambda(A - aI) \subset D(0, r) \Leftrightarrow$$

$$\lambda\left(\frac{A - aI}{r}\right) \subset D(0,1) \Leftrightarrow \lambda(I + TA_a) \subset D(0,1) \Leftrightarrow$$

lemma 3.

$$\begin{bmatrix} -X & (I + TA_a)X \\ X(I + TA_a)^{\mathrm{T}} & -X \end{bmatrix} < 0$$

By Schur complements lemma, the above inequality is equivalent to:

$$-X + X(I + TA_a)^{\mathrm{T}} X^{-1}(I + TA_a)X < 0$$

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Multiplying X^{-1} on the both sides of the inequality above, and let $P = X^{-1}$, then we get

$$\left(I + TA_a\right)^{\mathrm{T}} P\left(I + TA_a\right) - P < 0$$

which implies

$$(I + TA_a)^{\mathrm{T}} \frac{P}{T}(I + TA_a) - \frac{P}{T} < 0$$

Theorem 2 The delta operator system (5) is said to have a D-stable robust controller, if there exists a positive definite symmetric matrix $X > 0$, a matrix Y and a scalar quantity $\varepsilon > 0$ satisfying the following inequality

$$\begin{bmatrix} -X & \Pi_1^{\mathrm{T}} & \Pi_2^{\mathrm{T}} \\ \Pi_1 & -X + \frac{\varepsilon}{r}EE^{\mathrm{T}} & 0 \\ \Pi_2 & 0 & -\varepsilon I \end{bmatrix} < 0 \quad (7)$$

where $\Pi_1 = X + TA_a X + \frac{BY}{r}$, $\Pi_2 = G_1 X + G_2 Y$

Proof According to theorem 1, we know that the $\lambda\left(\bar{A}\right) \subset D\left(a, r\right)$ if and only if there exists a positive definite symmetric matrix $P \in R^{n\times n}$ such that

$$\left(I + T\bar{A}_a\right)^{\mathrm{T}} \frac{P}{T}\left(I + T\bar{A}_a\right) - \frac{P}{T} < 0$$

where $\bar{A}_a = \dfrac{\bar{A} - \left(a + r\right)I}{rT}$

Using Schur complements lemma; the above inequality is equivalent to:

$$\begin{bmatrix} -P & (I + T\bar{A}_a)^{\mathrm{T}} \\ (I + T\bar{A}_a) & -P^{-1} \end{bmatrix} < 0$$

Multiplying $\mathrm{diag}(P^{-1}, I)$ on both sides of the inequality above, let $X = P^{-1}$, $Y = KX$, it follows from the matrix inequality above that

$$\begin{bmatrix} -X & (\Delta)^{\mathrm{T}} \\ \Delta & -X \end{bmatrix} < 0$$

where $\Delta = X + TA_a + \dfrac{EFG_1 X + BY + EFG_2 Y}{r}$

which implies

$$\begin{bmatrix} -X & \Pi_2^{\mathrm{T}} \\ \Pi_1 & -X \end{bmatrix} +$$

$$\frac{1}{r}\begin{bmatrix} 0 \\ E \end{bmatrix} F\begin{bmatrix} \Pi_2 & 0 \end{bmatrix} + \frac{1}{r}\begin{bmatrix} \Pi_2 & 0 \end{bmatrix}^{\mathrm{T}} F^{\mathrm{T}} \begin{bmatrix} 0 \\ E \end{bmatrix}^{\mathrm{T}} < 0 \quad (8)$$

By lemma 2, for any positive scalar quantity $\varepsilon > 0$, the inequality (8) is equivalent to:

$$\begin{bmatrix} -X & \Pi_1^{\mathrm{T}} \\ \Pi_1 & -X \end{bmatrix} +$$

$$\frac{\varepsilon}{r}\begin{bmatrix} 0 \\ E \end{bmatrix}\begin{bmatrix} 0 \\ E \end{bmatrix}^{\mathrm{T}} + \frac{\varepsilon^{-1}}{r}\begin{bmatrix} \Pi_2 & 0 \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} \Pi_2 & 0 \end{bmatrix} < 0 \quad (9)$$

Then by using Schur complements lemma, the inequality (9) is equivalent to the inequality (7).

Therefore, the controller is obtained, which can ensure $\lambda(\bar{A}) \subset D(a, r)$ and $K = YX^{-1}$.

Theorem 3 The delta operator system (6) has a D-stable robust reliable controller, if there exists a positive definite symmetric matrix X, a diagonal matrix $U > 0$, a matrix Y and a scalar quantity $\varepsilon > 0$ satisfying the inequality (10)

$$\begin{bmatrix} -X & \Theta_{21}^{\mathrm{T}} & \Theta_{31}^{\mathrm{T}} & 0 & \Theta_{51}^{\mathrm{T}} \\ \Theta_{21} & \Theta_{22} & 0 & \Theta_{42}^{\mathrm{T}} & 0 \\ \Theta_{31} & 0 & \frac{\varepsilon}{r}I & \Theta_{43}^{\mathrm{T}} & 0 \\ 0 & \Theta_{42} & \Theta_{43} & -U & 0 \\ \Theta_{51} & 0 & 0 & 0 & -U \end{bmatrix} < 0 \quad (10)$$

where $\Theta_{21} = X + TA_a X + \dfrac{BM_0 Y}{r}$

$$\Theta_{31} = G_1 X + G_2 M_0 Y$$

$$\Theta_{22} = -X + \frac{\varepsilon}{r}EE^{\mathrm{T}}$$

$$\Theta_{42} = UM_1^{\frac{1}{2}}B^{\mathrm{T}} \quad \Theta_{43} = UM_1^{\frac{1}{2}}G_2^{\mathrm{T}} \quad \Theta_{51} = M_1^{\frac{1}{2}}Y$$

Then the control gain matrix is given by $K = YX^{-1}$

Proof According to theorem 1, we know that the $\lambda(\tilde{A}) \subset D(a, r)$ if and only if there exists a positive definite symmetric matrix $P \in R^{n\times n}$ such that

$$(I + T\tilde{A}_a)^{\mathrm{T}} \frac{P}{T}(I + T\tilde{A}_a) - \frac{P}{T} < 0$$

where $\tilde{A}_a = \dfrac{\tilde{A} - \left(a + r\right)I}{rT}$

Using Schur complements lemma; the above inequality is equivalent to:

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$$\begin{bmatrix} -P & (I+T\tilde{A}_a)^{\mathrm{T}} \\ (I+T\tilde{A}_a) & -P^{-1} \end{bmatrix} < 0$$

Multiplying $\mathrm{diag}(P^{-1}, I)$ to the above inequality from the left and right respectively, let $X = P^{-1}$, $Y = KX$, it follows from the matrix inequality above that

$$\begin{bmatrix} -X & \Gamma^{\mathrm{T}} \\ \Gamma & -X \end{bmatrix} < 0$$

where

$$\Gamma = X + TA_a + \frac{EFG_1X + BMY + EFG_2MY}{r}$$

which implies that

$$\begin{bmatrix} -X & \Sigma^{\mathrm{T}} \\ \Sigma & -X \end{bmatrix} + \frac{1}{r}\begin{bmatrix} 0 \\ E \end{bmatrix} F \begin{bmatrix} \Lambda & 0 \end{bmatrix} + \frac{1}{r}\begin{bmatrix} \Lambda & 0 \end{bmatrix}^{\mathrm{T}} F^{\mathrm{T}} \begin{bmatrix} 0 \\ E \end{bmatrix}^{\mathrm{T}} < 0 \quad (11)$$

where $\Sigma = X + TA_aX + \frac{BMY}{r}$, $\Lambda = G_1X + G_2MY$

According to lemma 2, for any positive scalar quantity $\varepsilon > 0$, we have

$$\begin{bmatrix} -X & \Sigma^{\mathrm{T}} \\ \Sigma & -X \end{bmatrix} + \frac{\varepsilon}{r}\begin{bmatrix} 0 \\ E \end{bmatrix}\begin{bmatrix} 0 \\ E \end{bmatrix}^{\mathrm{T}} + \frac{\varepsilon^{-1}}{r}\begin{bmatrix} \Lambda & 0 \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} \Lambda & 0 \end{bmatrix} < 0 \quad (12)$$

From Schur complements lemma, the inequality (12) is equivalent to:

$$\begin{bmatrix} -X & \Sigma^{\mathrm{T}} & \Lambda^{\mathrm{T}} \\ \Sigma & \Theta_{22} & 0 \\ \Lambda & 0 & -\frac{\varepsilon}{r}I \end{bmatrix} < 0 \quad (13)$$

Replacing $M = M_0 + M_1^{\frac{1}{2}}\Sigma M_1^{\frac{1}{2}}$ in (13), then the inequality (13) can be described as:

$$\begin{bmatrix} -X & \Theta_{21}^{\mathrm{T}} & \Theta_{31}^{\mathrm{T}} \\ \Theta_{21} & \Theta_{22} & 0 \\ \Theta_{31} & 0 & -\frac{\varepsilon}{r}I \end{bmatrix} + \begin{bmatrix} 0 \\ BM_1^{\frac{1}{2}} \\ G_2M_1^{\frac{1}{2}} \end{bmatrix}\Sigma\begin{bmatrix} \Theta_{51} & 0 & 0 \end{bmatrix} + \begin{bmatrix} \Theta_{51} & 0 & 0 \end{bmatrix}^{\mathrm{T}}\Sigma^{\mathrm{T}}\begin{bmatrix} 0 \\ BM_1^{\frac{1}{2}} \\ G_2M_1^{\frac{1}{2}} \end{bmatrix}^{\mathrm{T}} < 0 \quad (14)$$

From lemma 1, there exists a positive definite symmetric matrix $U \in R^{m\times m}$, such that

$$\begin{bmatrix} -X & \Theta_{21}^{\mathrm{T}} & \Theta_{31}^{\mathrm{T}} \\ \Theta_{21} & \Theta_{22} & 0 \\ \Theta_{31} & 0 & -\frac{\varepsilon}{r}I \end{bmatrix} + \begin{bmatrix} 0 \\ BM_1^{\frac{1}{2}} \\ G_2M_1^{\frac{1}{2}} \end{bmatrix} UU^{-1}U \begin{bmatrix} 0 \\ BM_1^{\frac{1}{2}} \\ G_2M_1^{\frac{1}{2}} \end{bmatrix}^{\mathrm{T}} + \begin{bmatrix} \Theta_{51} & 0 & 0 \end{bmatrix}^{\mathrm{T}} U^{-1} \begin{bmatrix} \Theta_{51} & 0 & 0 \end{bmatrix} < 0 \quad (15)$$

by Schur complements lemma, the inequality (15) is equivalent to the inequality (10), the proof is completed. Therefore, the controller is obtained, which can ensure $\lambda(\tilde{A}) \subset D(a,r)$ and $K = YX^{-1}$.

Remark As the model of discretization of delta operator tends to continuous system model when the sampling interval $T = 0$, and then this would enable the controlled studies of continuous systems and the discrete systems on the issue boil down to study for delta operator systems. Therefore, the theorems in this paper are suitable for both continuous system and discrete system.

4. NUMERICAL EXAMPLE

Consider the delta operator system (5) and (6) with parameters:

$$A = \begin{bmatrix} -1.6 & 0.5 & -1.8 \\ 0.6 & -2.5 & 0.3 \\ 0 & 0.3 & -2 \end{bmatrix}, B = \begin{bmatrix} 0.5 & -0.1 & 0.7 & 0.3 \\ 0.2 & -1.1 & 0.4 & 0.6 \\ 0.7 & -0.2 & 1.3 & 0.9 \end{bmatrix}$$

$$E = \begin{bmatrix} 0.2 & -0.1 \\ -0.2 & 0.3 \\ 0.1 & -0.2 \end{bmatrix}, G_1 = \begin{bmatrix} -0.9 & 0.1 & 0.5 \\ 6 & -0.3 & 0.8 \end{bmatrix}$$

$$G_2 = \begin{bmatrix} 0.4 & 0.5 & -0.7 & 0.2 \\ -0.1 & 0.8 & 0.3 & 0.9 \end{bmatrix}$$

$$M_0 = \mathrm{diag}(0.65, 0.85, 0.55, 0.9)$$

$$M_1 = \mathrm{diag}(0.45, 0.45, 0.45, 0.3) \quad x_0 = [1,1,1]^{\mathrm{T}}$$

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The circular region is chosen to be $D(a,r) = D(3,2)$ and the sampling interval $T = 0.2$. By computing, we can obtain

$$X = \begin{bmatrix} 4.6867 & 1.5663 & 5.8136 \\ 1.5663 & 16.0118 & 2.9055 \\ 5.8136 & 2.9055 & 7.7683 \end{bmatrix}$$

$$Y = \begin{bmatrix} 22.6307 & -2.2102 & 28.4492 \\ -7.2924 & -2.9898 & -6.9251 \\ -0.4807 & -0.6733 & 0.3807 \\ -24.2081 & -2.0117 & -33.5490 \end{bmatrix}$$

$$\varepsilon = 10.2480$$

$$K = \begin{bmatrix} 2.6241 & -0.7541 & 1.9805 \\ -6.8331 & -0.3052 & 4.3364 \\ -2.5676 & -0.1593 & 2.0301 \\ 4.2695 & 0.8799 & -7.8430 \end{bmatrix}$$

for system (5), and we also get

$$X = \begin{bmatrix} 200.6513 & 30.7523 & 92.3671 \\ 30.7523 & 165.5072 & 19.7571 \\ 92.3671 & 19.7571 & 109.3019 \end{bmatrix}$$

$$Y = \begin{bmatrix} 24.6513 & -15.0061 & -0.8992 \\ -21.8165 & 9.1494 & -6.8921 \\ -29.6271 & -10.2168 & -28.4365 \\ -105.2655 & -47.4640 & -144.0314 \end{bmatrix}$$

$$U = \text{diag}(159.2589, 16.4131, 75.7912, 138.3129)$$

$$\varepsilon = 445.2022$$

$$K = \begin{bmatrix} 0.2209 & -0.1108 & -0.1749 \\ -0.1398 & 0.0763 & 0.0413 \\ -0.0427 & -0.0276 & -0.2191 \\ 0.1515 & -0.1455 & -1.4195 \end{bmatrix}$$

for system (6).

Fig 1 shows the pole distribution of delta operator system (5) with D-stable robust controller. All of its poles are in the specified circular disc.

Fig 2 shows the pole distribution of delta operator system (5) with actuator continuous fault. The D-stable robust controller is out of work in this condition. The poles of system (5) are partly out of the specified circular disc.

Fig 3 shows the pole distribution of delta operator system (6) with D-stable robust reliable controller.

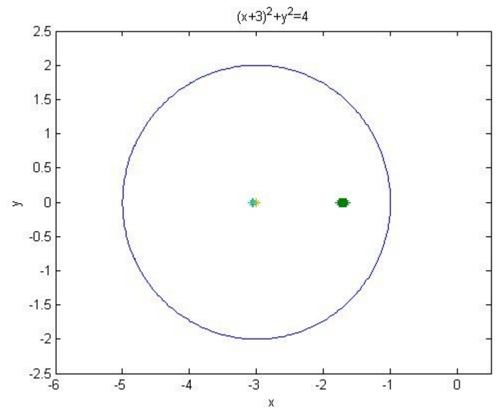

Fig 1. The Pole Distribution Of System (5)

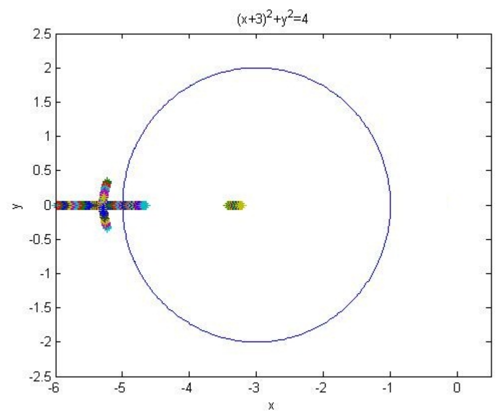

Fig 2. The Pole Distribution Of Delta Operator System (5) With Actuator Continuous Fault

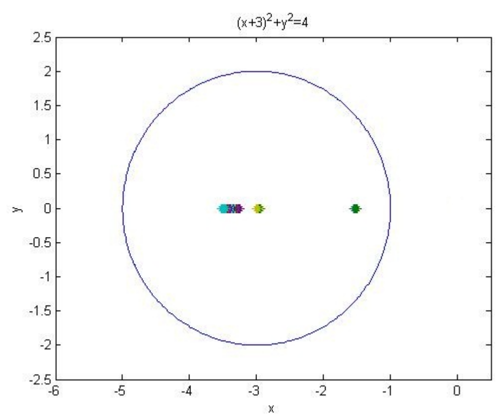

Fig 3. The Pole Distribution Of System (6)

5. CONCLUSION

Generally, in a practical system design, designers want the system as reliable as possible. And this paper gives a method to design the D-stable robust reliable controller of the linear system with actuator continuous fault. As the model of discretization of

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delta operator tends to continuous system model, the controller designed by this paper is suitable for both discrete systems and continuous systems. Finally, an example shows the effectiveness and the feasibility of this method.

REFERENCES

[1] Zhang D J and Yang C W, 2000. Delta operator theory for feed-back control systems: A survey. Control Theory & Applications, 15(2): 153-160.

[2] ZHANG Duan-jin, WANG Zhong-yong and WU Jie, 2003. Survey on system control and signal processing using the delta operator. Control and Decision, 18(4): 385-391.

[3] LIU Man, JING Yuan –wei and ZHANG Si-ying, 2004. D-Stable Robust Fault-Tolerant Control for Delta Operator Systems. Journal of Northeastern University(Natural Science), 25(8): 715-719.

[4] Yan Junrong, Guo XIjin and Gui Xia, 2006. Robust Control of Pole Restricted in Circular Region for Delta Operator System. Journal of Nanjing University of Aeronautics & Astronautics, 38(Suppl): 42-44.

[5] Xiao Min-qing, 2009. Robust D-stabilization for Delta operator systems with sensor failure. Control Theory & Applications, 26(2): 183-185.

[6] Veillette R J, Medanic J V and Perkins W R, 1992. Design of reliable control system. IEEE Trans Automat Control, 37(2): 770-784.

[7] Yang G H, Wang J L and Soh Y C, 2000 Reliable LQG control with sensor failures IEEE Proceedings-Part D, Control Theory and Applications, 47: 2188-2192.

[8] Yang G H, Wang J L and Soh Y C, 2001. Reliable H_∞ design for linear system. Automatica, 37: 717-725.

[9] Xiao Minqing, 2008. An Unified LMI Approach to Reliable D-Stabilization Controller Design for Linear Systems. Proceedings of the 27th Chinese Control Conference, pp. 36-40.

[10] I.R. Petersen, 1987. A stabilization algorithm for a class of uncertain linear Systems. Syst.Contr. Let, 8: 351-357.

[11] Chilali M and Gahinet P, 1996 . H_∞ design with pole placement constraints: an LMI approach. IEEE Trans. Automat Control, 41: 358-367.