Journal of Theoretical and Applied Information Technology
28th February 2013. Vol. 48 No.3

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

POLE RELIABLE ASSIGNMENT OF PARABOLIC REGION WITH ACTUATOR FAILURE

[1]BO YAO, [2]JIAN RONG, [3]HAO HU

[1]College of Mathematics and System Science, Shenyang Normal University, Shenyang, 110034, P.R. China

[2]College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing, 210016 P.R. China,

ABSTRACT

For a class of uncertain linear systems, this paper proposes the mixed fault model which is more general and more practical. This paper discusses the existing problems of the system reliable controller, considers the reliable pole assignment of parabolic region with actuator fault, and gives a new method to deal with the mixed fault matrix. For mixed fault model with actuator fault, it gives the sufficient condition of designing this type of controller which can make the pole of linear systems to be located in the parabolic region. It achieves the design of state feedback reliable controller by solving the LMI. The simulation proves the Design method in this paper is feasible.

Keywords: *Mixed Fault Model; Pole Assignment; Actuator Fault; Parabolic Region*

1. INTRODUCTION

The minimum requirement of a control system is the stability. A good controller not only transfers information fast, but also has a good response of damping events. A traditional method which can ensure system have satisfactory response, is that the poles of closed-loop system could be seated in left complex plane[1,2], and this method is called regional pole assignment. The main purpose of regional pole assignment: In the system analysis and design, the stability of the system should be considered first of all, and the instantaneous response of the linear system should be closely related with the poles' position. It guarantees system have certain dynamic and steady-state performance, as long as the poles of the closed-loop system assign in a proper region of the complex plane. M.Chilali and P.Gahinet put forward "LMI region" firstly at 1996, afterwards, with the development of reliable control, regional pole assignment theories go deep into the reliable control gradually, and set up regional pole reliable assignment.

In fact, accurate poles assignments are not needed, it is satisfied, as long as the pole of the closed-loop system assign in a specified position of complex plane. In recent years, the pole assignment theories of different areas are very active, for example, the pole assignment of sector region[4], circular disc region [3,6] and hyperbolic region. In fact, we can promote pole assignment of vertical or horizontal zone region to parabolic region. There is not much research about parabolic region.

Pole assignment of linear system is an important method of controller design. [3]discusses the pole assignment of circular disc with the actuator failure for linear system; [4]introduces the pole assignment of sector region for linear system. In view of the importance of regional pole assignment, it will obtain a very good development.

Discrete failure model was created by R.J.Veillette[5].[6]discusses the pole assignment of circular disc with discrete failure model for linear system. Continuous failure model was created by Yang[7];[8]discusses the robust pole assignment of circular disc for continuous interval system. But in the practical problems, some systems are not singly appeared discrete fault or continuous fault, that is to say, systems often exist both discrete fault and continuous fault at the same time. Mixed failure model appears in[9] first. So what this paper studies is to design the controller with mixed failure model to make the conservative smaller when design the reliable controller. Finally, it uses the LMI to

Journal of Theoretical and Applied Information Technology
28th February 2013. Vol. 48 No.3

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

complete the reliable controller design with mixed failure model.

2. MATERIALS AND METHODS

Consider an uncertain linear system of the form

$$\dot{x}(t) = (A+\Delta A)x(t) + (B+\Delta B)u(t) \qquad (1)$$

where $x(t) \in \mathrm{R}^n$ is the system state; $u(t) \in \mathrm{R}^m$ is the actuator failure control input； $A \in \mathrm{R}^{n\times n}$, $B \in \mathrm{R}^{n\times m}$ are known constant matrices of appropriate dimensions； ΔA、ΔB express the nondeterminacy of the system with the forms as follows :

$$[\Delta A \;\; \Delta B] = DH[E_1 \;\; E_2]$$

where D, E_1, E_2 are known constant matrices of appropriate dimensions， H is an uncertain constant matrix of appropriate dimension, which satisfies with $H^{\mathrm{T}}H \le I$, I is unit matrix.

A feedback control with the feedback gain matrix K :

$$u(t) = Kx(t) \qquad (2)$$

Similarly, the actuator failure model is adopted as follows:

$$u^f(t) = Fu(t) \qquad (3)$$

where $u^f(t) \in \mathrm{R}^m$ is the actuator failure control input , F is actuator failure model with the form as follow:

$$F = \mathrm{diag}\left(n_1,\cdots,n_p,m_{p+1}\cdots,m_n\right) \qquad (4)$$

Let (4) be defined as follow:

$$F = N_i + M \qquad (5)$$

where $N_i = \mathrm{diag}\left(n_1,n_2,\cdots n_p,0,\cdots,0\right)$ is called discrete failure matrix, $M = \mathrm{diag}\left(0,\cdots,0,m_{p+1},m_{p+2},\cdots,m_n\right)$ is called continuous failure matrix.

We find that the actuator failure matrix is composed of two parts, a discrete failure matrix N_i and continuous failure matrix M . This model is called mixed failure model.

For discrete failure matrix N_i , if $n_j = 0$, it means the complete failure of the j th actuator control signal; if $n_j = 1$, it means normal operation of the j th actuator control signal, $j = 1,2,\cdots,p$.

For continuous failure matrix M , where $0 \le m_{di} \le m_i \le m_{ui}$, $i = 1,2,\cdots,n$ with $m_{di} \le 1$ and $m_{ui} \ge 1$,if $m_i = 0$,it means the complete failure of the i th actuator control signal; if $m_i = 1$,it means normal operation of the i th actuator control signal; if $0 \le m_{di} \le m_i \le m_{ui}$, $m_{ui} \ge 1$ and $m_i \ne 1$,it corresponds to the case which partial failure of the i th control signal.

Introduce the following notations:

$$M_u = \mathrm{diag}\left(0,\cdots,0,m_{u,p+1},\cdots,m_{un}\right)$$
$$M_d = \mathrm{diag}\left(0,\cdots,0,m_{d,p+1},\cdots,m_{dn}\right)$$

$$M_0 = \frac{1}{2}\left(M_u + M_d\right), M_1 = \frac{1}{2}\left(M_u - M_d\right)$$

$$F_0 = N_i + M_0$$

Then we have

$$F = F_0 + M_1\Sigma \;\; , \;\; |\Sigma| \le I \qquad (6)$$

where $\Sigma = \mathrm{diag}\left(\sigma_1,\sigma_2,\cdots\sigma_n\right)$, $i = 1,2,\cdots n$,and the uncertain diagonal matrix Σ stands for all the diagonal matrix which can satisfy with $\Sigma^{\mathrm{T}}\Sigma \le I$.

3. RESULTS AND DISCUSSIONS

Definition [10] For a region D in the complex plane, if there exist a symmetric matrix $L \in \mathrm{R}^{n\times n}$ and a matrix $M \in \mathrm{R}^{n\times n}$,which can make

$$D(l) = \left\{z \in C : L + zM + \overline{z}M^{\mathrm{T}} < 0\right\}$$

then D is called linear matrix inequality region (LMI region).

Lemma 1[11] Let D be LMI region, and $A \in \mathrm{R}^{n\times n}$, the necessary and sufficient condition of $\lambda(A) \subset D$ is that there exists a positive definite symmetric matrix $X \in \mathrm{R}^{n\times n}$, which makes the following inequality holds:

$$L \otimes X + M \otimes (AX) + M^{\mathrm{T}} \otimes (AX)^{\mathrm{T}} < 0$$

where $\otimes$ is the symbol of Kronecker product.

Theorem 1 Set $A \in \mathrm{R}^{n\times n}$, there exists $\lambda(A) \subset D$, if there exists a positive definite symmetric

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

matrix $X \in \mathrm{R}^{n\times n}$, it makes the following inequality holds:

$$\begin{pmatrix} AX + XA^{\mathrm{T}} + 2aX & -AX + XA^{\mathrm{T}} \\ AX - XA^{\mathrm{T}} & -\frac{2}{b}X \end{pmatrix} < 0$$

Proof Considering the parabola l of complex plane

$$x + a = -by^2$$

where $a, b > 0$ are known constant.

The algebraic expression of parabolic region $D(l)$ is:

$$D(l) = \left\{ z = x + \mathrm{i}y : x + a < -by^2, x, y \in \mathrm{R} \right\}$$

By $z = x + \mathrm{i}y$ and $\overline{z} = x - \mathrm{i}y$, we can get

$$x = \frac{z + \overline{z}}{2}, \; y = \frac{z - \overline{z}}{2\mathrm{i}}$$

Therefore parabolic region $D(l)$ can be described as

$$D(l) = \left\{ z \in \mathrm{C} : (z + \overline{z}) + 2a - \frac{b}{2}(z - \overline{z})^2 < 0 \right\}$$

As $(z + \overline{z}) + 2a - \frac{b}{2}(z - \overline{z})^2 < 0$ is equivalent to

$$\begin{pmatrix} z + \overline{z} + 2a & -\mathrm{i}(z - \overline{z}) \\ \mathrm{i}(z - \overline{z}) & -\frac{b}{2} \end{pmatrix} < 0 \qquad (7)$$

Let $\begin{pmatrix} \mathrm{i} & 0 \\ 0 & 1 \end{pmatrix}$ and $\begin{pmatrix} -\mathrm{i} & 0 \\ 0 & 1 \end{pmatrix}$ left multiply and right multiply matrix inequality (7) respectively, then (7) is equivalent to

$$\begin{pmatrix} z + \overline{z} + 2a & -z + \overline{z} \\ z - \overline{z} & -\frac{b}{2} \end{pmatrix} < 0 \qquad (8)$$

Let

$$L = \begin{pmatrix} 2\mathrm{a} & 0 \\ 0 & -\frac{2}{b} \end{pmatrix}, M = \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix} \qquad (9)$$

Then matrix inequality (8) can be described as:

$$L + zM + \overline{z}M^{\mathrm{T}} < 0$$

Because of the definition of LMI region, we know parabolic region $D(l)$ is LMI region.

By Lemma 1, there exists a positive definite symmetric matrix $X \in \mathrm{R}^{n\times n}$, which makes the following inequality holds:

$$L \otimes X + M \otimes (AX) + M^{\mathrm{T}} \otimes (AX)^{\mathrm{T}} < 0$$

Then

$$\begin{pmatrix} 2aX & 0 \\ 0 & -\frac{2}{b}X \end{pmatrix} + \begin{pmatrix} AX & -AX \\ AX & 0 \end{pmatrix} + \begin{pmatrix} (AX)^{\mathrm{T}} & (AX)^{\mathrm{T}} \\ -(AX)^{\mathrm{T}} & 0 \end{pmatrix} < 0$$

So we can get

$$\begin{pmatrix} AX + XA^{\mathrm{T}} + 2aX & -AX + XA^{\mathrm{T}} \\ AX - XA^{\mathrm{T}} & -\frac{2}{b}X \end{pmatrix} < 0$$

This paper is in order to control a state feedback

$$u(t) = Kx(t)$$

for all the pole of the closed-loop system with the form as follow:

$$\dot{x}(t) = \left[(A + \Delta A) + (B + \Delta B)K \right] x(t) \qquad (10)$$

$$\dot{x}(t) = \left[(A + \Delta A) + (B + \Delta B)FK \right] x(t) \qquad (11)$$

are seated in parabolic LMI region.

$$D(l) = \left\{ z \in C : L + zM + \overline{z}M^{\mathrm{T}} < 0 \right\}$$

Lemma 2[12] X, Y are known constant matrices with appropriate dimensions, for any constant $\varepsilon > 0$, the following inequality holds:

$$XFY + Y^{\mathrm{T}}F^{\mathrm{T}}X^{\mathrm{T}} \le \varepsilon XX^{\mathrm{T}} + \varepsilon^{-1}Y^{\mathrm{T}}Y$$

where $F^{\mathrm{T}}F \le I$.

Lemma 3[13] Let F, E and Σ be real matrices of appropriate dimensions, and $\Sigma = \mathrm{diag}(\sigma_1, \sigma_2, \cdots \sigma_r)$ with $\sigma_i^{\mathrm{T}}\sigma_i \le I$, $i = 1, 2, \cdots, r$. Then, for any real matrix $\Lambda = \mathrm{diag}(\lambda_1 I, \lambda_2 I, \cdots, \lambda_r I) > 0$, the following inequality holds:

$$F\Sigma E + E^{\mathrm{T}}\Sigma^{\mathrm{T}}F^{\mathrm{T}} \le F\Lambda F^{\mathrm{T}} + E^{\mathrm{T}}\Lambda^{-1}E$$

Considering system (1), when the actuator works normally, we have the following theorem:

Theorem 2 All the pole of the closed-loop system (10) will be seated in parabolic region $D(l)$, if there exists a positive definite symmetric matrix

Journal of Theoretical and Applied Information Technology
28th February 2013. Vol. 48 No.3

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

$X \in R^{n\times n}$ and a matrix $Y \in R^{m\times n}$, which can make the following inequality holds:

$$\begin{pmatrix} \Psi+\Psi^{T}+2aX+(\delta_1+\delta_2)DD^{T} & \Psi^{T}-\Psi+\delta_1 DD^{T} & \Xi^{T} & 0 \\ \Psi-\Psi^{T}+\delta_1 DD^{T} & \delta_1 DD^{T}-\frac{2}{b}X & 0 & \Xi^{T} \\ \Xi & 0 & -\delta_1 I & 0 \\ 0 & \Xi & 0 & -\delta_2 I \end{pmatrix}<0$$

where $\Psi = AX + BY$, $\Xi = E_1X + E_2Y$ and the state feedback gain matrix is $K = YX^{-1}$.

Proof From Theorem 1, all the pole of the closed-loop system (10) will be seated in parabolic region $D(l)$, if there exists a positive definite symmetric matrix $X \in R^{n\times n}$, which makes the following inequality holds:

$$\begin{pmatrix} \Psi+\Psi^{T}+2aX & \Psi^{T}-\Psi \\ \Psi-\Psi^{T} & -\frac{2}{b}X \end{pmatrix}+$$

$$\begin{pmatrix} D \\ D \end{pmatrix} H\left(E_1X+E_2Y,0\right)+\left(E_1X+E_2Y,0\right)^{T} H^{T}\begin{pmatrix} D \\ D \end{pmatrix}^{T}+$$

$$\begin{pmatrix} -D \\ 0 \end{pmatrix} H\left(0,E_1X+E_2Y\right)+\left(0,E_1X+E_2Y\right)^{T} H^{T}\begin{pmatrix} -D \\ 0 \end{pmatrix}^{T}<0$$

(12)

where $Y = KX$, $\Psi = AX + BY$.

From lemma 2, there exists $\delta_1,\delta_2 > 0$, inequality (12) can be included in the following inequality

$$\begin{pmatrix} \Psi+\Psi^{T}+2aX & \Psi^{T}-\Psi \\ \Psi-\Psi^{T} & -\frac{2}{b}X \end{pmatrix}+$$

$$\delta_1\begin{pmatrix} D \\ D \end{pmatrix}\begin{pmatrix} D \\ D \end{pmatrix}^{T}+\delta_1^{-1}\left(E_1X+E_2Y,0\right)^{T}\left(E_1X+E_2Y,0\right)+$$

$$\delta_2\begin{pmatrix} -D \\ 0 \end{pmatrix}\begin{pmatrix} -D \\ 0 \end{pmatrix}^{T}+\delta_2^{-1}\left(0,E_1X+E_2Y\right)^{T}\left(0,E_1X+E_2Y\right)<0$$

(13)

Using the Schur complement, matrix inequality (13) is equivalent to the conclusion of Theorem2. And it is obvious that the state feedback gain matrix is $K = YX^{-1}$ by the process of proof above.

Considering the system (1) with actuator failure, we have the following theorem:

Theorem 3 All the pole of the closed-loop system (11) will be seated in parabolic region $D(l)$, if there exists a positive definite symmetric matrix $X \in R^{n\times n}$, a matrix $Y \in R^{m\times n}$ and positive definite symmetric matrices $U,V \in R^{m\times m}$, which make the following inequality holds:

$$\begin{pmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} \\ \Phi_{12}^{T} & \Phi_{22} & \Phi_{23} & \Phi_{24} \\ \Phi_{13}^{T} & \Phi_{23}^{T} & \Phi_{33} & 0 \\ \Phi_{14}^{T} & \Phi_{24}^{T} & 0 & \Phi_{44} \end{pmatrix}<0$$

where

$$\Phi_{11}=\begin{pmatrix} \Pi+\Pi^{T}+(\varepsilon_1+\varepsilon_2)DD^{T}+2aX & \Pi^{T}-\Pi++\varepsilon_1 DD^{T} \\ \Pi-\Pi^{T}+\varepsilon_1 DD^{T} & \varepsilon_1 DD^{T}-\frac{2}{b}X \end{pmatrix}$$

$$\Phi_{12}=\begin{pmatrix} X^{T}E_1^{T}+Y^{T}F_0^{T}E_2^{T} & 0 \\ 0 & X^{T}E_1^{T}+Y^{T}F_0^{T}E_2^{T} \end{pmatrix},$$

$$\Phi_{13}=\begin{pmatrix} BM_1U & Y^{T} \\ BM_1U & 0 \end{pmatrix} \quad \Phi_{14}=\begin{pmatrix} BM_1V & 0 \\ 0 & -Y^{T} \end{pmatrix},$$

$$\Phi_{22}=\begin{pmatrix} -\varepsilon_1 I & 0 \\ 0 & -\varepsilon_2 I \end{pmatrix}, \quad \Phi_{23}=\begin{pmatrix} E_2M_1U & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Phi_{24}=\begin{pmatrix} 0 & 0 \\ -E_2M_1V & 0 \end{pmatrix}, \quad \Phi_{33}=\begin{pmatrix} -U & 0 \\ 0 & -U \end{pmatrix},$$

$$\Phi_{44}=\begin{pmatrix} -V & 0 \\ 0 & -V \end{pmatrix}$$

$$\Pi = AX + BF_0Y$$

and the state feedback gain matrix is $K = YX^{-1}$.

Proof From Theorem 1, all the pole of the closed-loop system (11) will be seated in parabolic region $D(l)$, if there exists a positive definite symmetric matrix $X \in R^{n\times n}$ .And let $Y = KX$, we can get

$$\begin{pmatrix} AX+BFY+XA^{T}+Y^{T}F^{T}B^{T}+2aX & XA^{T}+Y^{T}F^{T}B^{T}-AX-BFY \\ AX+BFY-XA^{T}-Y^{T}F^{T}B^{T} & -\frac{2}{b}X \end{pmatrix}+$$

$$\begin{pmatrix} D \\ D \end{pmatrix} H\left(E_1X+E_2FY,0\right)+\left(E_1X+E_2FY,0\right)^{T} H^{T}\begin{pmatrix} D \\ D \end{pmatrix}^{T}+$$

$$\begin{pmatrix} -D \\ 0 \end{pmatrix} H\left(0,E_1X+E_2FY\right)+\left(0,E_1X+E_2FY\right)^{T} H^{T}\begin{pmatrix} -D \\ 0 \end{pmatrix}^{T}<0$$

(14)

From lemma 2, there exist $\varepsilon_1,\varepsilon_2 > 0$, inequality (14) can be included in the following inequality:

Journal of Theoretical and Applied Information Technology
28th February 2013. Vol. 48 No.3

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

$$\begin{pmatrix} AX+BFY+XA^{T}+Y^{T}F^{T}B^{T}+2aX & XA^{T}+Y^{T}F^{T}B^{T}-AX-BFY \\ AX+BFY-XA^{T}-Y^{T}F^{T}B^{T} & -\frac{2}{b}X \end{pmatrix}+$$

$$\varepsilon_1\begin{pmatrix}D\\D\end{pmatrix}\begin{pmatrix}D\\D\end{pmatrix}^{T}+\varepsilon_1^{-1}\left(E_1X+E_2FY,0\right)^{T}\left(E_1X+E_2FY,0\right)+$$

$$\varepsilon_2\begin{pmatrix}-D\\0\end{pmatrix}\begin{pmatrix}-D\\0\end{pmatrix}^{T}+\varepsilon_2^{-1}\left(0,E_1X+E_2FY\right)^{T}\left(0,E_1X+E_2FY\right)<0$$

(15)

Using the Schur complement and (6), matrix inequality (15) is equivalent to can be describe as

$$\begin{pmatrix} \Pi+\Pi^{T}+(\varepsilon_1+\varepsilon_2)DD^{T}+2aX & \Pi^{T}-\Pi+\varepsilon_1DD^{T} & X^{T}E_1^{T}+Y^{T}F_0^{T}E_2^{T} & 0 \\ \Pi-\Pi^{T}+\varepsilon_1DD^{T} & -\frac{2}{b}X+\varepsilon_1DD^{T} & 0 & X^{T}E_1^{T}+Y^{T}F_0^{T}E_2^{T} \\ E_1X+E_2F_0Y & 0 & -\varepsilon_1I & 0 \\ 0 & E_1X+E_2F_0Y & 0 & -\varepsilon_2I \end{pmatrix}+$$

$$\begin{pmatrix}BM_1\\BM_1\\E_2M_1\\0\end{pmatrix}\Sigma\left(Y,0,0,0\right)+\left(Y,0,0,0\right)^{T}\Sigma^{T}\begin{pmatrix}BM_1\\BM_1\\E_2M_1\\0\end{pmatrix}^{T}+$$

$$\begin{pmatrix}BM_1\\0\\0\\-E_2M_1\end{pmatrix}\Sigma\left(0,-Y,0,0\right)+\left(0,-Y,0,0\right)^{T}\Sigma^{T}\begin{pmatrix}BM_1\\0\\0\\-E_2M_1\end{pmatrix}^{T}<0 \quad (16)$$

where $\Pi = AX + BF_0Y$.

From lemma 3, there exist positive definite symmetric matrices $U,V \in R^{m\times m}$ which make inequality (16) be included in the following inequality:

$$\begin{pmatrix} \Pi+\Pi^{T}+(\varepsilon_1+\varepsilon_2)DD^{T}+2aX & \Pi^{T}-\Pi+\varepsilon_1DD^{T} & X^{T}E_1^{T}+Y^{T}F_0^{T}E_2^{T} & 0 \\ \Pi-\Pi^{T}+\varepsilon_1DD^{T} & -\frac{2}{b}X+\varepsilon_1DD^{T} & 0 & X^{T}E_1^{T}+Y^{T}F_0^{T}E_2^{T} \\ E_1X+E_2F_0Y & 0 & -\varepsilon_1I & 0 \\ 0 & E_1X+E_2F_0Y & 0 & -\varepsilon_2I \end{pmatrix}$$

$$+\begin{pmatrix}BM_1\\0\\0\\-E_2M_1\end{pmatrix}V\begin{pmatrix}BM_1\\0\\0\\-E_2M_1\end{pmatrix}^{T}+\left(0,-Y,0,0\right)^{T}V^{-1}\left(0,-Y,0,0\right)$$

$$+\begin{pmatrix}BM_1\\BM_1\\E_2M_1\\0\end{pmatrix}U\begin{pmatrix}BM_1\\BM_1\\E_2M_1\\0\end{pmatrix}^{T}+\left(Y,0,0,0\right)^{T}U^{-1}\left(Y,0,0,0\right)<0 \quad (17)$$

Using the Schur complement, we conclude matrix inequality (17) is equivalent to the conclusion of Theorem 3. And it is obvious that the state feedback gain matrix is $K = YX^{-1}$ by the process of proof above.□

4. NUMERICAL EXAMPLE

In this section, we present a numerical example for actuator failure to illustrate the proposed design method.

Consider the linear system (1) with the following parameters

$$A=\begin{pmatrix}-0.9 & 1.7 & 2.2\\ 0.5 & -3.7 & -1.6\\ 2.5 & -1.6 & -4.2\end{pmatrix},\ B=\begin{pmatrix}-0.2 & -1 & 0.3\\ -3.5 & 0.8 & -1.6\\ -5 & -1.8 & -1.5\end{pmatrix}$$

$$D=\begin{pmatrix}0.1 & 0 & 0.2\\ 0 & 0.15 & 0.1\\ 0 & 0 & 0.4\end{pmatrix},\ E_1=\begin{pmatrix}0.5 & 0 & 0\\ 0 & 0.5 & 0\\ 0 & 0 & 0.5\end{pmatrix}$$

$$E_2=\begin{pmatrix}0.4 & 0.1 & 0.2\\ 0.2 & 0.5 & 0.2\\ 0.6 & 0.6 & 1\end{pmatrix},\ f_{u3}=2.4,\ f_{d3}=0.8$$

$$a=0.2,\ b=0.5 \quad (18)$$

The poles of the system which is determined by (19) set for $\{-6.6510, 0.2322, -2.3812\}$, so nominal system is unstable.

(I) When the actuator of system (18) works normally, according to theorem 2, the reliable control gain matrix is:

$$K=\begin{pmatrix}0.4560 & 0.0246 & 0.4777\\ 0.4910 & 0.5061 & 0.5133\\ -0.6881 & -0.6435 & -1.3519\end{pmatrix}$$

By the reliable controller which designed in theorem 2, the pole of closed-loop system (10) will be seated in the region, as in Figure 1:

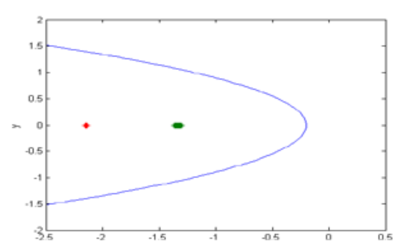

Fig 1

(II) When The Actuator Of System (18) Has Failure, Let Discrete Fault Matrix

$N_1 = \mathrm{diag}(1,0,0)$, $N_2 = \mathrm{diag}(0,1,0)$, $N_3 = \mathrm{diag}(1,1,0)$

Journal of Theoretical and Applied Information Technology
28th February 2013. Vol. 48 No.3

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

By the controller which designed in theorem 2, the pole of closed-loop system (11) will not be seated in the region, as in Figure 2, 3, 4:

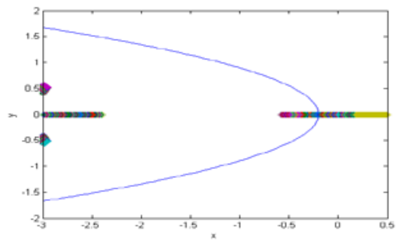

Fig 2. For Closed-Loop System (11), Distribution Of The Pole With $N = N_1$

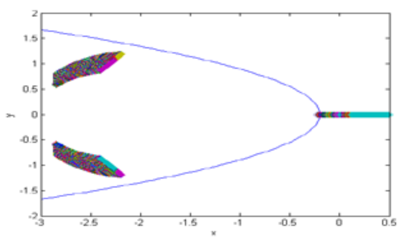

Fig 3. For Closed-Loop System (11), Distribution Of The Pole With $N = N_2$

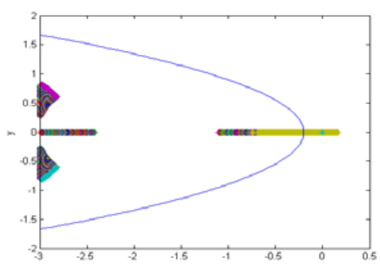

Fig 4. For Closed-Loop System (11), Distribution Of The Pole With $N = N_3$

(III) When the actuator of system (18) has failure, according to theorem 3, the reliable control gain matrix is:

$$K = \begin{pmatrix} 0.5293 & 0.3573 & 0.5982 \\ 1.3324 & 0.2401 & 0.7605 \\ 0.1673 & -0.0393 & -0.1003 \end{pmatrix}$$

The pole of closed-loop system (11) will be seated in the region, as in Figure 5, 6, 7:

Simulation results show that, when uncertain linear system (1) does not have failure, it is stable by the controller designed in theorem 2. However when system (1) has actuator failure, it is unstable

as the controller designed in theorem 2. But the reliable controller with mixed failure model which designed by theorem 3 will have a good and stable performance, not only when all control components are operational, but also in case of some admissible control component outages. It ensures the system's asymptotic stability in the event of actuator failure.

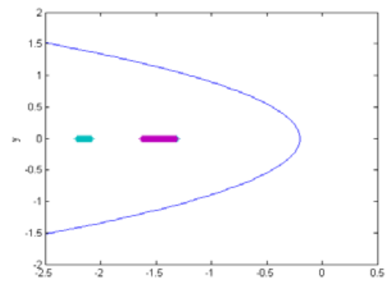

Fig 5. For Closed-Loop System (11), Distribution Of The Pole With $N = N_1$

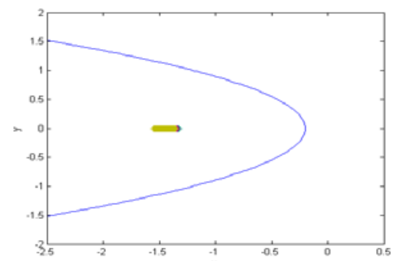

Fig 6. For Closed-Loop System (11), Distribution Of The Pole With $N = N_2$

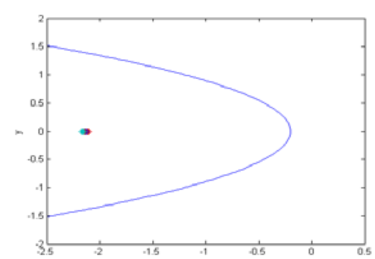

Fig 7. For Closed-Loop System (11), Distribution Of The Pole With $N = N_3$

5. CONCLUSION

In this paper, we have considered the reliable design problem for linear system with actuator failure. A more general and practical mixed failure model of actuator failures is adopted, and gives the sufficient condition for existence of a reliable controller. By solving the LMI, it proposes a reliable controller design method. A numerical example is also given to illustrate the design procedures and their effectiveness.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

REFERENCES

[1] B.C.Kuo, 1982. Automatic Control Systems. Englewood Cliffs, NJ: Prentice-Hall.

[2] Hu Hao and Yao Bo,2011.Robust pole assignment of circular disc for a class of continuous interval system. Computing Technology and Automation, 30(2):17-20.

[3] Hu Hao,Yao Bo and Huang Shan,2011.Robust Circular Pole Assignment for A Class of Discrete Interval Systems. Proc. of CCDC2011, pp:575-580.

[4] I.R.Petersen, 1987, A stabilization algorithm for a class of uncertain linear System.Syst., 08:351- 357.

[5] J.Ackermann, 1993. Robust Control: Systems with Uncertain Physical Parameters. London, Springer-Verlag.

[6] M.Chiali and P.Gahinet, 1996. H_∞ design with pole placement constraints: an LMI approach.IEEE Transactions on Automatic Control, 41(3): 358-367.

[7] R.J.Veillette,J.V.Medamic and W.R.Perkins, 1992. Design of reliable control systems.IEEE Transactions on Automatic Control, 37(3):770-784.

[8] Wang Jianhua and Yao Bo, 2010.Design of reliable controller with mixed fault model in linear systems.IEEE, Proc.of CCDC2010, pp:1709- 1712.

[9] Yao Bo and Sun Xin, 2004.Reliable Control Pole Assignment of Quadrant Region.Proc.of the 5th World Congress on Intelligent Control, pp: 39 - 42.

[10] Yao Bo, Wang Fuzhong and Zhang Siyang, 2008 .Pole Reliable Assignment of Circular Disc With Actuator Fault. CCDC2008, pp: 463 -466.

[11] Yang G H,Wang J L and Soh Y C,2001.Reliable H_∞control for linear systems . Automatica, 37(3):717-725

[12] Yu Li, 2002.Robust Control-Linear Matrix Inequality Approach.Beijing, pp: 96-100.

[13] Y.S.Lee,Y.S.Moon,W.H.Kwon and P.G.Park,2004. Delay-dependent robust H_∞ control for uncertain systems with a state-delay. Automatica, pp.65-72.