

# RESEARCH ON FAULT-TOLERANT ROUTING WITH HIGH SUCCESS PROBABILITY IN MESH INTERCONNECTION NETWORKS AND IMAGE

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## ABSTRACT

Mesh networks have been applied to build large scale multicomputer systems and Network-on-Chips (NoCs) extensively. Mesh networks perform poorly in tolerating faults in the view of worst-case analysis, so it is practically important for multicomputer systems and NoCs manufactures to determine that how much success probability to construct a fault-free path between two non-faulty nodes for the mesh network when the node failure probability and the network size are given. In this paper, we mainly focus on fault-tolerant routing algorithm on mesh networks from probabilistic view, and provide a probabilistic method for studying routing algorithm. We propose two simple and novel routing algorithms based on the concept of k-submesh. We apply probabilistic analysis on the fault tolerance of our routing algorithms. Suppose that each node fails independently with given probability, we can derive the probability that our routing algorithms successfully return a fault-free routing path. For example, we formally prove that our routing algorithms succeed in finding a fault-free routing path with success probability at least 99% for a mesh network of up to twenty-thousand nodes as long as the node failure probability is bounded by 1.87%. Our routing algorithms run in liner time. Simulation results show that the length of the routing paths constructed by our algorithms is very close to the optimal length. On the other hand, the results also show that the multicomputer systems and NoCs based on mesh networks are quite reliable and trustable in theory and practice.

**Keywords:** *Mesh Networks; K-Submesh; Fault-Tolerant Routing; Probabilistic Analysis; Image*

## 1. INTRODUCTION

With the size of computer networks increasing rapidly, dealing with networks with faulty components has become unavoidable. Fault tolerant routing is to construct a fault-free path between source and destination nodes, and carry out reliable point to point communication and data exchange pattern in a network. In this paper, we will concentrate on the fault tolerant routing of mesh networks. The advantages of mesh networks include their simplicity, regularity and good scalability, so mesh networks are among the most important and attractive interconnection network topologies for large multicomputer systems. A number of large research and commercial multicomputer systems have been built based on mesh topologies, including Infiniband, Myrinet, Illiac IV, Intel Paragon, Stanford DASH, Goodyear MPP, Tera Computer System, Intel Touchstone Delta, MIT Alewife, Cray T3D, Blue Gene Supercomputer, and MasPar series [1], [2],

[3], [4]. In particular, current VLSI technology allows to fabricate large scale Network-on-Chips (NoCs) devices, integrating thousands of cores into a single chip based on mesh interconnection networks, such as Intel Corp's TeraFLOPS and Tiler Corp's TILE64 [5],[6]. It operates as a wireless interface that forwards Internet data packets to/from other stations. Many current academic papers and industry deployments assume that stations are within one hop radio transmission range of such an access point. In ad-hoc mode there is no centralized device. All stations, or nodes, operate in a peer-to-peer mode, and they compete for the shared wireless channel. In this way, they are able to communicate among the domain, but are unable to access outer networks.

In practical use, however, another scenario appears in which all users in a local area network try to connect to Internet, but some of them are beyond one hop transmission range of the access points. This happens when wireline Internet access is too

expensive to deploy for various reasons, including low utilization or expense of cabling. For example, in existing buildings, cable deployment is the major portion of the cost for network setup. Similarly, in a conference there will be high utilization, but only for the period of the conference. The cost of deployment just for the conference is expensive. In such situations, the stations have relatively fixed positions (within one room, for example), and are required to forward others' packets in a peer-to-peer mode, while they communicate to Internet via access points. In such cases, the access point that is connected to the Internet is more frequently referred to as a gateway, and the network is called a wireless mesh network (WMN) [2] [6] [8].

In this paper we discuss the unique aspects of wireless mesh networks, and their differences from ad-hoc networks. In particular, we propose an algorithm for routing in such networks that is able to take advantage of the capabilities of such networks that are not present in ad-hoc networks. We provide some evidence that the approach we propose is likely to perform noticeably better than existing ad-hoc routing protocols.

This remainder of this paper is organized as follows. In Section II we introduce wireless mesh networks and our diverse-routing algorithm. We describe its advantages, and give evidence that it will be a superior approach. In Section III we study the power-aware network connectivity problem. Section IV briefly discusses the impact of gateway selection on network performance. Finally, in Section V, we conclude the paper.

## 2. CONCEPTS AND DEFINITIONS

Wireless mesh networks have the potential to play a critical role as an alternative technology for last-mile broadband Internet access. They can be viewed as a special case of wireless multi-hop ad-hoc networks, in which each node operates both as a host and as a router. However, WMNs have a number of features that distinguish them from pure ad-hoc networks. First, the positions of different nodes of a WMN are relatively fixed. By relatively fixed position, we mean that, although the nodes may not be absolutely immobile, any change of position is limited within certain range. The implication of this is that routing paths can be created that are likely to be stable. This substantially reduces the need for routing packet overhead. Indeed, such routing packets are likely only needed at initialization and when traffic volume is sufficiently low that a node cannot be sure that its neighbour is

still present, as opposed to having crashed. Second, unlike pure ad-hoc networks, where the traffic flows between arbitrary pairs of nodes, in WMN, all traffic is either to or from a designated gateway, which connects the wireless mesh network to the Internet. The relevance of this point is that the traffic may be split over multiple gateways, so as to reduce the load within any given portion of the network. Third, the nodes will typically have access to a power source, and so power consumption is not a critical issue. Finally, such systems can be created within a single domain of authority, and so many security issues present in ad hoc networks are no longer relevant.

The most commonly used topology for wireless mesh networks is a grid layout, due to the layout of buildings. Since each node would communicate with the gateway, it must do so either directly, if it is within the radio transmission range, or indirectly, which requires other nodes to forward packets. In order to minimize the collision probability, each node should adjust its power to a level that is able to reach its four direct neighbors, and no more. This, thus, forms a grid network. Therefore, we can adopt a quasi-xy-routing algorithm in WMN. Xy-routing is commonly used in mesh or torus topology parallel computers to avoid deadlock in wormhole routing [1]. In WMN with this grid topology, each node routes to its direct neighbours. For example, a node  $(x, y)$  in Fig. 1 has direct neighbors  $(x-1, y)$ ,  $(x+1, y)$ ,  $(x, y-1)$ ,  $(x, y+1)$ . Each node performs packet forwarding for its neighbors to and from the gateway.

Packet delay is caused by various reasons, including collision resolution during packet forwarding, packet buffering, and different scheduling algorithms. However, the most critical cause is packet delay in WMN is path length. Under the same traffic intensity, a smaller number of hops would lead to less packet delay. For two nodes,  $S(x_S, y_S)$  and  $D(x_D, y_D)$ , in a grid network, their shortest distance is given by:

$$d = |x_S - x_D| + |y_S - y_D| \quad (1)$$

To minimize packet delay we wish to use the shortest path. However, this must be done in the context of minimizing collisions, since highly-contended paths that are shortest are not necessarily ideal [9]. We therefore propose a shortest-path load-balancing diverse routing protocol. Our protocol is as follows:

S	1	1	1	1	1	
1	2	3	4	5	6	
1	3	6	10	15	21	
1	4	10	20	35	56	
1	5	15	35	70	126	
1	6	21	56	126	252	D

Fig. 1. Diverse Route Calculation.

1. if the next hop is a gateway, compete for transmission with it; else
2. determine neighbour nodes' load;
3. select a lightly-loaded path for next hop and transmit;
4. go to step 1.

Step 2 enables the current node to acquire a picture of local network traffic. We presume this may be achieved by promiscuous snooping of the medium. While in a pure ad-hoc network the cost of such snooping may be too high, in terms of energy consumption, in the WMN context this should be quite feasible. Step 3 is then a simple matter of selecting the lightest-load node. There will, in general, be just two choices for any given destination, presuming that a shortest path route is desired. Alternately, the current node can skip step 2 and simply randomly alternate between the two choices (e.g. right or down in Fig. 1). In this manner our protocol achieves diverse routing. The number of paths available is then determined according to the following theorem.

Theorem 1: For any two given node S (x<sub>S</sub>, y<sub>S</sub>) and D (x<sub>D</sub>, y<sub>D</sub>) in a wireless mesh network, there exists

$$\binom{|x_S - x_D| + |y_S - y_D|}{|x_S - x_D|}$$

Different routes that have distance d, given in Eq. 1.

Proof: We prove the theorem by induction. Without loss of generality, assume x<sub>S</sub> ≤ x<sub>D</sub> and y<sub>S</sub> ≤ y<sub>D</sub>.

Step 1. From S (x<sub>S</sub>, y<sub>S</sub>) to (x<sub>S</sub>, y<sub>S</sub>+1), there is only one path; Likewise for S (x<sub>S</sub>, y<sub>S</sub>) to (x<sub>S</sub>+1, y<sub>S</sub>). The number of shortest paths between S (x<sub>S</sub>,

y<sub>S</sub>) to (x<sub>S</sub>+1, y<sub>S</sub>+1) is 2, which is a summation of the above 2.

Step 2. Suppose from S to T (x<sub>T</sub>, y<sub>T</sub>), the number of shortest paths is

$$\binom{x_T - x_S + y_T - y_S}{x_T - x_S}$$

Again, suppose x<sub>S</sub> ≤ x<sub>T</sub> and y<sub>S</sub> ≤ y<sub>T</sub>.

Step 3. From S to (x<sub>T</sub>+1, y<sub>T</sub>), the number of shortest paths can be calculated recursively as

$$\frac{(x_T - x_S + 1 + y_T - y_S)!}{(x_T - x_S + 1)! \cdot (y_T - y_S)!}$$

From S to (x<sub>T</sub>, y<sub>T</sub>+1), the number of paths is

$$\frac{(x_T - x_S + y_T - y_S + 1)!}{(x_T - x_S)! \cdot (y_T - y_S + 1)!}$$

Therefore, from S to (x<sub>T</sub>+1, y<sub>T</sub>+1), the paths are a sum of the above two, because the paths must go through either (x<sub>T</sub>+1, y<sub>T</sub>) or (x<sub>T</sub>, y<sub>T</sub>+1). That is,

$$\frac{(x_T - x_S + y_T - y_S + 2)!}{(x_T - x_S + 1)! \cdot (y_T - y_S + 1)!}$$

which is

$$\binom{x_T - x_S + 1 + y_T - y_S + 1}{x_T - x_S + 1}$$

The question then arises as to how useful our approach would be. Jones [9] has performed extensive experiments in multipath WMN routing algorithms, using source-based routing. In particular, his work demonstrated the following. First, single-flow multipath routing to/from separate gateways can improve the performance by up to a factor of two over single-path routing, as is used in AODV and DSR. Second, in grid networks of 10\*10 nodes, with sources and destinations selected randomly, using multipath routing aggregate throughput increased by between 5% and 61%, with an average increase of 27%. This indicates that the multipath routing can improve the performance of grid networks. We expect that our approach will yield better results than Jones because we dynamically adjust the path on route, based on current load.

Apart from routing issues, many researchers are concerned with scheduling algorithms in WMN [4] [5]. Jakubczak et al. [3] observed that nodes close to a gateway tend to have better chance for transmission when competing for the shared wireless channel with others that are further away from the gateway. Both Jakubczak et al. [3] and Munawar [7] offer scheduling algorithms that achieve both fairness and high throughput.

### 3. A NOVEL FAULT-TOLERANT ROUTING ALGORITHM

In the previous section, we discuss routing issues for wireless mesh networks. For a relatively stationary topology, it is easy to find a route from an individual node to a gateway, as compared to ad-hoc network routing. This section addresses the route-maintenance problem.

In ad-hoc networks, route failure is mainly caused by node mobility or power-off. Most routing algorithms would produce a route-error message, and trigger re-routing. In wireless mesh networks, where nodes tend not to move, route failure is most probably caused by power-off or system failure. Under this circumstance, we may re-route with another diverse path. Note that for stations on the boundary of a mesh network, we do not need to strictly follow the shortest-distance diverse path. If a node's only adjacent neighbor fails, the node becomes an island. We would then increase its power level so that it can reach other neighbors. This scenario is illustrated in Fig. 2.

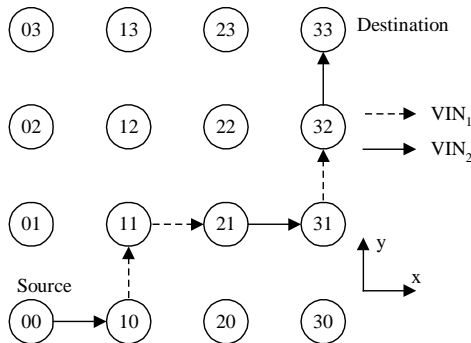


Fig. 2. Message Holding Virtual Channel Of  $VIN_1$  &  $VIN_2$ .

Our scheme is to keep the power level as low as possible. Although a higher power level can reach a longer distance, and thus require fewer hops, it will also lead to more interference with other nodes, increasing the collision probability. In an extreme situation, where each node can hear every other node, a lot of collision will happen, and will have to be resolved with a much longer back-off time, especially in heavy-traffic situations. There is always a trade-off between network capacity and throughput [4]. Therefore, we try to keep the power to a low level. When the gateway is not the performance bottleneck, multiple packet-forwarding paths with fewer collisions can improve network throughput.

However, if a station is unable to reach other nodes, it will have to increase its power level to find some neighbours. In the same way, if a node joins the network, it will first look for its neighbors. Some island nodes might restore their power upon a new node's appearance, which could connect them to gateway in a normal mesh. How to find an alternative path during network failure is critical in wireless mesh networks.

### 4. GATEWAY'S EFFECT ON PERFORMANCE

Due to the traffic pattern, most of the data packets are to or from designated gateway. As such, it is difficult or impossible to balance the load between nodes close to gateway and other nodes. With diverse routing, we have tried to balance the load among different routing paths to the gateway in order to avoid interference. Further, we presume nodes can use multi gateways. Finally, we note that placement of gateways at different positions in the mesh can have a direct effect on network throughput. For example, in Fig. 1, a gateway at a corner, rather than at the center, will more likely result in a higher delay and lower throughput for the mesh.

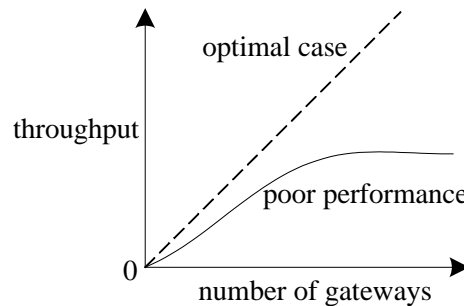


Fig. 3. Throughput Increase With Number Of Gateways

When traffic increases to certain amount that the existing gateways cannot handle any more, adding new gateways in mesh network could greatly alleviate congestion. For example, in a conference center when several conferences are held simultaneously, the organizers might place additional gateways to meet the increasing Internet traffic. In this situation, maintaining a balanced load among all gateways is important. The throughput of a load-balanced network would ideally grow linearly with the increment of gateway numbers.

Specifically, we propose to build some intelligent gateways that can perform virtual private network (VPN) functions. Because of the simplified routing issue, nodes might be able to use a local address, say, (x, y), to route Internet packets, and the packets are encapsulated at intelligent gateway and for-

warded by other nodes. This would further allow multiple WMN subscribers to share limited number of IP addresses, if gateways can do address conversion.

Table 1. Node Failure Probability When The Success Probability Of The Algorithm MPR Is  $Q_0 = 99\%$

mesh size	#nodes	node failure prob.
50 × 50	2500	≤ 2.96%
100 × 100	10000	≤ 2.88%
200 × 200	40000	≤ 2.55%
400 × 400	160000	≤ 2.04%
500 × 500	250000	≤ 1.87%

We are able to obtain a lower bound  $(1 - (1 - C_1)2(1 - C_2)2) \cdot C_2$  which is the success probability of the algorithm MPR. Let  $(1 - (1 - C_1)2(1 - C_2)2) \cdot C_2 \geq Q_0$  if we require that the success probability of the algorithm MPR is more than a value  $Q_0$ . Therefore, we will calculate an upper bound of node failure probability  $p$  and guarantee the success probability  $Q_0$  of the algorithm MPR. Let  $Q_0 = 99\%$ , we calculate node failure probability  $p$  for different size mesh networks, the results are shown in Table 1.

Table 1 provides strong evidence that the success probability is further improved. For example, the success probability of the algorithm MPR is at least 99% if the node failure probability is bounded by 1.87% (which should be considered as a very big failure probability in theory and practice) in mesh network with 25000 nodes (e.g.  $M_{500 \times 500}$ ). All these results, which are formally proved based on mathematical analysis, show the efficiency of our algorithms in mesh networks with very large size. On the other hand, our discussion and conclusion are also based on precise and quantitative calculation and analysis.

Clearly, being able to compute the connectivity provability of submesh for further larger  $k$  will lead to even better conclusions for the success probability of the algorithms SPR and MPR.

In the aspect of multicomputer systems, R. Boppana and S. Chalasani proposed the most classical fault block model and fault-tolerant routing algorithm with two virtual channels [9]. Subsequently, the results have further been extended, fault block model maybe solid faulty shape, such as “+”, “L” and “T”, but the new algorithm must use four virtual channels without overlapping fault chain. C. Chen and C. Chiu improved the results, and their algorithm only uses three virtual channels with overlapping faults chain [10]. R. L. Hadas proposed a

fault-tolerant model called “Origin” concept for mesh networks [11]. The algorithm can tolerate at least  $(k - 1)^{n-1}$  faulty nodes in  $n$ -dimensional mesh networks. Glass and Ni considered an adaptive fault-tolerant routing algorithm based on a “Turn” model [12]. The Turn model produces routing algorithms which are deadlock-free, adaptive, minimal or non-minimal, and livelock-free for direct networks no matter whether they involve virtual channels or not. In addition, the routing algorithms produced by the Turn model can deal with dynamic faulty nodes. Practical deadlock-free fault-tolerant routing is proposed based on the planar network fault model [13].

In the aspect of NoCs, the authors utilize the region concept to design fault tolerant algorithms in [14], [15]. In [16], the authors discussed the implementation of techniques for detection and recovery of faults in a NoC. In [17], a routing scheme called MinFT is proposed, which adapts as per-link failures while following the minimal path and reserving bandwidth as per QoS requirement in NoCs. The highlight of the scheme is the continuation of functioning of NoCs even in case of link failure as well as node failures for different types of traffic. In [18], the authors enhanced available message-based approach for NoCs architectures without using virtual channels. In [19], the authors proposed a novel routing control algorithm for non-VC router of irregular 2D-mesh NoCs. In [20], the authors proposed a distributed routing algorithm for NoCs, allowing a network to reconfigure around faulty components. Experimental results showed an average reliability of over 99.99% when 10% of the network **links** have failed under different networks sizes.

In general, fault-tolerant routing mechanism depends on the following factors: (1) Fault-tolerant model, such as fault block model, Origin-based, Turn model and extended safety levels; (2) Faulty nodes distribution information, such as algorithm is based on local-information, global-information or limited-global-information; (3) Path length by fault-tolerant algorithms constructed is optimal, minimal or not. (4) Fault-tolerant algorithms need virtual channels or not and how many number of faults can be tolerated by algorithms. For existing algorithms, adding virtual channels is not free, it would involve more buffer space and complicated logic control to the nodes. In fact, the extra buffer space and logic circuits not only make mesh's nodes to be prone to failure and become more unreliable, but also virtual channels can't be efficiently utilized and at the same time result in increasing the cost to



construct fault-tolerant routing. And that, many non-faulty nodes will be sacrificed if the shape of faulty nodes block is restricted.

## 5. CONCLUSIONS

Mesh networks are a kind of very important network topologies in massively multiprocessor parallel systems and NoCs. Many researchful and commercial multicomputer systems and NoCs have been built based on mesh networks up to thousands of processors. For this kind of systems with a large number of nodes, fault tolerance and fault-tolerant routing are among the most important and interesting topic.

In this paper, we mainly focus on fault-tolerant routing algorithm on mesh networks. Based on the concept of k-submesh, we propose two simple and novel routing algorithms for mesh networks. We apply probabilistic model on the fault tolerance of our routing algorithms. Suppose each node fails independently with given probability, we are able to derive the probability that our routing algorithms successfully return a fault-free routing path. Our research provides formally proven results that show the success probability of our algorithms is very high. For example, we formally prove that our routing algorithms succeed in finding a fault-free routing path with success probability at least 99% for a mesh network of up to twenty-thousand nodes as long as the node failure probability is bounded by 1.87%. It is entirely possible that the node failure probability is controlled within 1.87% under modern integrated circuit technology.

In fact, the scheme established in this paper is obviously not only restricted to lower dimensional mesh networks. For high dimensional mesh, such as 3-D mesh networks, the scheme can get the same efficient fault tolerant routing algorithms. Moreover, the technique developed here is very general and can be applied to any hierarchical network structures (i.e., the network structures in which larger networks can be decomposed into smaller sub-networks of similar structure). Finally the techniques given in the current paper are also applicable to the study of network fault tolerant routing algorithm under other probability distributions of node failures.

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