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# A MODEL OF GRANULAR COMPUTING ON BINARY RELATIONS BASED ON GENERALIZED ROUGH SET AND ITS GRANULATION IN INFORMATION SYSTEM

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**ABSTRACT**

Granular computing is a new information processing method. Many models and methods of granular computing have been proposed and studied. Nowadays, the granular computing has been appeared in many areas of information processing. In this paper, a study of the model of granular computing on Binary Relations based on Generalized Rough Set is presented. We hope to expand the range of application of rough sets and provide ideas for dealing with the problems of uncertain information.

**Keywords:** *Rough sets, Granular Computing, Generalized Rough Set, Information Granulation*

## 1. INTRODUCTION

An important concept in the artificial intelligence is the knowledge of the intelligence demand. We have witnessed a rapid development in the topic. Rough set theory has been successfully applied to several data analysis tasks in the field of artificial intelligence [1], such as data mining, knowledge discovery, pattern recognition, decision analysis, process control, image processing and medical diagnosis [2-9].

Since his 1994 keynote speech at the Third International Workshop on Rough Sets and Soft Computing, Lotfi Zadeh has been continuously pointing out the importance of information granulation [10, 11]. Granulation is a very natural concept and appears almost everywhere in different names, such as chunking, clustering, data compression, divide and conquer, information hiding, interval computations, and rough set theory, just to name a few[12,13]. A granule may be interpreted as one of the numerous small particles forming a larger unit. Collectively, it provides a representation of the unit with respect to a particular level of granularity. A family granule contains every object in the universe. There are many granulated views of the same universe. Different views of the universe can be linked together by binary relations.

The theory of rough sets can be generalized in several directions. Within the set-theoretic framework, generalizations of the element based definition can be obtained by using non-equivalence binary relations [14-18], generalizations of the granule based definition can be obtained by using coverings [19, 20, 21].The generalized rough set expand the range of application of rough set. It can use common binary relation to define rough set by converting common binary relation into equivalent relation [22]. Based on the different knowledge bases, it can get the upper approximation and lower approximation to discover the relation.

This article is organized as follows. In Section 2, we recall the model of granular computing based on partition of the universe. Section 3 introduces the framework for the model of granular computing on binary relation. Section 4 looks at how the information could be granulated. In Section 5, we conclude this paper and discuss the outlook for further work.

## 2. MODEL OF GRANULAR COMPUTING BASED ON PARTITION OF THE UNIVERSE

A simple granulation of the universe can be defined based on an equivalence relation or a partition. Let $U$ denote a finite and non-empty set called the universe. Suppose $E \subseteq U \times U$ denote an

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equivalence relation on $U$ , where $\times$ denotes the Cartesian product of sets. That is, $E$ is reflexive, symmetric, and transitive. The pair $(U, E)$ is called an approximation space. The equivalence relation $E$ partitions the universe $U$ into disjoint subsets called equivalence classes. The partition of the universe is referred to as the quotient set and is denoted by $U/E$ .The equivalence relation is the available information or knowledge about the objects under consideration. It represents a very special type of similarity between elements of the universe. For two elements $x, y$ in $U$ belong to the same equivalence class. If $xEy$ , we say that $x$ and $y$ are indistinguishable.

A granule of a partition model is an equivalence class defined by an equivalence relation. The internal structure of a granule and of a granulated view is captured by the equivalence relation. An arbitrary set $X \subseteq U$ may not necessarily be a union of some equivalence classes. This implies that one may not be able to describe $X$ precisely using the equivalence classes of $E$ . In this case, one may approximate $X$ by a pair of operators named lower and upper approximations:

$$apr_{-}(X) = \bigcup\{[x]_E \mid x \in U, [x]_E \subseteq X\},$$

$$apr^{-}(X) = \bigcup\{[x]_E \mid x \in U, [x]_E \cap X \neq \phi\}$$
(1)

where

$$[x]_E = \{y \mid y \in U, xEy\}$$
(2)

is the equivalence class containing $x$ . Both lower and upper approximations are unions of some equivalence classes. More precisely, the lower approximation $apr_{-}(X)$ is the union of those equivalence granules that are subsets of $X$ . The upper approximation $apr^{-}(X)$ is the union of those equivalence granules that have a non-empty intersection with $X$ .

By definition, a definable set has the same lower and upper approximations. In terms of equivalence classes, lower and upper approximations can be expressed by:

$$apr_{-}(X) = \{x \mid x \in U, [x]_E \subseteq X\}$$

$$apr^{-}(X) = \{x \mid x \in U, [x]_E \cap x \neq \phi\}$$
(3)

where

$$[x]_E = \{y \mid xEy\}$$
(4)

The above rough set model is known as the partition model of granular computing [23, 24].

There is another view for interpreting rough set theory, i.e., a particular set-oriented view characterized by rough membership functions. The rough membership functions can be seen as a special type of fuzzy membership functions.

For a subset $X \subseteq U$ , we have the following well defined rough membership function:

$$\mu_x(x) = \frac{Card([x]_E \cap X)}{Card([x]_E)}$$
(5)

where $Card(*)$ denotes the cardinality of a set.

Rough membership functions may be interpreted as a special type of fuzzy membership functions interpretable in terms of probabilities defined simply by cardinalities of sets. One may view the fuzzy set theory as an un-interpreted mathematical theory of abstract membership functions. The theory of rough set thus provides a more specific and more concrete interpretation of fuzzy membership functions [16]. The source of the fuzziness in describing a concept is the indiscernibility of elements. In the theoretical development of fuzzy set theory, fuzzy membership functions are treated as abstract mathematical functions without any constraint imposed.

## 3. A MODEL OF GRANULAR COMPUTING ON BINARY RELATION

A binary relation is a subset, $R \subseteq V \times U$ . For each object $p \in V$ , we associate a subset $N_p \subset U$ , where

$N_p = \{u \in U \mid pRu\}$ consists of all elements $u$ that are related to $p$ by $R$ , called a binary neighborhood. If the binary relation is an equivalence relation, then $N_p$ is the equivalence class containing $p$ .

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In stead of using a binary relation, we can define neighborhoods directly, that is, we can consider a map:

$B:V\to 2^u, p\to B_p$ , is called binary granulation.

The map $B$ or the collection $\{B_p\}$ is referred to as a binary neighborhood system for $V$ on $U$ .We called $B_p$ a basic neighborhood and $B$ or $\{B_p\}$ a basic neighborhood system respectively [23].

Given a binary neighborhood system $\{B_p\}$ for $V$ on $U$ , there is a binary relation R $R\subseteq V\times U$ , such that

$N_p = B_p$.

It is easy to check that binary relation $(BR)$, binary neighborhood system $(BNS)$ or binary granulation $(BG)$ are equivalent to each other.

Since a binary granulation ( $BG, BR, BNS$ ) is a map, we have a partition, called the induced partition (equivalence relation) of $B$, and denoted by $E_B$ .In this case, we may approximate $X$ by a pair of operators named lower and upper approximations:

$$apr_{-}(X)=\bigcup\{[x]_{E_B} \mid x\in U, [x]_{E_B}\subseteq X\},$$

$$apr^{-}(X)=\bigcup\{[x]_{E_B} \mid x\in U, [x]_{E_B}\cap X\neq\phi\} \quad (6)$$

where

$$[x]_{E_B}=\{y \mid y\in U, xEy\}$$

For a subset $X\subseteq U$ , we have the following well defined rough membership function:

$$\mu_x(x)=\frac{Card([x]_{E_B}\cap X)}{Card([x]_{E_B})} \quad (7)$$

Where $Card(*)$ denotes the cardinality of a set.

## 4. INFORMATION GRANULATION BASED ON BINARY RALATION IN INFOAMATION STSTEM

Granular computing is a new information processing method. Many models and methods of granular computing have been proposed and studied. Nowadays, the granular computing has been appeared in many areas of information processing. Granules are regarded as the primitive notion of granular computing. It is a key issue how to construct information granules in granular computing. A granule may be interpreted as one of the numerous small particles forming a larger unit. It provides a representation of the unit with respect to a particular level of granularity. A family granule contains every object in the universe. There are many granulated views of the same universe. This idea inspired our initiative of studying attribute reduction in information systems by utilizing information granules created on attributes set. Now, we study the information granulation based on binary relation in information system. And further we will study attribute reduction method by the granular computing model[26].

An information system is composed of a 4-tuple as following:

$S=(U,A,F,V)$ , where

$U=\{x_1,x_2,\cdots,x_n\}$ is a non-empty finite set of objects;

(2) $A=\{a_1,a_2,\cdots,a_m\}$ is a non-empty finite set of attributes; It is composed of C and $D$;

(3) $V$ is attribute values 1 or 0; For every $a\in A$, there is a mapping $F$ , $F\subseteq U\times A$,

$F:U\times A\to V_a$, where $V_a$ is called the value set of $a$ .

An object $x_i\in U$ has the attribute $a$ , we write $xFa$ or $(x,a)\in F$ .

Information granulation is a grouping of elements based on their indistinguishability, similarity, proximity or functionality. For $\forall x_i\in U, a_i\in A$, the set $(x_i,a_i)$ is called an information granule, we note $GR=(x_i,a_i)$ . For $\forall x_1,x_2\in GR$ , we have $d(x_1)=d(x_2)$. An

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information system $S=(U,A,F,V)$ could be defined to be $S=(GR,D)$.

If the object $x_i \in U$ could be described by the attribution $a_i \in A$, $(x_i,a_i)$ is called as sufficient information granule, note $GR_{suf}(x_i,a_i)$.

If the object $x_i \in U$ could not be described completely by the attribution $a_i \in A$ and $a_i$ is indispensable, $(x_i,a_i)$ is called as necessary information granule, note $GR_{nec}(x_i,a_i)$.

If the attribution $a_i \in A$ is sufficient attribution and necessary attribution, i.e.

$$GR(x_i,a_i)=GR_{suf}(x_i,a_i)\bigcap GR_{nec}(x_i,a_i),$$

$GR(x_i,a_i)$ is called as information granule.

To evaluate its performance, the proposed model was implemented in an example. The data set comes from the reference [27].

| $U$ | $C$ | | | | $D$ |
|---|---|---|---|---|---|
| | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $d$ |
| $x_1$ | 1 | 1 | 3 | 2 | 1 |
| $x_2$ | 2 | 1 | 2 | 1 | 1 |
| $x_3$ | 1 | 1 | 3 | 2 | 1 |
| $x_4$ | 1 | 2 | 2 | 1 | 2 |
| $x_5$ | 1 | 2 | 2 | 1 | 2 |
| $x_6$ | 1 | 2 | 1 | 1 | 3 |
| $x_7$ | 1 | 2 | 2 | 1 | 2 |
| $x_8$ | 1 | 2 | 1 | 1 | 3 |

$$U/R_C=\{\{x_1,x_3\},\{x_2\},\{x_4,x_5,x_7\},\{x_6,x_8\}\}$$
$$U/R_D=\{\{x_1,x_2,x_3\},\{x_4,x_5,x_7\},\{x_6,x_8\}\}$$

$$GR_{suf}(x_i,a_i)=\{a_1,a_2,a_3,a_4\}$$

$$GR_{nec}(x_i,a_i)=\{\{a_1,a_3\},\{a_2,a_3\}\}$$

$$GR(x_i,a_i)=\{\{a_1,a_3\},\{a_2,a_3\}\}$$

We may also obtain its reduction set of the information system, $red(C)=\{a_3\}$.

## 5. CONCLUSIONS

Granular computing is viewed as an effective tool for dealing with the problems of uncertain information. In this study, we are concerned with an extension of the range of application of rough sets. In this paper, a study of the model of granular computing on Binary Relations based on Generalized Rough Set is presented. The information granulation in information system is discussed. And we could also use it to solve the problems of attribute reduction and classification in the further study.

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