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WORKSPACE AND SINGULARITY ANALYSIS OF 3/3-RRRS PARALLEL MANIPULATOR

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ABSTRACT

The workspace of manipulator can be defined as the operational area of the end effector. Because the moving platform of the parallel manipulator with six degree-of-freedom have six independence degree-of-freedoms, it means the workspace has six-dimensional, the six-dimensional workspace is difficult to be directly calculated and unable to do graphic description. For the project of this article-3/3-RRRS parallel manipulator, this article will analyze the workspace of the manipulator in two cases. A discretization method is proposed for the computation of the reachable position/orientation workspace of the 3/3-RRRS parallel manipulator, respectively. Four examples of a 3/3-RRRS parallel manipulator are given to demonstrate these theoretical results. As a special position and inherent attributes of the mechanism, singularity has a serious impact on the mechanism's motion control performance and drive performance. The method of searching algorithm in three-dimensional space is applied to the singularity analysis, coordinate of all possible stable positions are substituting into the Jacobian, position-singular points under given orientation condition are conformed.

Keywords: *Parallel Manipulator, Workspace Analysis, Singularity Analysis*

1. INTRODUCTION

The size and shape of the workspace are important index to measure the performance of the mechanism and the important process of the end effector's design [1]. And as a special position and inherent attributes of the mechanism, singularity has a serious impact on the mechanism’s motion control performance and drive performance. Much work has been done at home and abroad during recent three decades.

In terms of the application of numerical method, Fichter fixed three orientation parameters in six location parameters and a position parameter of the parallel manipulator and set the remaining two position parameters as variables. Then the numerical method is applied to the study of the workspace of six degree-of-freedom (DoF) Stewart parallel manipulator [2]. Gosselin chooses to fix the orientation parameters of the parallel manipulator moving platform and use approach of arc intersects to determine the position workspace of the six DoF parallel manipulators. Since the purpose of approach is to solve the boundary of the workspace, the scanning is efficient and the volume of workspace can be worked out [3]. Similarly, Chen applied the genetic algorithm to the problem of solving process of the six DoF space parallel robot manipulators [4]. Hunt and Fichter first use the screw theory to analyze the singularity of two different types of Steward parallel manipulator [5] and et al. [6-12]. Especially, Cao et al. discussed the orientation-singularity and the orientation-workspace of the Stewart-Gough manipulators [13, 14, 15, 16].

2. WORKSPACE ANALYSIS OF 3/3-RRRS PARALLEL MANIPULATOR

3/3-RRRS parallel manipulator is a new type of realizing arbitrary three-dimensional spatial motion of the parallel manipulator. The manipulator consists of a moving platform and fixed platform as well as three completely identical branched chains, among them, both the moving platform and fixed platform are equilateral triangle, P is the centroid of the moving platform, O is the centroid of the fixed platform; Between the moving and fixed platform the three branched chain by A_iB_i, B_iC_i, C_iD_i (i=1, 2, 3) respectively in turn are connected together, between the fixed platform and A_iB_i, A_iB_i and B_iC_i, B_iC_i and C_iD_i, the connection of rotation pair; S_{i1}, S_{i2}, S_{i3} the rotation axis respectively, between C_iD_i

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and the moving platform the connection of spherical hinge pair, S_{i4}, S_{i5}, S_{i6} the rotation axis (see 3/3-RRRS parallel manipulator structure Figure 1).

Branched chain $A_iB_iC_iD_i$ are always whole rotation in the same plane round the rotation pair at the point A_i, the angle between centroid in moving and fixed platform and vertex ligature PD_i as well as OA_i, is 120°mutually and respectively.

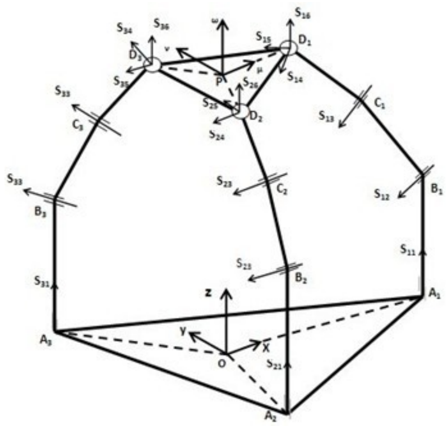

Figure 1: Schematic Representation of a 3/3-RRRS Parallel Manipulator

2. 1 The Influencing Factors of the Workspace Analysis

The constraint of drive rotation angle directly affects the workspace of the manipulator. The six revolute joint in A_i and B_i serve as drive pair of the 3/3-RRRS parallel manipulator. Constituting the links of each of the drive pairs A_iB_i and B_iC_i are the crankshaft type structure. So that the range of the rotation angle of the drive pairs can be expanded. So, in the solving process of workspace, define range of the rotation angle of six drive pairs γ_i, α_i and constraint as follow:

$$\gamma_{imin} \le \gamma_i \le \gamma_{imax}\ ,\ \alpha_{imin} \le \alpha_i \le \alpha_{imax} \tag{1}$$

In the formula γ_{imin}, γ_{imax} and α_{imin}, α_{imax} are given limit angle in engineering practice.

Set the vector $\overrightarrow{Nvp}$ as the direction vector which is a vertical plane of the moving platform, to indicate the vector about Euler angles η, φ and δ of the moving platform location:

$$\begin{aligned}\overrightarrow{Nvp} = (&\sin\delta\cdot\sin\eta + \cos\delta\cdot\cos\eta\cdot\cos\varphi,\\ &-\cos\eta\cdot\sin\delta + \cos\delta\cdot\sin\eta\cdot\sin\varphi,\\ &\cos\delta\cdot\cos\varphi)^T\end{aligned} \tag{2}$$

According to the geometry relationship, rotation angle β_i of the spherical hinge is:

$$\beta_i = a\cos(\overrightarrow{Nvp}.\overrightarrow{C_iD_i}) / \left\|\overrightarrow{C_iD_i}\right\| \tag{3}$$

In the equation, (.), acos and norm are all function language of MATLAB, (.) indicates dot product between vector $\overrightarrow{Nvp}$ and $\overrightarrow{C_iD_i}$, acos indicates to return to the arccosine and norm indicates the modulus of the vector $\overrightarrow{C_iD_i}$.

Then the spherical hinge should meet the follow constraints:

$$\beta_{imin} \le \beta_i \le \beta_{imax} \tag{4}$$

In the formula, β_{imin}, β_{imax} are the limiting rotation angle given for engineering practice.

According to the kinematics analysis of 3/3-RRRS parallel manipulator, the manipulator structure parameters have little influence to drive. It's not the emphasis.

2. 2 The Solving Method of the Workspace

By the symbolic solution of location inverse solution, we can obtain the drive joint variables corresponding to the output location of the moving platform quickly. And judge the obtained joint variables with the constrains to search out the manipulator's workspace boundary. It means that we can obtain the drive input α_i (α_i has two solution), γ_i (i=1, 2, 3) what the point P's position variables a, b, c and orientation variable η, φ, δ on the moving platform is homologous by position inverse solution. The location point can be judged whether it belong to the workspace by judging whether α_i, γ_i, β_i consistent with the constrains of Eq. 1 and Eq. 4 at the same time.

The three-dimensional limit boundary of the parallel manipulator workspace searching shows follow:

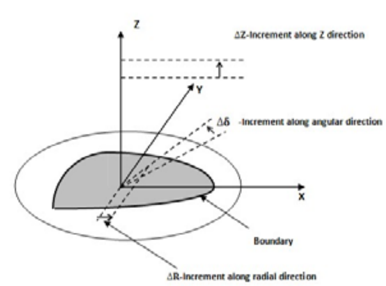

Figure 2: Workspace Limit Boundary Searching Diagram

2. 3 Position Workspace Analysis

Set the structural parameters of the manipulator as: the circumcircle radius of the fixed platform *R=200mm*, the circumcircle radius of the moving platform *r=200mm*. The length of each links are:

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$|A_iB_i|$=200*mm*, $|B_iC_i|$=150*mm*, $|C_iD_i|$=100*mm*. Set the solid league coordinate system origin *P* on the moving coordinate system as the research object, and the position coordinates of the point *P* meets the Eq. 5:

$$\begin{cases} X = R_1 \cdot \cos\sigma \\ Y = R_1 \cdot \sin\sigma \\ Z = Z \end{cases} \quad (5)$$

In the equation, R_1, α, Z are respectively inner loop variables, middle loop variable and outer loop variable of the three-dimensional search.

2. 4 Position Workspace Examples

The accuracy and efficiency of workspace three-dimensional search greatly depend on the calculated step value. In order to obtain the position workspace of the 3/3-RRRS parallel manipulator quickly and effectively and ensure the accuracy of the position workspace, we set the steps respectively as ΔR_1=1*mm*, $\Delta\alpha$=3°, ΔZ=1*mm*. Might given to a group of moving platform orientation: $\eta=0$, $\varphi=0$, $\delta=0$.

We propose two groups of range of the rotation angle of the drive angle. The first group: $-\pi/4\leq\gamma_1\leq\pi/4$, $-7\pi/16\leq\gamma_2\leq9\pi/16$, $-\pi/8\leq\gamma_3\leq7\pi/16$; $-5\pi/8\leq\alpha_1\leq 5\pi/8$, $-5\pi/8\leq\alpha_2\leq5\pi/8$, $-5\pi/8\leq\alpha_3\leq5\pi/8$; $\beta_1\leq3\pi/10$, $\beta_2\leq 3\pi/10$, $\beta_3\leq3\pi/10$ and the second group: $-3\pi/4\leq\gamma_1\leq3\pi/4$, $-3\pi/4\leq\gamma_2\leq3\pi/4$, $-3\pi/4\leq\gamma_3\leq3\pi/4$, $-3\pi/4\leq\alpha_1\leq 3\pi/4$, $-3\pi/4\leq\alpha_2\leq3\pi/4$, $-3\pi/4\leq \alpha_3\leq3\pi/4$, $\beta_1\leq7\pi/20$, $\beta_2\leq7\pi/20$, $\beta_3\leq7\pi/20$.

According to the above parameters and constraint conditions, using three-dimensional limit boundary search method, we can get the three-dimensional boundary contour figure of the manipulator position workspace as shown in Figure 3 and Figure 4

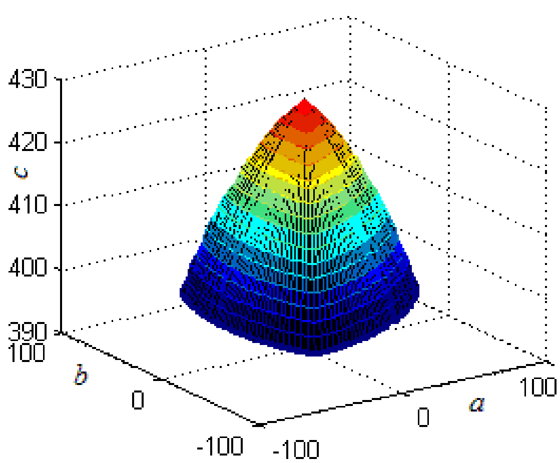

Figure 3: Position-Workspace under the Constraints of First Group

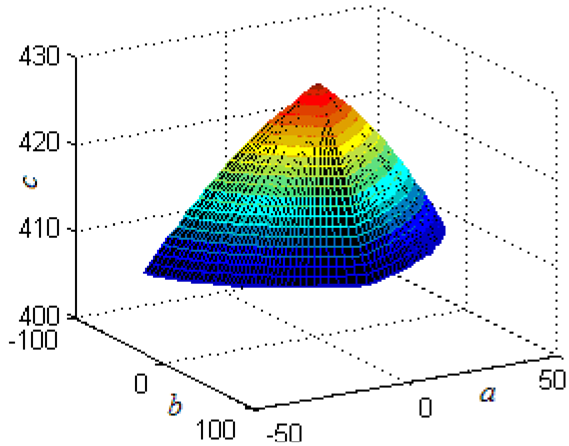

Figure 4: Position-Workspace under the Constraints of Second Group

2. 5 Solving the Orientation Workspace

Keep the manipulator's structure parameters accordance with former article, set the solid league coordinate system origin *P* on the moving coordinate system as the research object, and the position coordinates of the point *P* meets the Eq. 6:

$$\begin{cases} \eta = R_1 \cdot \cos\sigma \\ \varphi = R_1 \cdot \sin\sigma \\ \delta = \delta \end{cases} \quad (6)$$

In the equation, R_1, α, Z are respectively inner loop variables, middle loop variable and outer loop variable of the three-dimensional search.

As well, in order to obtain the orientation workspace of the 3/3-RRRS parallel manipulator quickly and effectively and ensure the accuracy of the orientation workspace, we set the steps respectively as ΔR_1=0.001*mm*, $\Delta\alpha$=3°, $\Delta\delta=\pi/360$.

2. 6 Orientation Workspace Examples

We propose two groups of the workspace and range of the rotation angle of the drive angle as follow:

The third group: the moving platform location: a=0, b=0, c=410. The range of the rotation angle of the drive angle: $-\pi/4\leq\gamma_1\leq3\pi/4$, $\pi/12\leq\gamma_2\leq7\pi/12$, $5\pi/12 \leq\gamma_3\leq11\pi/12$, $-3\pi/4\leq\alpha_1\leq5\pi/8$, $-3\pi/4\leq\alpha_2\leq3\pi/4$, $-3\pi/4\leq \alpha_3\leq3\pi/4$, $\beta_1\leq7\pi/20$, $\beta_2\leq7\pi/20$, $\beta_3\leq7\pi/20$.

The fourth group: the moving platform location: a=0, b=0, c=415. The range of the rotation angle of the drive angle: $-\pi/4\leq\gamma_1\leq\pi/4$, $\pi/8\leq\gamma_2\leq7\pi/12$, $-\pi/8\leq\gamma_3 \leq5\pi/12$, $-5\pi/8\leq\alpha_1\leq5\pi/8$, $-5\pi/8\leq\alpha_2\leq5\pi/8$, $-5\pi/8\leq\alpha_3\leq 5\pi/8$, $\beta_1\leq3\pi/10$, $\beta_2\leq3\pi/10$, $\beta_3\leq3\pi/10$.

According to the above parameters and constraint conditions, using three-dimensional limit boundary search method, we can get the three-

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dimensional boundary contour figure of the manipulator orientation workspace as shown in Figure 5 and Figure 6.

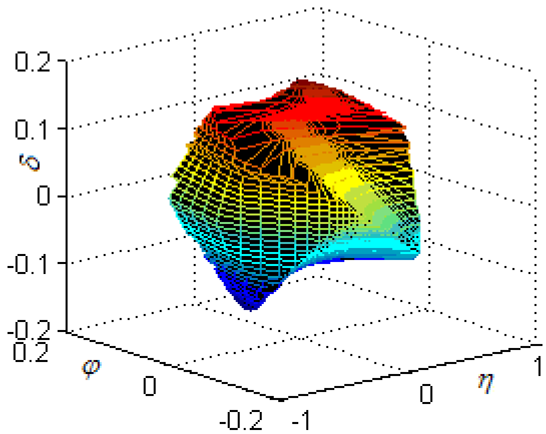

Figure 5: Orientation-Workspace under the Constraints of the Third Group

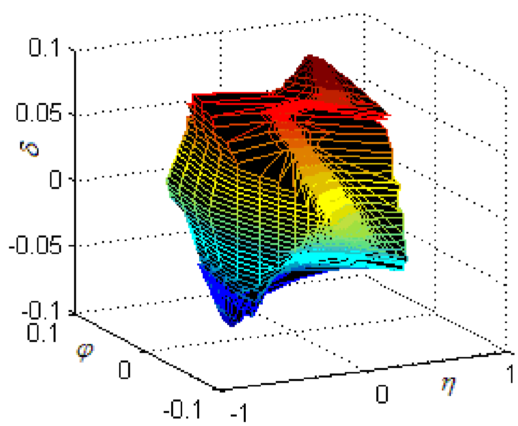

Figure 6: Orientation-Workspace under the Constraints of the Fourth Group

3. SINGULARITY ANALYSIS OF THE 3/3-RRRS PARALLEL MANIPULATOR

In this paper, on the basis of the kinematics analysis, according kinematics equals of the manipulator, we try to obtain Jacobian matrix's symbolic expressions about the six moving platform location parameters. Using the three-dimensional point searching method, traverse three-dimensional space completely covered by the functional requirements of the decision of all task trajectory point, obtain Jacobian matrix singular point, under the conditions of the given moving platform orientation, and draw out all the singular point in the MATLAB work environment.

As the description above, the drive input of the 3/3-RRRS parallel manipulator are α_i, γ_i (i=1, 2, 3). The motion parameters of the moving platform include position parameters a, b, c, and orientation parameters η, φ, δ. These parameters are functions of time t. In the 3/3-RRRS parallel manipulator, according to the geometry relationship, there is:

Link length constraint equation:

$$\left\|\overrightarrow{C_iD_i}\right\|^2 = L^2_{2,i} \tag{7}$$

Two vector vertical equation:

$$\overrightarrow{C_iD_i}.(\cos\gamma,\ \sin\gamma, 0) = 0 \tag{8}$$

where, in the coordinates of each of the hinge point D_i in the fixed coordinate system O-xyz can be expressed as D_i:(D_{iX}, D_{iY}, D_{iZ}). Vector (cosγ_i, sinγ_i , 0) is the vertical vector about plane $A_iB_iC_iD_i$.

Might set:

$$f_i = \left\|\overrightarrow{C_iD_i}\right\|^2 - L^2_{2,i} \tag{9}$$

$$F_i = \overrightarrow{C_iD_i}.(\cos\gamma,\ \sin\gamma, 0) \tag{10}$$

Take the partial derivative of the Eqs. 9 and 10 with respect to time t and transpose, we can obtain the Eqs. 11 and 12:

$$\frac{\partial f_i}{\partial\alpha}\alpha' + \frac{\partial f_i}{\partial b}b' + \frac{\partial f_i}{\partial c}c' + \frac{\partial f_i}{\partial\eta}\eta' + \frac{\partial f_i}{\partial\varphi}\varphi' + \frac{\partial f_i}{\partial\delta}\delta' = -(\frac{\partial f_i}{\partial\alpha_i}\alpha_i' + \frac{\partial f_i}{\partial\gamma_i}\gamma_i') \tag{11}$$

$$\frac{\partial F_i}{\partial\alpha}\alpha' + \frac{\partial F_i}{\partial b}b' + \frac{\partial F_i}{\partial c}c' + \frac{\partial F_i}{\partial\eta}\eta' + \frac{\partial F_i}{\partial\varphi}\varphi' + \frac{\partial F_i}{\partial\delta}\delta' = -(\frac{\partial F_i}{\partial\alpha_i}\alpha_i' + \frac{\partial F_i}{\partial\gamma_i}\gamma_i') \tag{12}$$

Put the velocity equations of all three branch chains on the 3/3-RRRS parallel manipulator together, written in matrix form:

$$\begin{pmatrix} \partial_a f_1 & \partial_b f_1 & \partial_c f_1 & \partial_\eta f_1 & \partial_\varphi f_1 & \partial_\delta f_1 \\ \partial_a F_1 & \partial_b F_1 & \partial_c F_1 & \partial_\eta F_1 & \partial_\varphi F_1 & \partial_\delta F_1 \\ \partial_a f_2 & \partial_b f_2 & \partial_c f_2 & \partial_\eta f_2 & \partial_\varphi f_2 & \partial_\delta f_2 \\ \partial_a F_2 & \partial_b F_2 & \partial_c F_2 & \partial_\eta F_2 & \partial_\varphi F_2 & \partial_\delta F_2 \\ \partial_a f_3 & \partial_b f_3 & \partial_c f_3 & \partial_\eta f_3 & \partial_\varphi f_3 & \partial_\delta f_3 \\ \partial_a F_3 & \partial_b F_3 & \partial_c F_3 & \partial_\eta F_3 & \partial_\varphi F_3 & \partial_\delta F_3 \end{pmatrix} \begin{pmatrix} a' \\ b' \\ c' \\ \eta' \\ \varphi' \\ \delta' \end{pmatrix}$$

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$$= -\begin{pmatrix} \partial_{\alpha_1} f_1 & \partial_{\gamma_1} f_1 & 0 & 0 & 0 & 0 \\ \partial_{\alpha_1} F_1 & \partial_{\gamma_1} F_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \partial_{\alpha_2} f_2 & \partial_{\gamma_2} f_2 & 0 & 0 \\ 0 & 0 & \partial_{\alpha_2} F_2 & \partial_{\gamma_2} F_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \partial_{\alpha_3} f_3 & \partial_{\gamma_3} f_3 \\ 0 & 0 & 0 & 0 & \partial_{\alpha_3} F_3 & \partial_{\gamma_3} F_3 \end{pmatrix} \begin{pmatrix} \alpha_1' \\ \gamma_1' \\ \alpha_2' \\ \gamma_2' \\ \alpha_3' \\ \gamma_3' \end{pmatrix} \quad (13)$$

For convenience of description, might set:

$$[P] = \begin{pmatrix} \partial_a f_1 & \partial_b f_1 & \partial_c f_1 & \partial_\eta f_1 & \partial_\varphi f_1 & \partial_\delta f_1 \\ \partial_a F_1 & \partial_b F_1 & \partial_c F_1 & \partial_\eta F_1 & \partial_\varphi F_1 & \partial_\delta F_1 \\ \partial_a f_2 & \partial_b f_2 & \partial_c f_2 & \partial_\eta f_2 & \partial_\varphi f_2 & \partial_\delta f_2 \\ \partial_a F_2 & \partial_b F_2 & \partial_c F_2 & \partial_\eta F_2 & \partial_\varphi F_2 & \partial_\delta F_2 \\ \partial_a f_3 & \partial_b f_3 & \partial_c f_3 & \partial_\eta f_3 & \partial_\varphi f_3 & \partial_\delta f_3 \\ \partial_a F_3 & \partial_b F_3 & \partial_c F_3 & \partial_\eta F_3 & \partial_\varphi F_3 & \partial_\delta F_3 \end{pmatrix} \quad (14)$$

$$[Q] = \begin{pmatrix} \partial_{\alpha_1} f_1 & \partial_{\gamma_1} f_1 & 0 & 0 & 0 & 0 \\ \partial_{\alpha_1} F_1 & \partial_{\gamma_1} F_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \partial_{\alpha_2} f_2 & \partial_{\gamma_2} f_2 & 0 & 0 \\ 0 & 0 & \partial_{\alpha_2} F_2 & \partial_{\gamma_2} F_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \partial_{\alpha_3} f_3 & \partial_{\gamma_3} f_3 \\ 0 & 0 & 0 & 0 & \partial_{\alpha_3} F_3 & \partial_{\gamma_3} F_3 \end{pmatrix} \quad (15)$$

Then [***P***], [***Q***] are called Jacobian matrix of this manipulator. Therefore, the Eq. 13 can also be written as follows:

$$[P]\begin{pmatrix} a' \\ b' \\ c' \\ \eta' \\ \varphi' \\ \delta' \end{pmatrix} = -[Q]\begin{pmatrix} \alpha_1' \\ \gamma_1' \\ \alpha_2' \\ \gamma_2' \\ \alpha_3' \\ \gamma_3' \end{pmatrix} \quad (16)$$

According to Eq. 16, we can define two types of singular points as follows:

1): The first kind of singular point: the point of meeting $Det(\boldsymbol{P})=0$ and $Det(\boldsymbol{Q})\neq 0$.

2): The second kind of singular point: the point of meeting $Det(\boldsymbol{Q})=0$ and $Det(\boldsymbol{P})\neq 0$.

3. 1 Solving the Singularity

The solution process of singular configuration will be based on kinematics inverse solution and manipulator geometry relations. By using the method of three-dimensional search, search method is shown in Figure 2. Similarly, since the taken step size of the calculate will decide whether all the singular points can be searched, in order to quickly and effectively obtain all singular points of the 3/3-RRRS parallel manipulator, might set the step as: $\Delta R_1=0.01mm$, $\Delta\alpha=3°$, $\Delta Z=1mm$.

In order to maintain the continuity of analysis of the 3/3-RRRS parallel manipulator, we might still set the manipulator structure parameters as above. And set the moving platform orientation $\eta=0$, $\varphi=0$, $\delta=0$.

3. 2 Solve the First Kind Singular Points

By kinematics inverse solution, every group moving platform location corresponds to one group γ_i and eight groups α_i. That is, there are eight groups C_i coordinate combination. So according to the eight groups different C_i coordinate combination, we need to obtain eight different corresponding Jacobian matrixes, and work out the value of the corresponding Jacobian matrix's determinant. The obtained points which make $Det(\boldsymbol{P})=0$ are all singular points. The first kind singular points figure in 3/3-RRRS parallel manipulator is shown as Figure 7.

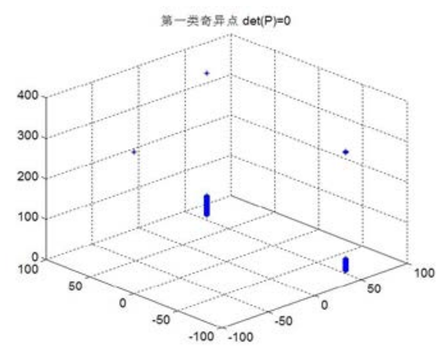

Figure 7: The First Kind Singular Points

It can be known easily, each singular points above is not in the position workspace which is determined by the condition of the moving platform orientation $\eta=0$, $\varphi=0$, $\delta=0$.

3. 3 Solve the Second Kind Singular Points

Similar to the solving process of the first singular points, the position output value of the moving platform need to plugged in eight different Jacobian matrixes. The obtained points which make $Det(\boldsymbol{Q})=0$ are all singular points. The number of the second singular point of the 3/3-RRRS parallel manipulator is zero by solving.

We know, through the analysis of the singular figure in example, that almost all the corresponding singular points under the condition of the moving platform orientation $\eta=0$, $\varphi=0$, $\delta=0$ are below the

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height of 100*mm* in the 3/3-RRRS parallel manipulator. And the two straight lines formed by these points are both parallel to the *Z* axis, and these points are all not in the corresponding space of the orientation. Therefore, the workspace in the condition of meeting the preset constraints is the final actual workspace.

4. CONCLUSIONS

This paper mainly analyses the workspace and singularity of the 3/3-RRRS parallel manipulator. In the different conditions of a given position and orientation, according to the different actuator angle constraint and other structural constraints, this paper has successfully solved the workspace figures having a certain geometry of the three-dimensional space. At the same time, all the singular point is obtained in this orientation, and shown in the three-dimensional figure.

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