

# A MODEL OF THE URBAN ROAD INTERSECTION LEFT-TURNING RESTRICTION

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## ABSTRACT

Intersection left-turning restriction is one of the important ways to solving the road congestion of the built-up area; however, how to set scientifically and rationally is the key. A bi-level programming model was making under the minimum the premise entire traffic network access time by using the stochastic traffic assignment model which is more appropriate. The upper planning described the intersection left-turning restriction by 0-1 variables, and the combination of Frank-Wolfe algorithm and genetic algorithm was built for solving the bi-level programming model. An example for solving the traffic organization of a road network in a city's old town was given. The result shows that the algorithm can solve this kind of problem high efficiently, and the scheme was gotten with certain scientificity and rationality.

**Keywords:** *Urban Road Intersection Left-turning Restriction, Stochastic Traffic Assignment, Bi-level programming, GA*

## 1. INTRODUCTION

As an important part of the urban road network, intersections have become the key restricting the ability of the entire road network. The improving methods of the intersection of the urban road network have been extensively studied, especially the left-turning restriction method. Of course, not all restriction measures are conducive to the improvement of traffic capacity; unreasonable settings will lead to delays, and aggravate the congestion in the network level. Therefore, the reasonable layout of intersections must be considered from the viewpoint of the entire network.

The viewpoints of the existing researches are either from qualitative analysis, or practical experiences [1-3], or quantitative analysis on a single intersection [4-6]; all these studies tend to analyze the issues empirically, but lack the consideration of internal mechanism. Jian Zhang et al.[7] established the bi-level programming model, discussed a numerical example as well as the solving method based on UE theory, pointing out the limitation for users obtaining the entire network information determines the limitation of the UE theory. Hailong Wang et al. [8] proposed the bi-level programming model based on stochastic user equilibrium, but the rationality for the mathematical expressions of the lower level needs to be verified. Thus, establishing a reasonable model which could

describe the problem effectively is the key to solve the problem.

## 2. ANALYSIS ON THE PROBLEM OF INTERSECTION LEFT-TURNING RESTRICTION

Essentially, the purpose of intersections is to complete a variety of steering actions. Reason for turn restriction should be explained from the traffic flow priority at intersections firstly. The traffic flow priority of intersections of roads with different levels may vary at the same direction, indicating turn restriction should satisfy certain conditions.

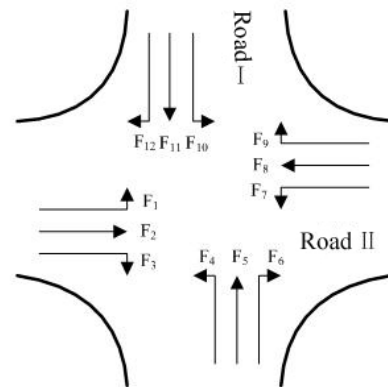


Figure 1: Traffic Flow In The Intersection

In the figure above, it is assumed that road I is the main road and road II is the branch road. The traffic is organized in accordance with the no signal light intersection, the traffic priority levels are shown in Table 1.

Table 1  
Traffic Flow Priority In The Intersection

priority level	level 1	level 2	level 3	level 4
Direction of the traffic flow	Go straight and turn right on the main road	turn left on the main road and the turn right on the branch road	Go straight on the branch road	turn left on the branch road
Direction in the figure	5,11,6,12	4,10,3,9	2,8	1,7

In the actual theoretical research and running process, left-turning restrictions must satisfy several conditions, which are the premise of obtaining good results. These preconditions including macro and micro aspect[1-4], the macro aspect refers to the requirements on the level of point, line and surface; the micro aspect includes traffic flow, traffic environment, as well as other traffic facilities. There are many classification methods for left-turning restriction from different viewpoints: fixed and temporary left-turning restriction from the viewpoint of time; left-turning restriction according to vehicle model and vehicle number from the viewpoint of vehicle type; left-turning restriction according to driving direction and turning direction from the viewpoint of range. On the left-turning restriction intersection, vehicle can takes many ways to accomplish the purpose of left turn. Combined with the characteristics of the road network and road segment, these ways can be divided into road network, road segments and nodes.

For a traffic network, the alternative set of intersections that can implement left-turning restriction based on infrastructure conditions and traffic operating conditions. Only a portion of these intersections will perform left-turning restriction. Thus, the intersections that can implement left-turning restriction are decision-making sets that can be known in advance. Under the specific traffic requirements, through setting reasonable left-turning restriction program, minimizing the total running time of all vehicles and improving the traffic condition of the roads can be achieved.

### 3. PROBLEM MODEL

#### 3.1 Network Representation Of Road Segments And Intersections

The basic elements of the abstract network are nodes of zero-dimension and arcs of one-

dimension, which are linear; these features can be presented using the node-arc theory in graph theory. In the actual operation of urban traffic network, there are 5 basic elements: zero-dimensional nodes, one-dimensional road segments, steering, OD pairs and two-dimensional traffic zones. In general, this is a nonlinear system, and the difficulty of the data modeling is greatly increased. Complex network can be adopted to describe an intersection in order to overcome the disadvantages of the ignoring of the steering. According to the complexity of the extension, the representation of an intersection has three different forms. Figure 2 shows the three different representations taking cross intersection as an example [9].

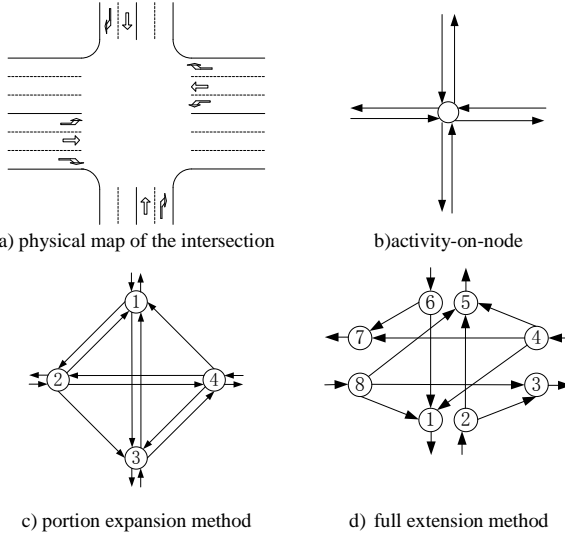


Figure 2: The Network Representation Method Of The Intersection

It can be seen from figure 2: graph b has the least nodes, but all of the turnings of the intersection are not shown; 4 virtual nodes are adopted in graph c, and the turnings are represented using virtual arcs, but the virtual nodes and arcs generate illegal paths that do not exist in reality, such as 4-3-1; graph d has the most nodes, overcoming the problem of illegal paths, but a large number of nodes and arcs make the storage space and time in the calculation increased largely. As mentioned above, the subject examined in the left-turning restriction program is several specific intersections, namely expansion only needs to be carried out on a portion of all intersections in the network, thus, the complexity of extended network is acceptable. In order to make reasonable modeling, the intersection will be processed based on full expansion method.

#### 3.2 Establishment Of The Model



Turning delay is the key to accurately quantify the problem, as a portion of travel cost difficult to be described. The existence of turning delay will affect the selection of the entire travel route, leading to travel cost function of the Jacobian matrix asymmetric, making it difficult to solve[10]. But the complexity of the problem can be alleviated through appropriate algorithms.

With a few assumptions, the stochastic traffic assignment (SUE) [8] is close to actual traffic conditions; therefore we in this work established a model based on stochastic traffic assignment to study the problem.

**3.2.1 Basic assumptions**

(1)The entire traffic network obeys specific assignment model, and the capacity of road segment is constant;

(2)Left-turning restriction is the only control measure of the network, and the set of lanes and signals adopting left-turning restriction is in line with the new traffic operation.

**3.2.2 Variable description**

Naming the traffic network contains many starting points and destinations, such as  $G$ , the connectivity of the network is very strong; the sets of all nodes, road segments, starting points and destinations are  $N$ ,  $A$ ,  $R$  and  $S$  respectively. Here are the variables involved in this article:

$r$  : Traffic generating node ,  $r \in R$  ;

$s$  : Traffic attraction node ,  $s \in S$  ;

$\Pi$  : Collection of network flows;

$\Pi_a$  : Collection of network flows related to road segment  $a$  ;

$x_{ab}$  : Flow from road segment  $a$  to road segment  $b$  ;

$K^{rs}$  : Collection of,  $|K^{rs}|$  is the number of the paths in the collection;

$f_k^{rs}$  : Flow of the  $k$ th path connecting  $O-D$  pair  $r-s$  ;

$\delta_{ak}^{rs}$  : 0-1 integer variable, 1 indicates  $a$  is on the  $k$ th path connecting  $O-D$  pair  $r-s$ , 0 indicates the opposite;

$\delta_{ab,k}^{rs}$  : 0-1 integer variable, 1 indicates segments  $a$  and  $b$  are on the  $k$ th path connecting  $O-D$  pair  $r-s$ , 0 indicates the opposite;

$P_k^{rs}$  : Probability for selecting the  $k$ th path connecting  $O-D$  pair  $r-s$  ;

$c_k^{rs}$  : Cost of the  $k$ th path connecting  $O-D$  pair  $r-s$  ;

$C(a)$  : Capacity of segment  $a$  ;

$q^{rs}$  : Travel demand on connecting  $O-D$  pair  $r-s$  ;

$t_a(x, y)$  : Cost function on segment  $a$  ;

$\Gamma$  : Collection of intersections with left-turning restriction;

$\mu = [\mu_l, \mu_h]$  : Left-turning is forbidden on segment  $\mu_l$  to enter segment  $\mu_h$ ,  $\mu_l, \mu_h \in A$ ,  $\mu \in \Gamma$ .

$y_\mu$  : 0-1 integer variable, 1 indicates left-turning restriction, 0 indicates the opposite;

$y$  : Vector form or collection of turning prohibition variable, i.e.  $y = [y_\mu]$  ;

$\phi_{\mu k}^{rs}$  : 0-1 integer variable, 1 indicates segments  $\mu_l$  and  $\mu_h$  are on the  $k$ th path connecting  $O-D$  pair  $r-s$ , 0 indicates the opposite.

**3.2.3 Lower model**

According to the theory proposed by Ying et al., we get the following lower model [11]:

$$F_{ab}(x, y) = x_{ab} - \sum_{rs} \sum_k q^{rs} \frac{\partial S^{rs}}{\partial c_k^{rs}} \delta_{ab,k}^{rs} = 0 \quad (1)$$

Where  $S^{rs}$  indicates satisfaction function, namely the minimum expectation cost for connecting  $O-D$  pair  $r-s$  :

$$S^{rs} = E[\min_{k \in K^{rs}} \{c_k^{rs}\}] \quad (2)$$

Besides, the travel cost function can be expressed as:

$$c_k^{rs} = \sum_a t_a(x, y) \delta_{ab,k}^{rs} + \sum_\mu y_\mu M \phi_{\mu k}^{rs} \quad (3)$$



Where is a sufficiently large positive constant, indicating the additional costs for not complying with the rule of left-turning restriction; assuming that this value is extremely large, and then no participant can accept this cost.

The above model can be used for multi-path selection; the satisfaction function will be examined using logarithmic form:

$$M S^{rs} = -\frac{1}{\theta} \ln \sum_k \exp(-\theta c_k^{rs}) \quad (4)$$

$$f_k^{rs} = q^{rs} P_k^{rs} \quad (5)$$

$$x_{ab} = \sum_{rs} \sum_k f_k^{rs} \delta_{ab,k}^{rs} = \sum_{rs} \sum_k q^{rs} P_k^{rs} \delta_{ab,k}^{rs} \quad (6)$$

The road segment formation time is generated by the extension of the impedance functions model given by Federal Highway Administration:

$$t_a(x, y) = t_a^0 \left( 1 + \chi_1 \left( \frac{x_{a,TH} + \phi_{RT} x_{a,RT} + \phi_{LT} x_{a,LT}}{s_a} \right)^{\chi_2} \right) \quad (7)$$

Where  $t_a^0$  is the travel cost on segment  $a$  in free flow state;  $x_{a,TH}$ ,  $x_{a,RT}$  and  $x_{a,LT}$  are the traffic flow of going straight, turning left and turning right respectively;  $s_a$  is the capacity of segment  $a$ ;  $\phi_{RT}$  and  $\phi_{LT}$  are the parameters of the effect of turning left and turning right on the flow of going straight respectively;  $\chi_1$  and  $\chi_2$  are the relative parameters of this formula. In this model, the travel time, as well as blocking delay and turning delay in free flow is fully considered.

### 3.2.4 Upper model

Based on the above assumptions, the following model is obtained [7, 12]:

$$\min G(x, y) = \sum_{ab} x_{ab}(y) t_{ab}(x(y), y) \quad (8)$$

$$\text{s.t} \quad y_\mu = \{0, 1\}, \forall \mu \in \Gamma \quad (9)$$

Where  $t_{ab}(x(y), y) = t_a(x(y), y)$ ;  $x_{ab}(y)$  is the implicit function of  $y$ , and can be obtained by solving the lower level programming.

In addition, take into account the purpose of left-turning restriction is to balance the traffic flow of the entire road network and to ease the saturated

intersections and road segments, therefore the following constraints must be satisfied:

$$S(a) = \frac{x(a, y)}{C(a)} \leq \bar{S}(a), \quad a \in A \quad (10)$$

$$S(a, y) = \frac{x(a, y)}{C(a, y)} \leq \bar{S}(a, y), \quad a \in \Gamma \quad (11)$$

Where  $\bar{S}(a)$  indicates the saturation degree (maximum) of segment  $a$  under given service level.

## 4. ALGORITHMS

The specific steps of the algorithm are shown in figure 3 [8, 13].

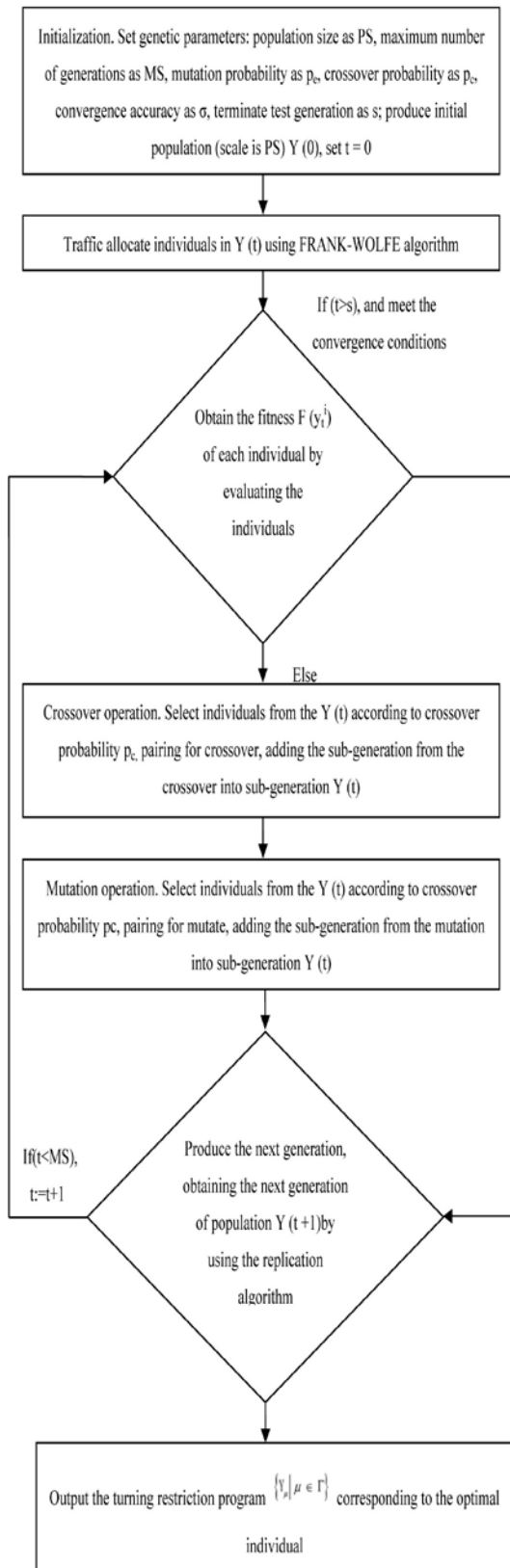


Figure 3: Algorithm Steps Diagram

5. A NUMERICAL EXAMPLE

The road network of the old city of a city is constituted of four vertical and four horizontal roads, with the increase in the number of motor vehicles, traffic jams often occur. According to the status quo of the road network of the old town, the research mainly focuses on the area inside the ring road (including the First Ring Road). The abstract network diagram of the roads in this area is shown in figure 4 (intersections marked with diamond-shaped box are alternative collection).

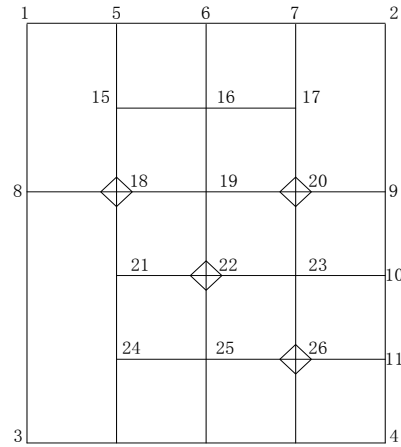


Figure 4: The Structure Of Road Network In Study Area

The network contains a total of 22 intersections and 37 road segments; all edges in figure 4 are bidirectional arcs (two-way road); four intersections in the network are set as left-turning restriction (generally should be four-phase signal control before prohibiting left-turning).

Traffic flow of each segment is obtained through traffic surveying on all the roads and intersections at different time [14], and the following OD table is calculated using relevant software.

Table 2: OD Distribution

OD	1	2	3	4
1	0	1200	1100	900
2	850	0	1050	950
3	750	1100	0	800
4	850	900	950	0

The road network of the old city was constructed in the last century, the width of each road segment is basically the same, and the capacity of each segment is about bidirectional 2000pcu/h. Therefore, the one-way traffic capacity is set as 1000pcu/h in this work. The form of the road network is a typical grid network, the density of the road is 5.78km/km<sup>2</sup>, and the average spacing of intersections is 400 m. Set the travel speed is 30km/h, then the green time of the road segment is

48s (approximately 50s). At segments considering the set of left-turning restriction, assume the intersection's average delay is 10s, and then the green time of common road segments is 60s. In addition, assume the impedance for right-turn traffic flow is 0, the impedance and capacity of the intersection under different restriction manners is shown in table 3:

Table 3: The Impedance And Capacity Of Each Direction In The Intersection

Restriction program	impedance /s		capacity /(pcu/h)	
	Straight vehicle	Left-turning vehicle	Straight vehicle	Left-turning vehicle
No restriction	20	25	600	150
Same direction	15	∞	720	0
Opposite direction	15	20	720	200
All directions	10	∞	980	0

Substitute the above parameters into the model, the chromosome of genetic algorithm will be coded in the order of 18,20,22,26 clockwise, taking north as the beginning (a total of 16 bits). The problem is a 0-1 decision problem; set the maximum number of generations is 100, the initial population is 20, the crossover rate is 0.7, the mutation probability for chromosome is 0.07, and the initial traffic flow is obtained using the all or none method.

The configuration of the computer used for simulation machine: P4 CPU with 3.0GHZ, WindowsXP operating system, and Microsoft Visual C++6.0 programming platform. The average calculating time is less than 5 seconds. The final output of the program is shown in figure 5.

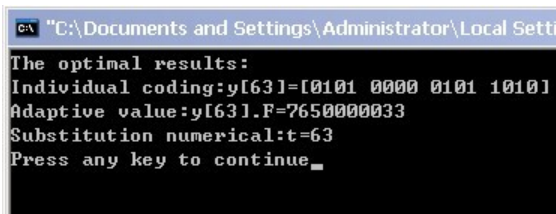


Figure 5: The Output Of The Algorithm

Figure 5 illustrates the best solution is to set left-turning restriction in the east and west of intersections 18 and 26, and the south and north of intersection 22. The total travel time is  $7.65 \times 10^8$  s (the original travel time is  $9.71 \times 10^8$  s).

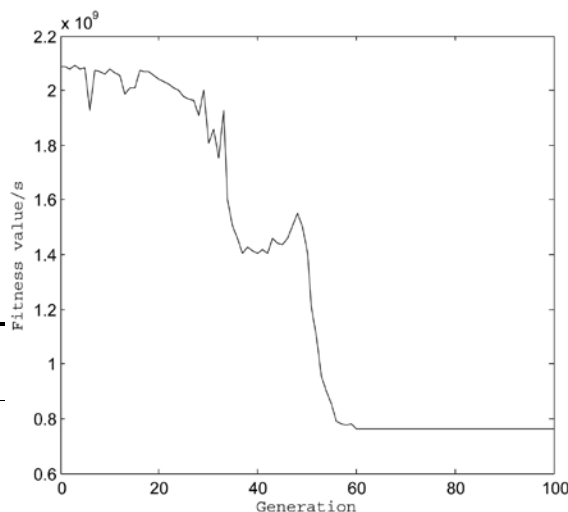


Figure 6: The Relationship Between The Adaptive Value And The Evolution

Figure 6 illustrates the optimization ability of the algorithm designed in this work is good: the descent gradient of convergence is very large, and the global optimization will be reached in the 63rd generation.

## 6. CONCLUSION

As a commonly used traffic organization manner, turning restriction at urban road intersections has the advantage of low cost and high efficiency. However, many factors need to be considered when taking this method, because unreasonable installation will reduce the efficiency of the entire traffic network. In this work, SUE method is introduced to examine the installation of left-turning restricted intersections. The upper programming model is built by using the minimum network green time, and the lower programming is the SUE traffic assignment. A combination algorithm is proposed based on existing algorithms, which is dominated by the Frank-Wolfe algorithm and accompanied with genetic algorithm. The rationality of the model and the effectiveness of the algorithm adopted in this work are verified through the programming of the left-turning restriction of the traffic network of an old town. The result of this article overcomes the subjective and arbitrary phenomenon in the decision of traffic organization plans, and provides certain basis to the decision-making of the management sector.

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