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OPTIMIZATION METHOD BASED ON DOE AND GRADIENT ALGORITHM

[1]XU YIN, *[2]AIMIN JI , [3] XINLEI HUANG, [4]PENGFEI SUN
[1,2,3,4] College of Mechanical & Electrical Engineering, Hohai University, Changzhou, Jiangsu Province 213022,P.R. China
E-mail: [1]hohaiyx@126.com, [2]jam@ustc.edu, [3]huangxinlei89@126.com,[4] sunpengfei30@163.com
* Corresponding Author

ABSTRACT

To solve problems that exist in optimal design such as falling into local optimal solution easily and low efficiency in multi-objective optimization, a new approach based on design of experiments (DOE) and gradient optimization (GO) was proposed. The new optimization method is called DPG (DOE Plus GO) which used DOE for preliminary analysis of the function model, and took the optimal values obtained in DOE stage as the initial values of design variables in GO stage so as to reduce the effect on the result of optimization made by the designers' decision. This paper gave two typical examples of optimization to confirm DPG global, efficient, and accurate with Isight code. Firstly, the bimodal problem was used to test DPG's global optimization ability, then the multi-objective optimization of the machine tool spindle, which required minimum quality, maximum stiffness, and strength was conducted. The results show the DPG optimization method could not only avoid falling into local solution, but also have an obvious superiority in treating the multi-objective optimization problems.

Keywords: *Local Solution, Multi-objective Optimization, Design of Experiments (DOE), Gradient Optimization (GO), DOE Plus GO (DPG)*

1. INTRODUCTION

Optimization technique is an application technology based on mathematics to solve different practical problems. It is widely used in industry, agriculture, national defense, engineering, etc. and produces great economic benefits in resource utilization, structure design, operation management, etc. As a powerful solving method, optimization has rapidly developed into an important applied mathematics discipline.

Although optimization technique seems mature nowadays, there are still optimization problems like falling into local optimal solution easily in the solving process. To solve this problem, WANG Meng[1] further expanded the searching depth of each dimension based on the complex method, and introduced GO thought into the optimization process; YAN Xueli[2] put forward a PSO algorithm with an increasing diversity in sharing particle information, and then combined the global optimal information of the previous rounds with current round to avoid the local solution; LI Min[3] took the directional migration into genetic algorithm, and promoted optimization accuracy by using the orientation of directional migration theory; WU Linli[4] rewrote the inertia weight and learning factor respectively by combining global and local optimum, simplified the velocity updating formula, then put forward the PSO with global-local best minimum optimization method.

In real life, optimization designs often expect a few objectives can simultaneously to achieve optimal values, which called the multi-objective optimization problems. While solving this kind of problem, people should coordinate between every subgoal optimal values and make a concession mutually to coupling the whole optimal solution. Usually, people can only get effective solution (also called Pareto solution or non-inferior solution) in multi-objective problem.

Along with the promotion of the product complexity, traditional multi-objective optimization methods like weighting method, goal programming method and non-inferior solution based on Pareto can no longer meet the efficiency need, so people started researching on methods that can obtain the Pareto solution with a high efficiency. Tappeta[5]

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integrated the weighting method and collaborative optimization method first, and then weighted the multiple targets into a single goal, at last used the collaborative optimization method for solving; Huang[6] put forward multi-objective pareto concurrent subspace optimization (MOPCSSO), which expanded the CSSO from single target to multi-objective; ZHANG Xiang[7] founded a new evaluation function with multi-objective mathematical meaning to deal with the disadvantage in the evaluation function; LEI Xin[8] put forward a new parametric method that regarded the goal of the evaluation function weight coefficient as parameters, pointed out the optimal solution depends on continuous variation of the parameters, and then gave parameters based on requirement to optimize the solution; GUAN Zhihua[9] introduced a method based on fuzzy preference, which used the fuzzy preference to determine the weight of the objective function.

Through the analysis of research status, the problems existed in optimization design can be concluded as falling into local solution easily and low efficient in multi-objective optimization. Section 2 puts forward a new optimization method based on DOE and GO called DPG (DOE Plus GO). In section 3, the bimodal problem with initial values sensitivity and local shape features is used to test the DPG's feasibility of preventing falling into local solution; then this paper uses the parameter optimization of the machine tool spindle, which is a typical multi-objective problem, to test its efficient and accuracy. Section 4 gives a conclusion that DPG is a method with obvious superiority in dealing with multi-objective optimization problems.

2. DPG OPTIMIZATION METHOD

The DPG in this paper combines DOE and GO algorithm. This method changes the thought of random optimization in the past. Firstly, it uses DOE to optimize feasible region evenly, then transfers the optimization solution in DOE stage to the initial values in GO stage, and uses GO to search for the global solution at last.

Because the DPG method has a variety of algorithmic combinations, this paper takes Latin hypercube design (LHD) and sequential quadratic programming (NLPQL) as the optimization strategy of the DOE and GO stage to verify the feasibility of DPG method.

The theory of LHD [10] is that the method evenly separates each one dimensional coordinate space $\left[x_k^{\min}, x_k^{\max}\right], k \in [1,n]$ into m blocks and records each block as $\left[x_k^{i-1}, x_k^{i}\right], i \in [1,m]$. Random select m points ensure that each level of the factor is researched only once, and a LHD design with n-dimensional space and m points is founded, recorded as $M \times N$ LHD.

NLPQL method expands the objective function by second order Taylor series, linearizes the constraint conditions and solves the quadratic programming to get the next design point; then begins a linear search according to the two alternative optimization functions. NLPQL is a stable algorithm and its Hessian matrix is updated by BFGS equation.

The theory of DPG algorithmic combination: LHD+NLPQL is as follows:

Optimization model can be described as follows:

$$\begin{aligned} &\min y = f(x) \\ &s.t. \quad c_i(x) = 0 \qquad i \in E = \{1, 2, \cdots, \ m_e\} \\ &\qquad\ \ c_i(x) \geq 0 \qquad i \in I = \{m_e + 1, \cdots, \ m\} \end{aligned} \tag{1}$$

LHD optimization stage:

Use the known cumulative distribution function $F_{x_i}(x)$ to describe each input parameter $x_i\,(i=1,2,\cdots,n)$, and then separate the $F_{x_i}(x)$ of x_i into n blocks which named as $S_{ij}\,(j=1,2,\cdots,N)$, each block is represented by probability P_{ij}, as Eq. (2) shows:

$$P_{ij} = P\left(x_i \in S_{ij}\right) \tag{2}$$

$$\sum_{j=1}^{N} P_{ij} = 1 \qquad (i = 1, 2, \cdots, n) \tag{3}$$

So under the equal probability interval $P_{ij} = 1/N$.

During the Latin hypercube sampling process, S_{ij} is on behalf of the representative parameters while the parameters are selected by random method. Generate n random numbers $U_j\,(j=1,2,\cdots,N)$ in interval (0, 1), then use Eq. (4) to change U_j to random number Q_j in interval *j*.

$$Q_j = \left[U_j + (j-1)\right]/N = \frac{U_j}{N} + \frac{j-1}{N} \qquad j = 1, 2, \cdots, N \tag{4}$$

The Eq. (10) is obviously founded:

$$(j-1)/N < Q_j < j/N \tag{5}$$

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$(j-1)/N$ is the lower bound and j/N is the upper bound, so only one random number Q_j will be generated in each interval.

$$x_{ji} = F_{ji}^{-1}(Q_j) \qquad i = 1, 2, \cdots, n \tag{6}$$

$F_{ji}^{-1}(\ )$ is the inverse cumulative distribution function of the input variable *i*.

Search for the x_i that makes $y's$ value become minimum in DOE stage, and set it as the initial values of all design variables $x_k\,(k = 1, 2, \cdots, n)$ in the GO stage.

NLPQL optimization stage:

A sub problem of quadratic programming is founded:

$$\begin{aligned} \min \quad & \nabla f(x_k)^T d + \frac{1}{2} d^T H_k d \\ s.t. \quad & c_i(x_k) + \nabla c_i(x_k)^T d = 0 \quad i \in E \\ & c_i(x_k) + \nabla c_i(x_k)^T d \ge 0 \quad i \in I \end{aligned} \tag{7}$$

①Hessian matrix H^k is updated by BFGS equation in NLPQL,

$$\begin{aligned} & p^k = a^k d \qquad q = \nabla^T f(x^{k+1}) - \nabla^T f(x^k) \\ & H^{k+1} = \left[I - \frac{p^k (q^k)^T}{(q^k)^T p^k} \right] H^k \left[I - \frac{q^k (p^k)^T}{(q^k)^T p^k} \right] + \frac{p^k (p^k)^T}{(q^k)^T p^k} \end{aligned} \tag{8}$$

②Search direction $S^{(k)}$ is solved by Eq. (9),

$$S^{(k)} = -H^k \nabla f(x) \tag{9}$$

③Step length $\alpha^{(k)}$ is searched along the $S^{(k)}$, as follows,

$$f\left(x^{(k)} + \alpha^{(k)} S^{(k)}\right) = \min_{\alpha \ge o} f\left(x^{(k)} + \alpha S^{(k)}\right) \tag{10}$$

So the next iteration point can be calculated by $X_{k+1} = X_k + \alpha^{(k)} S^{(k)}$. After several iterations, y becomes its minimum, and x_k is the global optimization solution.

Now the DPG optimization steps are as follows:

⑴Analyze the optimization problem, found function model after clearly ensuring the situation of specific variables, constants, constraints and objective functions;

⑵Use Isight code to found the model, and then the DOE analysis stage begins:①Confirm test plan: Define test design factors and their properties, select test method like LHD,②Implement test: Begin the operation according to the settled test plan,③Result analysis: Help designers to analyze the DOE test results and drew the corresponding conclusion;

⑶DOE optimization shows the Pareto diagram that can reflect the degree of response contributed by each effect factors, designers should take more attention to factors with greater contribution ;

⑷Set the optimal solution in DOE stage as the initial values of all design variables in GO stage, so that the local solution caused by the improper selection of initial values will be forbidden, and can also reduce the burden for the designers;

⑸Enter the GO optimization process. Search for new $X^{(1)}$ based on direction $S^{(0)}$ and step length $\alpha^{(0)}$ to cast down the value of f until global optimization solution x^* is founded。 The formula to determine the sequence is as follows:

$$X^{(k+1)} = X^{(k)} + \alpha^{(k)} S^{(k)} \tag{11}$$

⑹Confirm the formulas of search direction $S^{(k)}$ and step length $\alpha^{(k)}$;

⑺Every time that a group of optimal value is received in GO stage will conduct a feasibility verification, if it cannot meet the design requirement, return to Step. 4 and reset all initial values, then continue Step.4~Strp.7 until the feasibility meets the demand;

⑻View the optimization solution of DPG method, and the compare with the former value of parameters to verify the superiority of DPG method.

Fig.1 is the workflow of DPG optimization method.

3. DPG EXAMPLE VERIFICATION

3.1 Bimodal Problem [11]

Objective function:

$$\max Z = f_1 + f_2 + f_3 \tag{12}$$

$$\begin{aligned} & f_1 = 60/(1+(X+1)^2+(Y-3)^2) \\ & f_2 = 20/(1+(X-1)^2+(Y-3)^2) \\ & f_3 = 30/(1+X^2+(Y+4)^2) \end{aligned} \tag{13}$$

Constraints: $Z > 25$

The scope of design variables X and Y is $[-5, +5]$. The paper uses Isight to combine Excel and the optimizer to found the bimodal problem model

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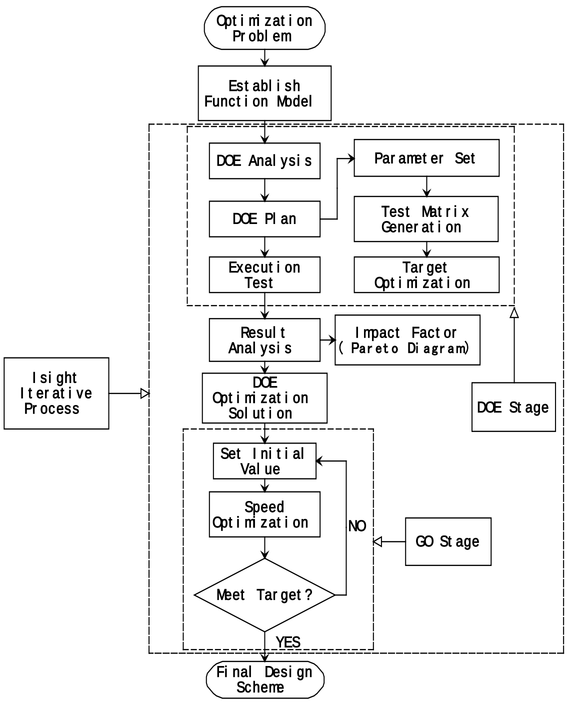

Figure 1 Workflow of DPG Optimization Method

First of all, the paper uses traditional method—NLPQL to solve this problem. Set the initial values of the design variable as $(0,-3.5)$, maximum iterations as 40, convergence accuracy as $1.0E-6$, while others are default. After 21 times iteration, the optimal solution $(0,-4), Z=31.57$ is received. In Fig.2, the value of Z is increased with a gradual change from green to yellow, and then the maximum is red. It is obvious that $(0,4)$ is just a local solution. It is mainly because the wrong choice of the initial point.

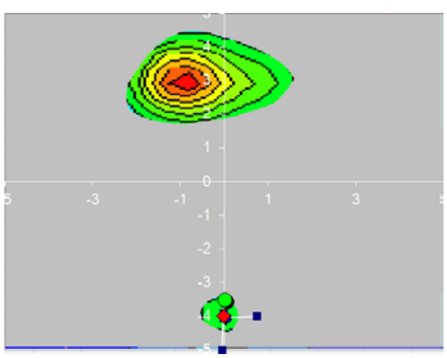

Figure 2 Contour Map of the Objective Functions

Then this paper uses DPG method to optimize the problem, LHD+NLPQL model is founded like Fig.3. The model is separated as a top task (DPG-1) and two subsystem tasks (LHD and NLPQL). The number of test points in LHD stage is set as 30 and NLPQL is the same as above.

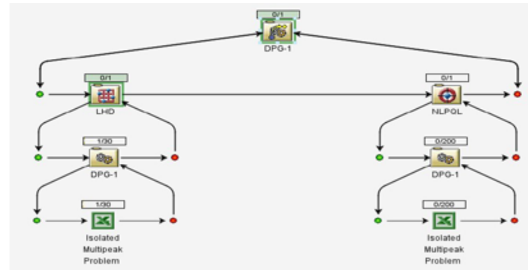

Figure 3 Process of Solving Bimodal Problem By DPG

After optimizing for 30 times, DOE gets the optimization solution $(-1.55, 4.31)$, going on iterating for 35 times in NLPQL stage, then the global solution $(-0.97,3), Z=64.63$ is received. The contour curve of the objective function in Fig.4 shows that after the DOE design, NLPQL searches for the solution only around the point $(-1.55, 4.31)$. The DPG method not only avoids falling into the local solution, but also provides reasonable initial values automatically to reduce the effect on the optimization caused by the designers' decision. Fig.5 and Fig.6 show the solving process of bimodal problem.

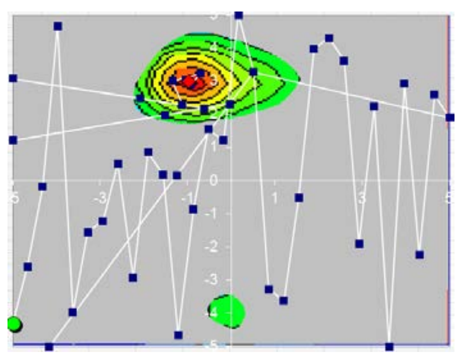

Figure 4 Contour Curve of the Objective Functions

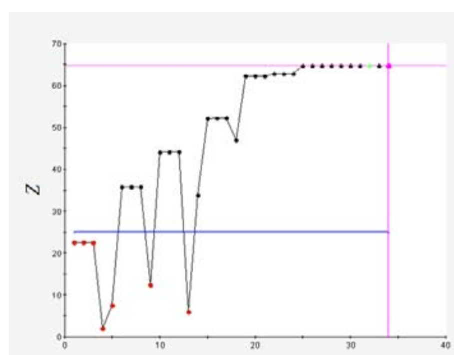

Figure 5 Optimization of Objective Function Value Z

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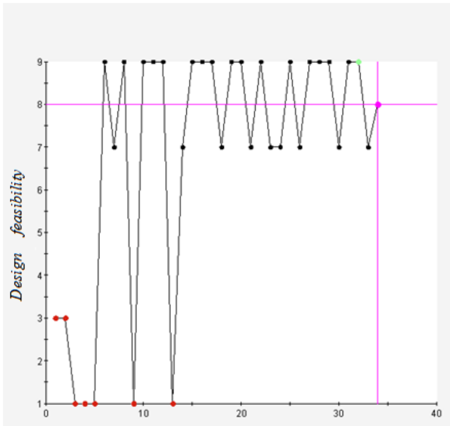

Figure 6 Design Feasibility (1-worst, 9-best)

3.2 Multi-objective Optimization of Machine Tool Spindle

The machine tool spindle optimization is a typical multi-objective problem. Its goal is to minimize the spindle quality and overhanging end deflection in the premise of satisfying the constraint conditions. Therefore, coordination is needed between the optimal values of the two goals to find the Pareto solution.

There are many researches on the optimization problems: LI Yangxing[12] and LU Haitao[13] took the quality of the spindle as the optimization objectives, and used fuzzy optimization algorithm and particle swarm optimization(PSO) for structural optimization; GUO Chenguang[14] completed the spindle parameter optimization by using genetic algorithm(GA) while doing research on the spindle structure and load deformation at the same time; Li[15] used the MATLAB code to analyze the optimization based on the lowest cost and the improved spindle model. LI Wanxiang[16] used MATLAB code to optimize the spindle and received a group of optimization solutions after several times of attempts. These researches above had improved the performance of the spindle to a certain extent, but there is still optimization space due to the limitation of method.

3.2.1 Objective function and constraint conditions

Machine tool spindle is generally a several support hollow step shaft; this paper simplifies the shaft into constant section. As Fig.7 shows, d is the spindle inner diameter, D is the external diameter, l is span, a is the length of overhanging end, y is deflection, and F is the load on the main shaft overhang end.

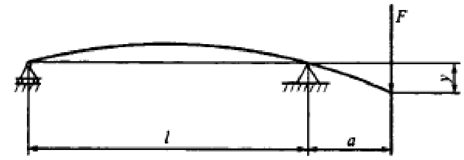

Figure 7 Machine Tool Spindle Deformation Diagram

In order to make the comparison between the DPG optimization and the existing methods easier, the example uses the value of parameters in literature [16]: $d = 45mm$, $F = 15000N$, allowable deflection $[y] = 0.125mm$, allowable shear stress $[\tau] = 220MPa$, allowable torsion angle $[\theta] = 0.02rad$. Main material density $\rho = 7800kg/m^3$, elastic modulus $E = 210GPa$, shear modulus $G = 80GPa$. Spindle speed $n = 80r/\min$, input power $P = 7.5kW$.

The mathematical model of optimization can be founded according to the above parameters; the design variable is elected as:

$$X = [x_1 \quad x_2 \quad x_3]^T = [D \quad l \quad a]^T \tag{14}$$

The first objective function minimizes the quality of spindle:

$$\min f_1(X) = \frac{1}{4\times 10^9}\pi(D^2 - d^2)(l+a)\rho \tag{15}$$

The second objective function minimizes the overhanging end deflection:

$$\min f_2(X) = \frac{64Fa^2(l+a)}{3000\pi E(D^4 - d^4)} \tag{16}$$

The third objective function minimizes the spindle shear stress:

$$\min f_3(X) = \frac{T}{W_n} \tag{17}$$

Among them, T is the shaft torque, $T = 9550P/n$; W_n is the torsional modulus of section, $W_n = \frac{\pi(D^4 - d^4)}{16D}$.

In the spindle designing stage, these following constraints will be used:

①Stiffness constraints:

$$g_1(X) = f_2(X) - [y] \le 0 \tag{18}$$

②The torsion angle of shaft overhang end cannot exceed the allowable value:

$$g_2(X) = \frac{32T(l+a)}{G\pi(D^4 - d^4)} - [\theta] \le 0 \tag{19}$$

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③Strength constraint:

$$g_3(X) = f_3(X) - [\tau] \le 0 \tag{20}$$

④Boundary constraint:

$$\begin{aligned} &D_{\min} \le D \le D_{\max} \\ &l_{\min} \le l \le l_{\max} \\ &a_{\min} \le a \le a_{\max} \end{aligned} \tag{21}$$

3.2.2 Calculation results and analysis

This paper uses DPG to solve the machine tool spindle optimization in Isight code. The initial value is $X = \left[100mm \quad 470mm \quad 110mm\right]^T$ and its constraints are $D_{\min} = 50mm$, $D_{\max} = 150mm$ $l_{\min} = 210mm$, $l_{\max} = 600mm$, $a_{\min} = 90mm$, $a_{\max} = 150mm$. The options of DOE and GO are just the same as in Example *1*.

After 30 rounds of iterations in DOE, the optimization solution $\left[100.34 \quad 84.48 \quad 210\right]$ is transferred into GO stage, and then through 73 rounds of gradient optimization iteration, the paper gets the final solution in Table.1.

Fig.8, Fig.9 and Fig.10 show the iterative process of the optimal objective functions in the GO stage. The increasing of f_1 will lead to the decreasing of f_2 and f_3, which is the feature of multi-objective problem. The figures prove that DPG method is a very efficient solution to solve the constraint relationship in multi-objective optimization.

Table 1 Comparison Between Optimization Results

Parameter	Optimization Method		Optimize Proportion
	DPG	Literature [16]	
D / mm	60.68	63.00	
l / mm	210.00	210.00	
a / mm	90.00	90.00	
Quality / kg	3.046	3.57	14.68%
Deflection /mm	0.1246	0.1247	0.08%
Shear stress /MPa	29.25	29.25	0

PS: Results are not round due to the contrast need

Comparing the optimization results of the DPG method with the previous ones', although there is no big change in overhanging end deflection and spindle shear stress, but the quality of spindle has been further reduced by a range reached 14.68%. So DPG makes a positive effect in the optimization.

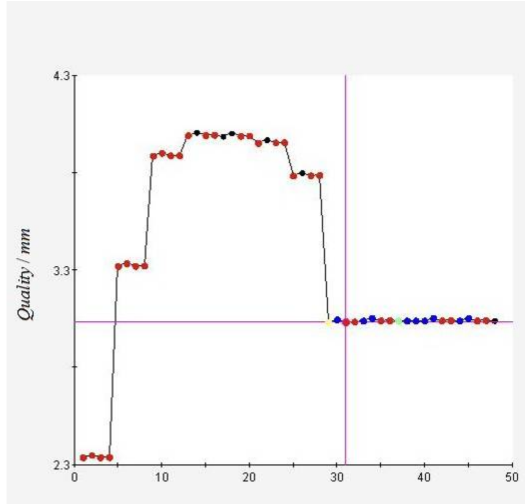

Figure 8 Optimization of Quality Function $\min f_1$

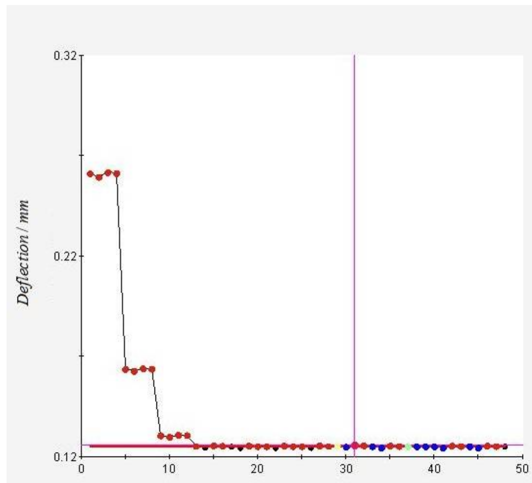

Figure 9 Optimization of Deflection Function $\min f_2$

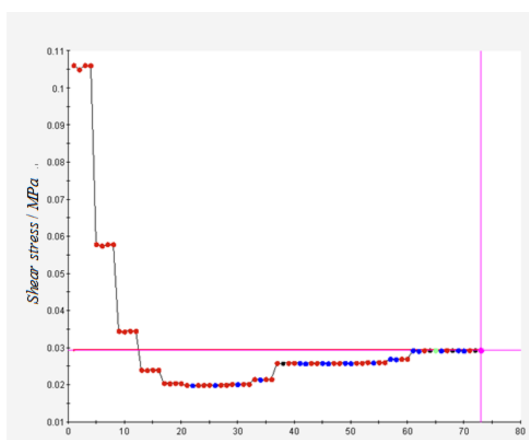

Figure 10 Optimization of Spindle Shear Stress $\min f_3$

In the meantime, DPG's effect on the convenience of the designer is obviously. Fig.11,

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Fig.12 and Fig.13 are the Pareto charts received in DOE stage [17], which reflect diameter D is occupying the absolute leading position in influencing the performance of spindle.

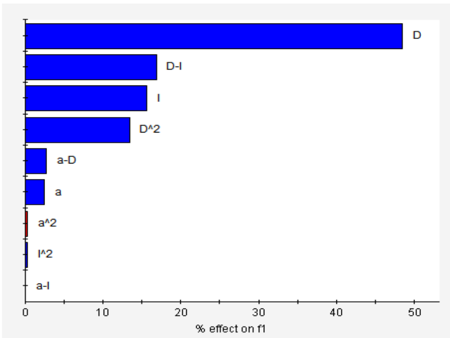

Figure 11 Pareto Chart of Optimization Function f_1

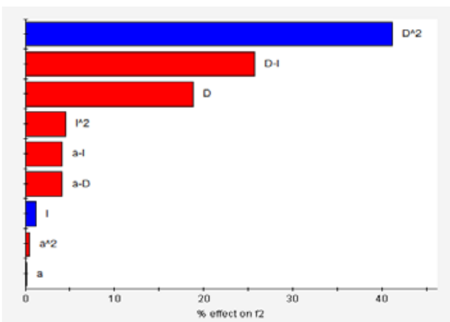

Figure 12 Pareto Chart of Optimization Function f_2

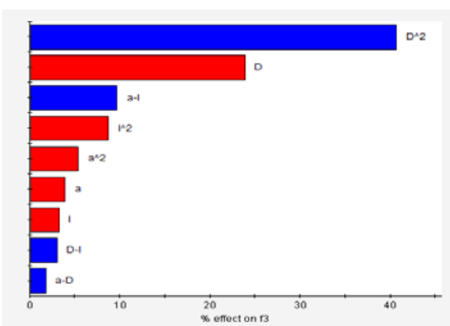

Figure 13 Pareto Chart of Optimization Function f_3

4. CONCLUSION

(1)Optimization technique, as an application discipline with highly practical significance, is troubled by problems like falling into local solution, low optimization efficiency, etc. The paper puts forward an optimization method named DPG based on DOE and GO. By using this method, Example.1 received the global solution and in Example.2 the quality of spindle was reduced by 14.68%. The whole optimization processes are proved more comprehensive, more efficient and more accurate.

(2)DPG method not only increases the robustness of optimization solutions, but also reduces the effect on the result made by designers' decision. DPG makes the transfer of initial values $X^{(0)}$ from DOE stage to GO stage automatic, and reflects the importance of each factor by Pareto chart. In the future, the DPG method should be further improved by using more complicated problems.

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