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A GREY DATA ENVELOPMENT ANALYSIS MODEL

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ABSTRACT

In this paper, grey system theory and DEA are combined to establish a grey DEA model. In traditional DEA model, Input and output values of decision-making groups are all real numbers and real variables. They are replaced by grey numbers and grey variables in grey DEA model respectively. Using positioning solution method of grey linear programming, we prove the necessary condition for grey effective of decision making unit. As a practical application, we evaluate the performance of eight listed companies in Qingdao City, Shandong Province, China. Listed companies are seen as decision-making unit, we analyze DEA effective of their performance.

Keywords: *Grey System, Data Envelopment Analysis, Grey Linear Programming, Grey Dea Effective*

1. INTRODUCTION

Grey system theory is a new method that studies problem of little research data and poor information with uncertainties. Its experimental observation data has no any special requirements and limitations, and therefore it is a very broad application area. Professor Deng Julong puts forward the grey linear programming (LPGP) theory. In GLP model, grey number was introduced to represent uncertainty information, and the grey information bring the grey solution.[1-3] After nearly three decades of development, taking “small sample”, “poor information” uncertain system as the research objects, the comparatively perfect theoretical framework, method groups and model groups have been gradually included in grey systems theory, and great superiority and vigorous vitality are shown when they are applied in various fields such as society, economy, life, production, etc. [4-10]

Data envelopment analysis (DEA) is a crossover study field in operations research, management science and mathematical economics. Based on the relative efficiency concept and multiple indicator inputs and multi-objective indicator of output data, data envelopment analysis evaluates the relative effectiveness of the decision-making unit. The significant advantage of DEA is that we can calculate the input-output efficiency of the decision-making unit without considering the functional relationship between inputs and outputs, no requiring pre-estimate parameters and assuming any weights, avoiding subjective factors. Especially DEA effectiveness is equivalent to Pareto effectiveness. Because of this unique advantage, in the past 30 years DEA has made great progress and a large number of theoretical studies and practical applications. DEA has become a mathematical analysis tools in management science and systems engineering. [11-16]

Data envelopment analysis (DEA) theory, taking efficiency analysis as the research starting point, nonparametric methods as characteristics, programming models as research tools, is a common system analysis method in management science research. When used for efficiency evaluation in uncertain systems, the traditional DEA models appear "stranded". DEA models with imprecise data which have been made some achievements but still not perfected need further in-depth study. There are the same application fields of grey system theory and data envelopment analysis from different research perspectives, and combined models with the advantages of the both theories have yet to be established and tested in practice.

2. PRELIMINARIES

The organization in the DEA is decision making unit abbreviated as DMU. Suppose there are n decision making units ($j=1,2,\cdots,n$), each decision making unit has m inputs ($i=1,2,\cdots,m$) and s outputs ($r=1,2,\cdots,s$). x_{ij} and y_{ij} represent the i-th input and the r-th output of the j-th decision making unit respectively. In the traditional DEA model, x_{ij} and y_{ij} are all real numbers. The following linear programming model is used to

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determine whether a decision making unit j_0 is DEA efficient.

$$\min E$$
$$st.$$
$$\begin{cases} \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rj_0} & (\text{r=1,}\cdots\text{s}) \\ \sum_{j=1}^{n} \lambda_j x_{ij} \geq E x_{ij_0} & (\text{i=1,}\cdots\text{m}) \\ \sum_{j=1}^{n} \lambda_j \geq 1 & (\text{j=1,}\cdots\text{n}) \end{cases} \quad (1)$$

When E <1, the j_0 decision making unit is non-DEA effective, otherwise it is DEA effective. Suppose there are n decision making units, each decision making unit has m types of inputs and s types of outputs ($j = 1, 2, \cdots, n$).The integrated DEA model is as follow.

$$\left(P^I\right)\begin{cases} \max \mu^T Y_0 - \delta_1 \mu_0 = V_{PI} \\ \omega^T X_j - \mu^T Y_j + \delta_1 \mu_0 \geq 0, j = 1, 2, \cdots, n \\ \omega^T X_0 = 0 \\ \omega \geq 0, \mu \geq 0, \delta_1 \delta_2 \left(-1\right)^{\delta_3} \mu_0 \geq 0 \end{cases} \quad (2)$$

$$\left(D^I\right)\begin{cases} \min \theta = V_{D^i} \\ \sum_{j=1}^{n} X_j \lambda_j \leq \theta X_0 \\ \sum_{j=1}^{n} Y_j \lambda_j \geq Y_0 \\ \delta_1 \left(\sum_{j=1}^{n} \lambda_j + \delta_2 \left(-1\right)^{\delta_3} \lambda_{n+1} \right) = \delta \\ \lambda_j \geq 0, j = 1, 2, \cdots, n, n+1 \end{cases} \quad (3)$$

Where $X_j = \left(x_{1j}, x_{2j}, \cdots, x_{mj}\right)^T$ are input vector and $Y_j = \left(y_{1j}, y_{2j}, \cdots, y_{sj}\right)^T$ are output vector. The corresponding production possibility set is as follow.

$$T = \{(X, Y) \mid \sum_{j=1}^{n} X_j \lambda_j \leq X, \sum_{j=1}^{n} Y_j \lambda_j \geq Y,$$
$$\delta_1 (\sum_{j}^{n} \lambda_j + \delta_2 (-1)^{\delta_3} \lambda_{n+1}) = \delta_1,$$
$$\lambda_j \geq 0, j = 1, 2, \cdots, n, n+1\} \quad (4)$$

Where the value of parameter δ_1 , δ_2 , δ_3 is 0 or 1.

3. GREY DATA ENVELOPMENT NALYSIS MODEL

In the traditional DEA model, Input and output values of decision-making groups are all real numbers. However, in many practical problems, evaluators often do not know their true value. The evaluation variables are indicated by grey number more appropriately.

Suppose there are n decision making units, each decision making unit has m types of inputs and s types of outputs ($j = 1, 2, \cdots, n$).The integrated grey DEA model is as follow.

$$\left(G - P^I\right)\begin{cases} \max \mu^T Y_0\left(\otimes\right) - \delta_1 \mu_0 = G - V_{PI} \\ \omega^T X_j\left(\otimes\right) - \mu^T Y_j\left(\otimes\right) + \delta_1 \mu_0 \geq 0, \\ \qquad j = 1, 2, \cdots, n \\ \omega^T X_0 = 0 \\ \omega \geq 0, \mu \geq 0, \delta_1 \delta_2 \left(-1\right)^{\delta_3} \mu_0 \geq 0 \end{cases} \quad (5)$$

$$\left(D - P^I\right)\begin{cases} \min \theta = G - V_{D^i} \\ \sum_{j=1}^{n} X_j\left(\otimes\right) \lambda_j \leq \theta X_0\left(\otimes\right) \\ \sum_{j=1}^{n} Y_j\left(\otimes\right) \lambda_j \geq Y_0\left(\otimes\right) \\ \delta_1 \left(\sum_{j=1}^{n} \lambda_j + \delta_2 \left(-1\right)^{\delta_3} \lambda_{n+1} \right) = \delta \\ \lambda_j \geq 0, j = 1, 2, \cdots, n, n+1 \end{cases} \quad (6)$$

Where $X_j\left(\otimes\right)$ are input vector and $Y_j\left(\otimes\right)$ are output vector.

$$X_j\left(\otimes\right) = \left(x_{1j}\left(\otimes\right), x_{2j}\left(\otimes\right), \cdots, x_{mj}\left(\otimes\right)\right)^T$$
$$Y_j\left(\otimes\right) = \left(y_{1j}\left(\otimes\right), y_{2j}\left(\otimes\right), \cdots, y_{sj}\left(\otimes\right)\right)^T$$

The numbers $x_{ij}\left(\otimes\right)$ and $y_{rj}\left(\otimes\right)$ are grey numbers. $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n$, $r = 1, 2, \cdots, s$ .The corresponding production possibility set are as follow.

$$T\left(\otimes\right) = \{(X\left(\otimes\right), Y\left(\otimes\right)) \mid \sum_{j=1}^{n} X_j\left(\otimes\right) \lambda_j \leq X\left(\otimes\right),$$
$$\sum_{j=1}^{n} Y_j\left(\otimes\right) \lambda_j \geq Y\left(\otimes\right), \delta_1 (\sum_{j}^{n} \lambda_j + \delta_2 (-1)^{\delta_3} \lambda_{n+1})$$
$$= \delta_1, \lambda_j \geq 0, j = 1, 2, \cdots, n, n+1\} \quad (7)$$

Where the value of parameter δ_1 , δ_2 , δ_3 is 0 or 1.

When $\delta_1 = 0$, $\left(G - P^I\right)$ is grey input model named C^2R .When $\delta_1 = 1, \delta_2 = 0$, $\left(G - P^I\right)$ is grey

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input model named BC^2 .When $\delta_1 = 1$, $\delta_2 = 1$, $\delta_3 = 0$, $\left(G - P^I\right)$ is grey input model named FG . When $\delta_1 = 1$, $\delta_2 = 1$ and $\delta_3 = 1$, $\left(G - P^I\right)$ is grey input model named ST .

Definition 1 When the optimal value of $\left(G - P^I\right)$ and $\left(D - P^I\right)$ are $G - V_{P^I} = G - V_{D^I} = 1$, we call DMU_{j_0} is weakly grey efficient.

Definition 2 If grey linear programming $\left(G - P^I\right)$ has optimal solution as follow,

$$\omega^0\left(\otimes\right), \mu^0\left(\otimes\right), \mu_0^0\left(\otimes\right)$$

Which satisfy $\omega^0\left(\otimes\right) > 0, \mu^0\left(\otimes\right) > 0$ and $\mu^{0T} Y_0\left(\otimes\right) - \delta_1 \mu_0^0 = 1$, we call DMU_{j_0} is grey efficient decision making unit of DEA.

Theorem 1 If DMU_{j_0} is (weakly) grey efficient decision making unit of grey input model named FG , it must be (weakly) grey efficient decision making unit of grey input BC^2 model.

Proof In integrated DEA model, when $\delta_1 = 1, \delta_2 = 1, \delta_3 = 0$, $\left(G - P^I\right)$ is grey input FG model. If DMU_{j_0} is (weakly) grey efficient decision making unit of grey input FG model, there must be $\omega^0\left(\otimes\right), \mu^0\left(\otimes\right), \mu_0^0\left(\otimes\right)$, which satisfy

$$\mu^{0T}\left(\otimes\right) Y_0\left(\otimes\right) - \delta_1 \mu_0^0\left(\otimes\right) = 1$$

and

$$\omega^0\left(\otimes\right) \geq 0, \mu^0\left(\otimes\right) \geq 0, \mu_0^0\left(\otimes\right) \geq 0$$

In integrated DEA model, when $\delta_1 = 1$, $\delta_2 = 0$, $\delta_3 = 0$, $\left(G - P^I\right)$ is grey input BC^2 model.

For grey input BC^2 model, if there is a solution $\omega^t\left(\otimes\right) \geq 0, \mu^t\left(\otimes\right) \geq 0, \mu_0^t\left(\otimes\right)$ which satisfies the formula $\mu^{tT}\left(\otimes\right) Y_0\left(\otimes\right) - \delta_1 \mu_0^t\left(\otimes\right) = 1$, then it is efficient.

Obviously, when

$$\omega^t\left(\otimes\right) = \omega^0\left(\otimes\right),$$
$$\mu^t\left(\otimes\right) = \mu^0\left(\otimes\right),$$
$$\mu_0^t\left(\otimes\right) = \mu_0^0\left(\otimes\right)$$

For grey input BC^2 model, we have

$$\mu^{0T}\left(\otimes\right) Y_0\left(\otimes\right) - \delta_1 \mu_0^0\left(\otimes\right) = 1$$

Then when DMU_{j_0} is (weakly) grey efficient decision making unit of grey input FG model, it must be (weakly) grey efficient decision making unit of grey input BC^2 model.

Theorem 2 If DMU_{j_0} is (weakly) grey efficient decision making unit of grey input FG model, and $\left(G - P_{FG}^I\right)$ has optimal solution which form is $\omega^0\left(\otimes\right), \mu^0\left(\otimes\right), \mu_0^0\left(\otimes\right)$, and $\mu_0^0\left(\otimes\right) = 0$, then DMU_{j_0} must be (weakly) grey efficient decision making unit of grey input C^2R model.

Proof Suppose DMU_{j_0} is (weakly) grey efficient decision making unit of grey input FG model, and $\left(G - P_{FG}^I\right)$ has optimal solution which form is $\omega^0\left(\otimes\right), \mu^0\left(\otimes\right), \mu_0^0\left(\otimes\right)$, and

$$\mu_0^0\left(\otimes\right) = 0$$

Then

$$\begin{aligned}
&\omega^{0T}\left(\otimes\right) X_j\left(\otimes\right) - \mu^{0T}\left(\otimes\right) Y_j\left(\otimes\right) \\
&= \omega^{0T}\left(\otimes\right) X_j\left(\otimes\right) - \mu^{0T}\left(\otimes\right) Y_j\left(\otimes\right) + \mu_0^0\left(\otimes\right) \\
&\geq 0 \\
&j = 1, 2, \cdots, n \\
&\omega^{0T}\left(\otimes\right) X_0\left(\otimes\right) = 1 \\
&\mu^{0T}\left(\otimes\right) Y_0\left(\otimes\right) = \mu^{0T}\left(\otimes\right) Y_0\left(\otimes\right) - \mu_0^0\left(\otimes\right) = 1
\end{aligned}$$

Thus $\omega^0\left(\otimes\right), \mu^0\left(\otimes\right)$ is optimal solution of $\left(G - P_{C^2R}^I\right)$ model.

As $\omega^0\left(\otimes\right) > 0, \mu^0\left(\otimes\right) > 0$ and $\mu^{0T} Y_0\left(\otimes\right) = 1$, DMU_{j_0} is grey efficient decision making unit of C^2R model.

When DMU_{j_0} is (weakly) grey efficient decision making unit of grey input FG model, and $\left(G - P_{FG}^I\right)$ has optimal solution which form are $\omega^0\left(\otimes\right), \mu^0\left(\otimes\right), \mu_0^0\left(\otimes\right)$, and $\mu_0^0\left(\otimes\right) = 0$, by similar method we wan have DMU_{j_0} must be (weakly) grey efficient decision making unit of grey input C^2R model.

Theorem 3 If DMU_{j_0} is (weakly) grey efficient decision making unit of grey input ST model, and $\left(G - P_{ST}^I\right)$ has optimal solution which form is $\omega^0\left(\otimes\right), \mu^0\left(\otimes\right), \mu_0^0\left(\otimes\right)$, and $\mu_0^0\left(\otimes\right) = 0$, then DMU_{j_0} must be (weakly) grey efficient decision making unit of grey input C^2R model.

The proof is similar to Theorem 2.

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4. SOLVING PROCESS OF GREY DATA ENVELOPMENT ANALYSIS MODEL

For grey linear programming

$$\max f(x) = C(\otimes)^T X$$
$$\begin{cases} A(\otimes) \le b \\ B(\otimes) \le d \end{cases} \tag{7}$$

$X = [x_1, x_2, \cdots, x_n]^T$

$C(\otimes) = [c_1(\otimes), c_2(\otimes), \cdots, c_n(\otimes)]^T$

$c_i(\otimes) \in [\underline{c}_i, \overline{c}_i], i = 1, 2, \cdots, n$

$A(\otimes) = [a_{\mu j}(\otimes)]_{m\times n}, a_{\mu j}(\otimes) \in [\underline{a}_{\mu j}, \overline{a}_{\mu j}]$

$B(\otimes) = [b_{\nu j}(\otimes)]_{p\times n}, b_{\nu j}(\otimes) \in [\underline{b}_{\nu j}, \overline{b}_{\nu j}]$

$b=[b_1,b_2,\cdots,b_m]^T, d=[d_1,d_2,\cdots,d_p]^T$

Set

$$\begin{cases} c_i(\otimes)=\underline{c}_i+\theta(\overline{c}_i-\underline{c}_i) \\ a_{\mu j}(\otimes)=\underline{a}_{\mu j}+\theta(\overline{a}_{\mu j}-\underline{a}_{\mu j}) \\ b_{\nu j}(\otimes)=\underline{b}_{\nu j}+\theta(\overline{b}_{\nu j}-\underline{b}_{\nu j}) \\ \theta=x_{n+1} \end{cases}$$

Using the above equation we can transform grey linear programming into ordinary linear programming as follow.

$$\max f(x)=\sum_{i=1}^{n}[\underline{c}_i+x_{n+1}(\overline{c}_i-\underline{c}_i)]x_i$$

$s.t.$

$$\begin{cases} \sum_{j=1}^{n}[\underline{a}_{\mu j}+x_{n+1}(\overline{a}_{\mu j}-\underline{a}_{\mu j})]x_i-b_\mu \le \theta \\ \sum_{j=1}^{n}[\underline{b}_{\nu j}+x_{n+1}(\overline{b}_{\nu j}-\underline{b}_{\nu j})]x_i-d_\nu \le \theta \\ 0 \le x_{n+1} \le 1 \\ \mu=1,2,\cdots,m; \nu=1,2,\cdots,p; p \le n \end{cases} \tag{8}$$

5. CASE STUDY

By the above grey DEA model, we evaluate the performance of listed companies. First of all listed companies are seen as different decision-making unit, we analyze DEA effective of their performance. The two more representative input factors, circulation Unit number and total assets are input variables. The output variables are main business income, net profit, operating activities cash flows and inventory turnover rate. We select eight listed companies in Qingdao as our object. The corresponding data are obtained from the Securities Star website and they are grey numbers. As input and output indicators have different dimensions, and there is negative number in raw data, the solution of the linear programming problem is difficult to get. It is normalized to a certain dimensionless interval by a certain functional relationship. Dimensionless input and output of DMU are shown in Table1.

Table 1 Nondimensional Data

DMU	company	input X_1	input X_2	output Y_1	output Y_2	output Y_3	output Y_4
1	Jiante Biological	[0. 12, 0. 13]	[0. 11, 0. 12]	[0. 10, 0. 13]	[0. 13, 0. 14]	[0. 10, 0. 12]	[0. 44, 0. 45]
2	Tsingtao Eastern	[0. 11, 0. 04]	[0. 11, 0. 13]	[0. 10, 0. 15]	[0. 11, 0. 16]	[0. 10, 0. 15]	[0. 12, 0. 17]
3	Tsingtao Brewery	[0. 41, 0. 45]	1	[0. 46, 0. 48]	[0. 28, 0. 31]	[0. 59, 0. 68]	[0. 19, 0. 23]
4	Hisense Electric	[0. 42, 0. 48]	[0. 45, 0. 48]	[0. 34, 0. 36]	[0. 21, 0. 22]	[0. 31, 0. 35]	[0. 13, 0. 15]
5	Tsingtao Double Star	[0. 17, 0. 19]	[0. 10, 0. 13]	[0. 12, 0. 16]	[0. 15, 0. 21]	[0. 16, 0. 17]	[0. 10, 0. 11]
6	Tsingtao Haier	1	[0. 75, 0. 84]	1	1	1	1
7	Tsingtao Soda	[0. 16, 0. 20]	[0. 23, 0. 25]	[0. 16, 0. 18]	[0. 17, 0. 20]	[0. 16, 0. 18]	[0. 28, 0. 34]
8	Aucma	[0. 16, 0. 17]	[0. 34, 0. 38]	[0. 15, 0. 18]	[0. 19, 0. 25]	[0. 52, 0. 56]	[0. 15, 0. 18]

We put the data in table1 into grey DEA model and establish grey input FG model to analyze DEA efficiency of each gray decision-making unit. The evaluation results of DEA efficiency are shown in Table 2.

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Table 2 The Evaluation Results Of DEA Efficiency

DMU	company	DEA efficiency value	slack variable					
			s_1^-	s_2^-	s_1^+	s_2^+	s_3^+	s_4^+
1	Jiante Biological	1	0	0	0	0	0	0
2	Tsingtao Eastern	0. 8276	0	0	0. 0056	0	0. 0028	0
3	Tsingtao Brewery	0. 9254	0	0. 2271	0	0. 2891	0	0. 1641
4	Hisense Electric	0. 7650	0	0	0	0. 1587	0. 0705	0. 1871
5	Tsingtao Double Star	1	0	0	0	0	0	0
6	Tsingtao Haier	1	0	0	0	0	0	0
7	Tsingtao Soda	0. 8635	0	0	0	0. 0127	0. 0296	0
8	Aucma	1	0	0	0	0	0	0

Table 3 shows that DEA efficiency values of Jiante Biological, Tsingtao Double Star, Tsingtao Haier and Aucma are all equal to 1. At least they are DEA grey weakly efficient. Furthermore, the slack variable s^- and s^+ are equal to 0. It indicates that the production and operation of the four listed companies are relative grey efficiency. The DEA efficiency values of other listed companies are all less than 1. It indicates that the other companies are DEA grey not weakly effective, less DEA grey effective.

6. CONCLUSION

In this paper, the grey system theory and DEA are combined to establish a new grey DEA model. The intersection of two different theories contributes to the development of the two disciplines theory. It provides a new perspective and an integrated approach to solve practical problems. We explore the deficiencies of grey system theory and DEA and integrate the advantages of the two theories. At last, we create a combination model. Our method has a more widely applications. It not only improves grey comprehensive evaluation method, but also enhances the grey comprehensive evaluation method holistic and systematic evaluation. It provides effective ideas and approaches. The study of this problem will improve the further integration and development of the grey comprehensive evaluation method and DEA method. It can provide more effective information for decision-makers. But it is just an attempt to form a new model by combining the two theories. There are more research need to further develop for inspection, correction and additional in practical applications. The problems need to be further studied are as follow. When the variable is a grey number, what is the path of decision-making unit promoted to DEA efficient from a non-DEA effective? If it is scale effective when it is effective technology? How to achieve scale effective? Solving these problems will procure DEA model to explain the reality more effective.

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