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GA-BASED H∞ CONTROL OF LINEAR SYSTEM WITH TIME-DELAY

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ABSTRACT

In this paper, in view of robust stability conditions relevant to linear time-delay systems, poses up an output feedback control method for linear system with time-delay based on genetic algorithm. Because of the system state in actuality is normally unpredictable, which studies time-delay related stability and H∞ control of linear system with time delay by combination of genetic algorithm and linear matrix inequality approach, and take the state feedback of time-delay system and static output feedback controller design into the design. Simulations are also given to illustrate its effectiveness and lesser conservatism, and the designed controller has less feedback gain. In addition, the method can be directly extended to robustness H∞ control of linear time-delay system with parameter uncertainty.

Keywords: *Terms-genetic Algorithm, Time-varying, Linear Matrix Inequalities, H∞ Control*

1. INTRODUCTION

Recently, attentions have been focused on delay-dependent H∞ filtering of linear time-delay systems, see for example References [1, 2, 3] which have adopted model transformation method, i.e. transforming the original system model into a new system equal or unequal to the original one, followed by acquisition of delay-dependent robust stability conditions and robust controller by application of Lyapunov-Krasovskii functional method and linear matrix inequality (LMI) method. Reference [4] adopts a new inequality to define the cross term, and introduces a state feedback, thus obtaining a comparatively less conservative delay-dependent stability condition and state feedback controller. In addition, as to H∞ control of linear time-delay system, Reference [5, 6] has constructed a new Lyapunov-Krasovskii method, avoiding the generally needed model transformation and estimation of the upper bound of cross terms, thus is less of conservatism, even compared to that of [3]. Wang et al [7], Palhares et al [8], and Xu et al [9] have also obtained results of less conservatism as to delay-dependent system stability and H∞ filtering issue.

In the above literatures, the matrix inequalities involved contain nonlinear items, which are normally addressed by application of iterative algorithm. But only suboptimal solutions can be figured out instead of global optimum solution, so the results obtained thereof are still very conservative in a large degree.

This paper utilizes genetic algorithm to substitute iterative algorithm for results of less conservatism, try to address the time-dependent stability and H∞ control issues of time-delay system based on genetic algorithm in combination with LMI method. Genetic algorithm is a computing model simulating biological evolution process, with major features of information exchange of individuals and population selection, which can overcome the drawback of coming out with local minimum values compared to other selection method, thus it's an effective global optimization selection algorithm, particularly suitable for complex nonlinear optimization problem [10, 11].

2. GA-BASED H∞ CONTROL OF LINEAR SYSTEM WITH TIME-DELAY

2.1 Problem Statement

Consider the following class of state time-delay linear systems:

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$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_w w(t) + A_1 x(t-\tau) + Bu(t) \\ z(t) &= \begin{bmatrix} C_0 x(t) + D_w w(t) \\ C_1 x(t-\tau) \\ Du(t) \end{bmatrix} \\ y(t) &= Cx(t) \\ x(t) &= \phi(t), t \in [-\tau, 0] \end{aligned} \quad (1)$$

Where, $x(t) \in R^n$ denotes the system state, $u(t) \in R^m$ is control input, $z(t)$ is measurement output, $w(t)$ is disturbance input vector, and given that $w(t) \in L_2[0,\infty)$, $\tau > 0$ denotes system state time-delay, $\phi(t)$ denotes the initial condition, while $A, A_1, B, B_w, C_0, C_1, C, D, D_w$ are constant real matrix with appropriate dimensions.

The goal of this study system (1) is to design an output feedback controller

$$u(t) = Ky(t) = KCx(t) \quad (2)$$

$K \in R^{m\times n}$, Is the feedback gain matrix. Two problems are discussed as to the closed-loop system: (I) stability analysis on System (1); (II) Delay-dependent H ∞ control of System (1).

2.2 Stability Analyses

In order to study the stability of System (1), assume $B_w = 0, C_0 = 0, D_w = 0, C_1 = 0, D = 0$, and design the controller (2) such that time delay τ is stable in the closed-loop system (1)

$$\dot{x}(t) = (A + BKC)x(t) + A_1 x(t-\tau) = \overline{A}x(t) + A_1 x(t-\tau) \quad (3)$$

As long as it meets $0 \le \tau \le \bar{\tau}$; $\bar{\tau}$ thereof is the upper bound of time-delay to be specified.

Several mathematical lemmas are introduced as follows:

Lemma 1 [12, 13, 14]: for given Vector a and b with appropriate dimensions and matrices N, X, Y, Y, and Z, where X and Z are symmetric matrices, if $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \ge 0$, then

$$-2a^T Nb \le \inf_{X,Y,Z} \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} X & Y-N \\ Y^T - N^T & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

Theorem 1: If there exist $P > 0, Q > 0, Z > 0$, matrices X and Y with appropriate dimensions such that:

$$\begin{bmatrix} (A+BKC)^T P + P(A+BKC) + \bar{\tau} x + Q + Y + Y^T & -Y + PA_1 & \bar{\tau}(A+BKC)^T Z \\ * & -Q & \bar{\tau} A_1^T Z \\ * & * & \bar{\tau} Z \end{bmatrix} < 0 \quad (4)$$

$$\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \ge 0 \quad (5)$$

Clearly, System (3) is asymptotically stable.

Proof: we assume that there exist positive definite matrices P, Q, Z and matrices X, Y, choosing the following Lyapunov function:

$$V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t)$$

Where $V_1 = x^T(t)Px(t)$

$$V_2 = \int_{-\tau}^{0}\int_{t+\beta}^{t} \dot{x}^T(\alpha) Z\, \dot{x}(\alpha)\, d\alpha\, d\beta$$

$$V_3 = \int_{t-\tau}^{t} x^T(\alpha) Q x(\alpha) d\alpha$$

As

$$x(t) - x(t-\tau) = \int_{t-\tau}^{t} \dot{x}(\alpha) d\alpha$$

So System (3) can be rewritten as：

$$\dot{x}(t) = (A + A_1)x(t) - A_1 \int_{t-\tau}^{t} \dot{x}(\alpha) d\alpha$$

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$$\dot{V}_1 = 2x^T(t)P(\bar{A}+A_1)x(t) - 2x^T(t)PA_1\int_{t-\tau}^{t}\dot{x}(\alpha)d\alpha \tag{6}$$

Define $a = x(t), b = \dot{x}(\alpha), N = PA_1$, apply Lemma 1,
We can obtain

$$-2x^T(t)PA_1\dot{x}(\alpha) \le \begin{bmatrix} x(t) \\ \dot{x}(\alpha) \end{bmatrix}^T \begin{bmatrix} X & Y - PA_1 \\ Y^T - (PA_1)^T & Z \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(\alpha) \end{bmatrix}$$

Replace the above into (6), one obtains the following:

$$\dot{V}_1 \le x^T(t)(\bar{A}^T P + P\bar{A} + \bar{\tau}X + Y + Y^T)x(t) + 2x^T(t)(PA_1 - Y)x(t-\tau) + \int_{t-\tau}^{t}\dot{x}^T(\alpha)Z\dot{x}(\alpha)\,d\alpha$$

$$\dot{V}_2 = \tau\dot{x}^T(\alpha)\,Z\dot{x}(\alpha)\,d\alpha + \int_{t-\tau}^{t}\dot{x}^T(\alpha)Z\dot{x}(\alpha)\,d\alpha$$

$$\dot{V}_3 = x^T(t)Qx(t) - x^T(t-\tau)Qx(t-\tau)$$

So

$$\dot{V} = \dot{V}_1+\dot{V}_2+\dot{V}_3 \le \begin{bmatrix} x(t) \\ x(t-\tau) \end{bmatrix}^T \left(\begin{bmatrix} M & PA_1 - Y \\ A_\tau^T P - Y^T & -Q \end{bmatrix} + \tau \begin{bmatrix} A^T \\ A_1^T \end{bmatrix} Z \begin{bmatrix} A & A_1 \end{bmatrix} \right) \begin{bmatrix} x(t) \\ x(t-\tau) \end{bmatrix}$$

$$M = \bar{A}^T P + P\bar{A} + \bar{\tau}X + Y + Y^T + Q$$

Based on Schur complement lemma, we can conclude that if inequality (4) holds, then $\dot{V} < 0$, so system (3) is asymptotically stable, thus the theorem is proved.

2.3 Delay-Dependent H ∞ Control

For system (1), we consider to design controller (2) such that the corresponding closed-loop system has the following two properties:

(I) system is asymptotically stable;

(II) For a given constant $\gamma(>0)$, under zero initial conditions, for any disturbance $w(t) \in L_2[0,\infty)$, the output of the closed-loop system meets H ∞ performance index:

$$\|z(t)\|_2 \le \gamma\|w(t)\|_2 .$$

Theorem 2: For a given constant γ , if there exist positive definite matrices P_1, Q, Z and matrices $P_2, P_3, X_{11}, X_{12}, X_{22}, Y_1, Y_2$ which satisfy the following inequality:

$$\begin{bmatrix} \phi_{11} & P^T\begin{bmatrix} 0 \\ A_1 \end{bmatrix} - \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} & P^T\begin{bmatrix} 0 \\ B_w \end{bmatrix} - \begin{bmatrix} C_0^T D_w \\ 0 \end{bmatrix} \\ * & -Q + C_1^T C_1 & 0 \\ * & * & -\gamma^2 I + D_w^T D_w \end{bmatrix} < 0 \tag{7}$$

$$\begin{bmatrix} X_{11} & X_{12} & Y_1 \\ * & X_{22} & Y_2 \\ * & * & Z \end{bmatrix} \ge 0$$

$$\phi_{11} = \begin{bmatrix} (A+BKC)^T P_2 + P_2^T(A+BKC) + \bar{\tau}X_{11} & (A+BKC)^T P_3 - P_2^T \\ +Q + C_0^T C_0 + (DKC)^T DKC + Y_1 + Y_1^T & +P_1^T + \bar{\tau}X_{12} + Y_2^T \\ P_3^T(A+BKC) + P_1 - P_2 + \bar{\tau}X_{12}^T + Y_2 & -P_3 - P_3^T + \bar{\tau}X_{22} + \bar{\tau}z \end{bmatrix}$$

$$P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}$$

Then we can conclude that the closed-loop System (1) is asymptotically stable and satisfies H ∞ performance γ .

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Proof: Choosing the following Lyapunov-Krasovskii function:

$$V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t)$$

Where

$$V_1 = \begin{bmatrix} x(t) \\ x(t) \end{bmatrix}^T HP \begin{bmatrix} x(t) \\ x(t) \end{bmatrix} \quad V_2 = \int_{-\tau}^{0} \int_{\tau+\beta}^{t} x^T(\alpha) Z\, x(\alpha)\, d\alpha\, d\beta$$

$$V_3 = \int_{t-\tau}^{t} x^T(\alpha) Q x(\alpha) d\alpha$$

$$V_4 = \int_0^t \int_{\beta-\tau}^{\beta} \begin{bmatrix} x(\beta) \\ x(\beta) \\ x(\alpha) \end{bmatrix}^T \begin{bmatrix} x_{11} & x_{12} & Y_1 \\ * & x_{22} & Y_2 \\ * & * & Z \end{bmatrix} \begin{bmatrix} x(\beta) \\ x(\beta) \\ x(\alpha) \end{bmatrix} d\alpha\, d\beta$$

Herewith

$$P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}, H = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, P_1 > 0, Z > 0, Q > 0$$

$$\begin{bmatrix} X_{11} & X_{12} & Y_1 \\ * & X_{22} & Y_2 \\ * & * & Z \end{bmatrix} \geq 0$$

Then

$$V + z^T(t) z(t) - \gamma^2 w^T(t) w(t) \leq \begin{bmatrix} x(t) \\ x(t) \\ x(t-\tau) \\ w(t) \end{bmatrix}^T \begin{bmatrix} \phi_{11} & P^T \begin{bmatrix} 0 \\ A_1 \end{bmatrix} - \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} & P^T \begin{bmatrix} 0 \\ B_w \end{bmatrix} - \begin{bmatrix} C_0^T D_w \\ 0 \end{bmatrix} \\ * & -Q + C_1^T C_1 & 0 \\ * & * & -\gamma^2 I + D_w^T D_w \end{bmatrix} \begin{bmatrix} x(t) \\ x(t) \\ x(t-\tau) \\ w(t) \end{bmatrix}$$

Therefore, in case (7) holds, we can obtain $V + z^T(t) z(t) - \gamma^2 w^T(t) w(t) < 0$

When $w(t) = 0$, which denotes that the close-loop System (1) is asymptotically stable

Under zero initial conditions, $V(x_t)|_{t=0} = 0$ and $V(x_t)|_{t=\infty} \geq 0$, we herewith obtain the following

$$\int_0^\infty (z^T(t) z(t) - \gamma^2 w^T(t) w(t)) dt \leq \int_0^\infty (z^T(t) z(t) - \gamma^2 w^T(t) w(t) + V) dt < 0$$

i.e.

$$\int_0^\infty (z^T(t) z(t) - \gamma^2 w^T(t) w(t)) dt < 0$$

So we have

$$\|z(t)\|_2 \leq \gamma \|w(t)\|_2 .$$

The theorem is proved.

2.4 GA-Based Feedback Controller Design

Genetic algorithm, an optimization algorithm based on population selection, by operation of selection, crossover, and genetic variation such that populations evolve so as to eventually converge to the global optimal solution. Herewith by combination of genetic algorithms and LMI, we locate for a suitable gain matrix K and time-delay upper bound $\bar{\tau}$ through addressing the following maximization issue:

$$\begin{aligned} & \max \bar{\tau} \\ & K \in R^{m \times n} \\ & s.t. LMIs \quad (4) and (5)\ or (7) \end{aligned} \qquad (8)$$

When applying a genetic algorithm, firstly a gain matrix $K \in R^{m \times n}$ is generated scholastically;

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then by targeting at (8), we adopt genetic operation such that it constantly evolves to obtain the optimal solution; herewith in case of a maximum time delay $\bar{\tau}$, and given that K satisfies (8), then a solution to (I) or (II) sets up. Note that once the gain matrix is given, Inequality (4) - (7) transform into linear matrix inequalities, which can be addressed by matlab LMI toolbox.

In light of the advantages of real-number encoding in its intuitiveness in expression, economic in time and space occupation, and highly-efficient in computation, in addition to its adoption of the real value of variables, we herewith adopt such method in the gain matrix K.

GA-based feedback control follows the steps as below:

(1) Scholastically generating initial populations and initial populations with N_p chromosomes (parent);

(2) Calculating the objective function value; specifying fitness values for individuals; applying dichotomy to locate for the maximum time delay $\bar{\tau}$ for each gain matrix $K_i, i=1,\ldots,N_p$ such that for K_i and $\bar{\tau}$, inequality (4)-(5) or (6) holds. In case that there is not a $\bar{\tau}$ for K_i such that (4)-(5) or (6) holds, then define a given value to $\bar{\tau}$ so that corresponding individual K_i would be eliminated during the evolutionary iteration process;

(3) Selecting: select the optimal individual by adoption of scholastic traversal method;

(4) Crossover and mutation: conducting crossover and mutation based on the crossover probability P_c and mutation probability P_m to generate a new generation of population (offspring);

(5) Replacement: calculating and specifying the fitness values to individuals of the new generation, followed by replacing the unfit individuals of parent generation based on fitness values;

(6) Check if the current population meets the algorithm termination condition (such as the maximum times of iterations), if yes, then obtain the optimal individual and algorithm terminates; otherwise return to the second step for repetitive evolutionary iteration until meeting the algorithm termination condition.

2.5 Simulations

In accordance with the above algorithm, we hereby conduct experiment simulations for problems (I) and (II) by application of genetic algorithm, for which we adopt the Matlab-based genetic algorithm toolbox GATBX developed by University of Sheffield, UK. The parameters are assumed as: the number of chromosomes $N_p = 20$, the maximum number of iterations $N_{it} = 100$, and mutation probability $P_m = 0.01$. When conducting crossover operations, we adopt discrete restructuring for real-number encoding of the GATBX toolbox, where, P_c is normally defined as 1.

Example 1: as to Problem (I), define that

$$A=\begin{bmatrix}0&0\\0&1\end{bmatrix}, A_1=\begin{bmatrix}-1&-1\\0&0.9\end{bmatrix}, B=\begin{bmatrix}0\\1\end{bmatrix}, C=\begin{bmatrix}1&0\\0&1\end{bmatrix},$$

then a feedback gain matrix K that meet the requirements as well as the corresponding maximum time delay $\bar{\tau}$ can be obtained by the abovementioned algorithm. A comparison is made with the aforementioned method, as shown in Table 1.

Table 1: Comparison of Different Stability Methods

Methods	Maximum $\bar{\tau}$ allowed	Feedback gain
Fridman et al (2002b)	1.51	[-58.31 -294.9]
Gao et al(2003)	3.2	[-7.964 -14..77]
Zhang et al(2005)	6	[-70.18 -77.67]
Our method	9.9405	[-96.1294 -97.4899]

From the above table, we can see that compared to the existed state feedback method, our method is of less conservatism, and can obtain a greater time delay upper bound.

Example 2: For Problem (II): H ∞ control of time-delay systems, letting

$$A=\begin{bmatrix}0&0\\0&1\end{bmatrix}, A_1=\begin{bmatrix}-1&-1\\0&0.9\end{bmatrix}, B=\begin{bmatrix}0\\1\end{bmatrix}, B_w=\begin{bmatrix}0\\1\end{bmatrix}, C=\begin{bmatrix}1&0\\0&1\end{bmatrix},$$

$$C_1=\begin{bmatrix}0&0.5\end{bmatrix}, C_0=\begin{bmatrix}0&1\end{bmatrix}, D_w=0, D=0.1$$

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The abovementioned algorithm can be utilized to address the problem and obtain the needed feedback gain matrix as well as the maximum time delay $\bar{\tau}$. A comparison has also been made with existing methods, with results shown as in Table 2:

Table 2: Comparison of Different H ∞ Control Methods

Methods	Given γ	Maximum $\bar{\tau}$ allowed	Feedback gain
Fridman et al (2002a)	0.1278	0.999	$\begin{bmatrix}0 & -1.0285\times10^{6}\end{bmatrix}$
Lee et al(2004)	0.1278	1.25	$\begin{bmatrix}0.6407 & -89.1149\end{bmatrix}$
Xu et al(2006)	0.1278	1.25	$\begin{bmatrix}0.1789 & -45.8572\end{bmatrix}$
Our method	0.1278	1.3649	$\begin{bmatrix}-0.0367 & -26.9640\end{bmatrix}$

From the above table, we can see that with the same γ, our method can locate for a state feedback control and static output feedback control with greater time delay compared to the existing method. Meanwhile, as pre-setting of the search scope of feedback gain matrix is allowed with genetic algorithm, the gain matrix proposed in this paper is smaller than those constructed by other methods.

3. CONCLUSIONS

In the light of time-delay system, this paper proposes a time-delay feedback control design method by combination of genetic algorithm and LMT, which based on LMT, locates for the optimal gain matrix K and maximum time delay $\bar{\tau}$ by application of genetic algorithm, and obtains the corresponding close-loop system that satisfies H ∞ performance. Examples presented in the last part indicate the effectiveness of the method and its advantage in less conservatism, On the hand, the method in this paper is posed up based on linear time-delay system with certain parameters, but can be extended directly to the study on robust H∞ control of linear time-delay system with parameter uncertainties.

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