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RESEARCH OF MULTI-ATTRIBUTE TRADE OFF OPTIMIZATION ON CRITICAL CHAIN MULTI-PROJECT SCHEDULING

[1] WANG WEIXIN *, [1] WANG XU , [2] GE XIANLONG ,[3]LIN YA ,[1] CHEN YANG , [1] GAO JIA
1 College of Mechanical Engineering, Chongqing University，Chongqing 400030,China;
2 School of Management, Chongqing Jiaotong University, Chongqing 400074,China
3 Deptment of Information Engineering, Chongqing Communication Institute, 400035, China
E-mail: xiaoxin301@126.com

ABSTRACT

For the multi-attribute tradeoff optimization of critical chain multi-project scheduling (CCMPS), multi-attribute utility function was adopted to conduct comprehensive optimization of time, cost, quality, robustness and the optimization model was established. Using randomness and stability of Normal Cloud Model, cloud genetic algorithm was designed to generate priority of multi-project scheduling activities and obtain dispatching plan finally. Effectiveness of the model and algorithm were verified by a case study.

Keyword : *Critical Chain ; Multi-Project Scheduling ; Time-Cost-Quality- Robustness ; Cloud Genetic Algorithms*

1. INTRODUCTION

Rabbania[1] put forward a schedule method for resource-constrained random network project; Bevilacqua. etc[2] illustrated that CCM can shorten the duration of the project and reduce operating costs in maximum through application instances. Peng[3] argued that the earlier the project begins, the lower risk of time delay is and presented improved ways of CCM. Wuliang Peng. Etc[4] analyzed resource constraints, multi-mode activities, uncertain activity duration and built up the model of multi-mode critical chain project scheduling. Jingwen Zhang. etc[5] used resource consumption to calculate activity cost, established schedule models for two categories of resource-constrained time-cost tradeoff project based on discrete time-cost tradeoff project scheduling and designed its two-stage hybrid optimizing algorithm. Qiong Liu. etc[6], proposed to build scheduling optimization model of critical chain multi-project aiming at the maximization of robustness and minimum of duration. Ali Salmasnia. etc[7] introduced quality parameters into the traditional time-cost tradeoff project scheduling problem. Based on analysis of research status on CCMPS problem, scholars of domestic and overseas have been tried to innovate theoretical model and solving algorithm to find out more optimal project scheduling scheme of CCM.Babak Abbasi[8] designed a multiple objective model for project scheduling, which aims at the minimum of duration and maximization of robustness.

There are mainly three types of existing research: the first is single objective optimization, that is to choose one attribute from time, cost and quality as objective function to build mathematical models; secondly, the three attributes are considered simultaneously in constructing multi-objective optimization models, which will be transformed into single-objective optimization and solved; third, the robustness of solution in critical chain project scheduling and quality robustness are studied. In practical, time, cost and quality are three major concerns for operators, and the robustness of multi-project scheduling is the key to success of multi-project operation mode. Therefore, the time-cost-quality-robustness tradeoff optimization of CCMPS will have considerable realistic significance.

2. PROBLEM DESCRIPTION

Suppose there are N projects in CCMPS, which are executed parallel without predecessor or successor relationship among them. Project i has J_i activities. Activity s_0 and e_0 are the virtual beginning and virtual ending, which represents the start and end of multi-project without occupying resources. The kinds of renewable and non-renewable resources are k and p separately, similarly, S_{ij} and E_{ij} mean starting and ending time of the project. A_{ij} represents the activity j in project i ; t_{ij} represents the planned project duration; P_{ij} and U_{ij} are sets of predecessor and successor activities of A_{ij} ; r_{ijk} is demand quantity of resource k in activity A_{ij} ; R_k $(k=1,2,\cdots p)$ is available quantity of k -th kind of renewable resource; R_p $(p=1,2,\cdots p)$ is total quantity of p -th kind of non-renewable resource; r_{ijk} stands for the quantity of renewable resource demanded by activity j of project i in unit duration; nr_{ijp} means the gross quantity of non-renewable resource demanded by activity j of project i ; q_{ij} is the quality index when activity j of project i is carried out.

Assume T , C , Q , L represents time, cost, quality and robustness to do tradeoff optimization of CCMPS, which can be decomposed into weighted polynomial as follows according to decomposition theorem of multi-attribute utility function.

$$\begin{cases} u(T,C,Q,L)=\alpha_T\bullet u(T)+\alpha_C\bullet u(C)+\alpha_Q\bullet u(Q)+\alpha_L\bullet u(L) \\ \alpha_T,\alpha_C,\alpha_Q,\alpha_L\geq 0 \\ \alpha_T+\alpha_C+\alpha_Q+\alpha_L=1 \end{cases} \quad (1)$$

$u(T)$, $u(C)$, $u(Q)$, $u(L)$ represent the utility function of time, cost, quality, robustness severally, whose weight coefficient is α_T , α_C , α_Q , α_L and the optimization objective is to obtain the maximum. Quadratic function form of utility function can be employed as the solution space and utility functions are all concave. Assume the utility value of total project duration D is 1, then

$$u(T)=\begin{cases} \varphi_T-\beta_T(T-D)^2, & T\in[0,2D] \\ 0, & T\notin[0,2D] \end{cases} \quad (2)$$

The total cost of multi-project management consists of the cost of renewable and non-renewable resources and when its utility value is 1, then

$$u(C)=\begin{cases} \varphi_C-\beta_C\left(C-(1-\eta)\bullet U\right)^2, & C\in[0,2(1-\eta)\bullet U] \\ 0, & C\notin[0,2(1-\eta)\bullet U] \end{cases} \quad (3)$$

If utility value of quality attribute is 1, then

$$u(Q)=\begin{cases} \varphi_Q-\beta_Q(Q-1)^2, & Q\in(0,1) \\ 0, & Q\notin(0,1) \end{cases} \quad (4)$$

When utility value of robustness equals 1, then

$$u(L)=\begin{cases} \varphi_L-\beta_L(L-1)^2, & L\in(0,1) \\ 0, & L\notin(0,1) \end{cases} \quad (5)$$

3. ESTABLISH THE MODEL OF CCMPS

3.1 Analysis Multi-Attribute Tradeoff Optimization Objective Of Ccmps.

The process of scheduling and controlling in critical chain multi-project management are very complicated and total duration is always the focus of project executer and proprietor, therefore, the objective pursued in critical chain multi-project management is the minimum of time, that is:

$$\min T = E_{e_0} + PB \tag{6}$$

The total cost of multi-project management consists of the cost of renewable and non-renewable resources. Setting c_p equals the unit price of p -th kind of non-renewable resource in unit duration and c_k equals the unit price of k -th kind of renewable resource in unit duration, then the objective function of project cost in critical chain multi-project management is:

$$\min C = \sum_{i=1}^{N}\sum_{j=1}^{J}\left(\sum_{k=1}^{K} r_{ijk} c_k t_{ij} + \sum_{p=1}^{P} nr_{ijp} c_p\right) \tag{7}$$

The quality attribute of project needs to be quantified for the purpose of doing systemic and deep research on CCMPS. Here, intermediate variable is introduced into each activity for representing earned quality value（EQV_{ij}）, whose computational formula is:

$$EQV_{ij} = EV_{ij} \times q_{ij} \tag{8}$$

EV_{ij} is the earned value of activity j in project i , q_{ij} is the quality index of activity j when executed in project i , which measures the actual quality level of activities and can be calculated by:

q_{ij} =actual quality level of activity j / prescribed quality level of activity $j \times 100\%$ （9）

Therefore, the total quality level of multi-project scheduling can be expressed by weighted average of quality level in all activities, which is:

$$\max Q = \frac{1}{\sum_{i=1}^{N}\sum_{j=1}^{J} EV_{ij}} \times \sum_{i=1}^{N}\sum_{j=1}^{J} EQV_{ij}$$

$$= \frac{1}{\sum_{i=1}^{N}\sum_{j=1}^{J} EV_{ij}} \times \sum_{i=1}^{N}\sum_{j=1}^{J} EV_{ij} \times q_{ij} \tag{10}$$

Quantization of robustness is the key in robustness research of CCMPS, however, there isn't mature study on this issue: maximizing minimum activity slack time can only increase the random time of activities in CCMPS rather than ensuring its abundance. For this reason, robustness will be quantified with the average value of slack time and minimum slack time of activities in this model and its calculation method is given:

$$L = \min s_{A_{ij}} + \left(\sum\sum s_{A_{ij}}\right)/\xi \tag{11}$$

$$s_{A_{ij}} = \min\left(S_{U_{ij}} - E_{ij}\right) \tag{12}$$

In the formula above, $s_{A_{ij}}$ is the free slack time of activity A_{ij} , $S_{U_{ij}}$ is the starting time of A_{ij} 's successor activity, ξ is the number of activities on non-critical chain. When free slack time of activities on critical chain equals zero (robustness of critical chain reaches the maximum), the free slack time of activities on non-critical chain will be maximized (flexible time of all activities on non-critical chain will be enough) which will improve stability of starting time for successor activity and then increase global robustness.

In order to guarantee the rate of completion on time and increase robustness of project quality, proper project buffer determined by the expectation probability of completion on time should be setting up in CCMPS according to self-adaptation which will set the value of buffer in line with RUF of each activity among the project. RUF is calculated as follows

$$RUF(k)=\sum_{i=1}^{N}\sum_{j=1}^{J}\left(r_{ijk}\times t_{ij}\right)/\left(T\times R_k\right)$$

$$k=1,2,,k \qquad (13)$$

T represents the length of critical chain and non-critical chain separately when calculating the project buffer and the time. The smaller value between buffer size and free float of the last activity on non-critical chain should be given to the buffer, in that way, the change of the critical chain because of oversized buffer can be avoided effectively.

3.2 Build The Tradeoff Optimization Model

On the basis of considering time, cost, quality and global robustness which are mostly concerned by organizers and contractors, multi-attribute utility function was adopted to conduct comprehensive optimization of time, cost, quality, robustness, and the optimization model was established as follows:

$$\max u(T,C,Q,L) \qquad (14)$$

$$s.t.\quad E_{ij}-E_{i(j-1)}\geq t_{ij} \qquad (15)$$

$$PB=(1+\max\{RUF(k)\})\times\sqrt{\sum_{i=1}^{N}\sum_{j=1}^{J}VAR_{ij}} \qquad (16)$$

$$\sum_{i=1}^{N}\sum_{j=1}^{J}r_{ijk}\leq R_k \qquad (17)$$

$$A_{ij}\in P_{pq}\cup\left\{A_{nm}\middle|E_{nm}=S_{pq},k_{nm}=k_{pq}\right\}$$

$$A_{ij}\in NCC, A_{pq}\in CC; \qquad (18)$$

In the model above:

VAR_{ij} is the variance of activity duration;

A_{pq} represents the activities on critical chain;

NCC is the set of activities on non-critical chain;

CC is the set of activities on critical chain;

be executed until the completion of current activity and activity can't be stopped once it started for the sake of continuity; formula (16) represents that project buffer value will be setup with self-adaptation; formula (17) expresses that resource consumption in one activity can't exceed its total amount; formula (18) shows the position to insert the buffer, namely, a feeding buffer PB will be inserted between one activity and critical chain when the activity is resource-constrained but doesn't belong to critical chain or it is a successor activity.

k_{nm} stands for the kinds of resources needed by activity A_{nm};

Formula (14) (The objective function) means to maximize total utility of multi-attribute tradeoff optimization in CCMPS; formula (15) reflects the predecessor and successor relationship among the project, which demands that successor activity can't

4. DESIGN CLOUD GENETIC ALGORITHM

Cloud Genetic Algorithm (CGA) introduces cloud model theory into traditional genetic algorithm and the randomness and stable tendency of a normal cloud model are adopted to design the probabilities of crossover and mutation, Pc and Pm. Greater Pc and Pm were set up in the initial stage for the purpose of generating superior individuals quickly, while in the later stage, smaller Pc and Pm were set up along with crossover operator of maximum retention mechanism to protect superior genes from damage . The reason for doing above operations is to improve global optimization ability. Adaptive operator is imported via cloud generator together with excellent characteristics of genetic algorithm which can avoid premature convergence to promote the adaptive adjustment of control parameters of Normal Cloud Model while population fitness changes, in this way, accuracy in searching precision and searching coverage can be improved. Crossover probability Pc and mutation probability Pm of CGA can make dynamic adjustment accord to fitness and Show a linear change with their self-fitness. The individual with higher fitness has lower probability of crossover and mutation, conversely, the lower fitness leads to higher probability.

The formula for adaptive generation of Pc and Pm is as follows:

Generating algorithm of P_c

$Ex := \overline{f}$, $\overline{f}$ is the average of population;

$En := \left(f_{\max} - \overline{f} \right) / C_1$, C_1 is control coefficient;

$He := E_n / C_2$; C_2 is control coefficient;

$Enn := Rand\left(En\right) * He + En$;

$P_c := \begin{cases} k_1 e^{-\frac{\left(f^{'} - Ex\right)^2}{2\left(Enn\right)^2}} & , f^{'} \geq \overline{f} \\ k_3 & f^{'} < \overline{f} \end{cases}$, $f^{'}$ is the bigger fitness relatively in crossover operation.

Generating algorithm of P_m

$Ex := f$, f is variation individual;

$En := \left(f_{\max} - \overline{f} \right) / C_3$, C_3 is control coefficient;

$He := En / C_4$, C_4 is control coefficient;

$Enn := Rand\left(En\right) * He + En$;

$$P_m := \begin{cases} k_2 e^{-\frac{\left(f - Ex\right)^2}{2\left(Enn\right)^2}} & f \geq \overline{f} \\ k_4 & f < \overline{f} \end{cases}$$

$k_1, \cdots, k_4$ are constants within $\left[0,1\right]$, let $k_1 = k_3 = 1, k_2 = k_4 = 0.5$. It can be seen that the higher initial value of P_c and P_m will decrease along with the population evolutionary.

There are three important control parameters in cloud model: Entropy En , reflecting the steepness of cloud model, Expectation Ex , indicating the horizontal position of cloud model and Hyper Entropy He , standing for the dispersion and fluctuation of cloud droplet. Stability of cloud model will drop if He is too high while randomness will descend when He is too low. The introduction of adaptive operator can effectively control the self-adaptation of parameters, adjust the convergence rate at the average value f_{avg} of population fitness and improve search capability of individuals with low fitness on the premise of maintaining the stable tendency of superior individuals for generating a larger solution space. Individuals will adapt to environment in accordance with distribution of population fitness when adaptive CGA is adopted to solve multi-attribute tradeoff optimization in CCMPS, which could improve the robustness of algorithm significantly. Control parameter of algorithm could adjust population fitness dynamically, urge the individuals around the average of self-fitness to drive evolutionary optimization of entirety and avoid getting into local optimality effectively. The concrete steps of adaptive CGA are as follows:

(1) Code design: Chromosome structure with multi-group code is selected in this article and each chromosome corresponds to a list of priority. Natural coding indicates the priority of mission which is a natural number within $[1, m]$ and a higher value means a greater priority.

(2) Initial population: Full permutation $W_u(t)$ of priority is generated randomly and the priority of each mission settled by chromosome is important basis of generating critical chain. The ultimate critical chain can be determined through the steps mentioned above, then calculate project buffer and the embedded buffer and confirm the insert position.

(3) Fitness function: Fitness function is the foundation to evaluate merits or defects of chromosome and let fitness function $fit(t) = 1/u(t)$. $u(t)$ is the desired value of individual, so the one, whose fitness is lower, is more superior.

(4) Selecting operator: Firstly, roulette-wheel selected operator is designed and relative fitness of individuals in species is calculated and ranked. Secondly, divide bets area into sectors according to fitness, which, in other words, will give outstanding chromosomes more chance to be passed into next generation.

(5) Crossover operator: Produced crossover probability Pc of population through X -generator of cloud model, calculate the certainty degree u, Ex, En and He, generate a pair of individuals by X -conditional cloud generator, employ double-crossing point operation and then chose one crossing point of gene to put the selected gene code at the first place of progeny chromosomes, delete the same gene code in paternal chromosome and copy other gene codes into progeny in order, adjust position of crossing point when some progeny chromosomes contrary to restraints is generated.

(6) Mutation operator: Produce mutation probability Pm of population via X -generator of cloud model, give the original individual to Ex and calculate En and He, create a new individual with cloud generator and update it when the random number, within (0, 1), meets $temp > u$.

(7) Repeat (2)-(6) and stop the process when $Maxgen = 50$ or terminal condition is met, then output results.

5. EXPERIMENT AND SIMULATION

Take the multi-project scheduling problem consisted of three parallel projects as an example to test and simulate tradeoff optimization model and its solving algorithm above. As shown in fig.1, there are 32 activities(including virtual starting and ending activity), 6 kinds of resources(4 of them are renewable and the other 2 are non-renewable) and the usage amount, total amount of resources and project duration has already been marked in fig.2, the rate of indirect cost $c = 4$,the single price of the 6 resources are:

$$c_p^1 = c_p^2 = c_p^3 = c_p^4 = 3, c_k^1 = c_k^2 = 2.$$

The utility function of time T, cost C, quality Q and robustness L are $u(T)$, $u(C)$, $u(Q)$ and $u(L)$, and specific value of parameters can be get according to respective upper and lower limits: $\varphi_T = 1$, $\beta_T = \frac{1}{19^2}$, $\varphi_C = 1$, $\beta_C = \frac{1}{1100^2}$, $\varphi_Q = 1$, $\beta_Q = 1$, $\varphi_L = 1$ $\beta_L = 1$. Setting weight to each parameter with frequently-used expert evaluation method or the actual need in enterprises and the weight coefficient are $\alpha_T = 0.4$, $\alpha_C = 0.2$, $\alpha_Q = 0.1$, $\alpha_L = 0.3$.

Multi-attribute utility function of CCMPS can be simplified as the following based on the analysis above:

$$u(T,C,Q,L)=\alpha_T\left(1-\frac{1}{19^2}(T-19)^2\right)+\alpha_C\left(1-\frac{1}{1100^2}(C-1100)^2\right)+\alpha_Q\left(1-(Q-1)^2\right)+\alpha_L\left(1-(L-1)^2\right)$$

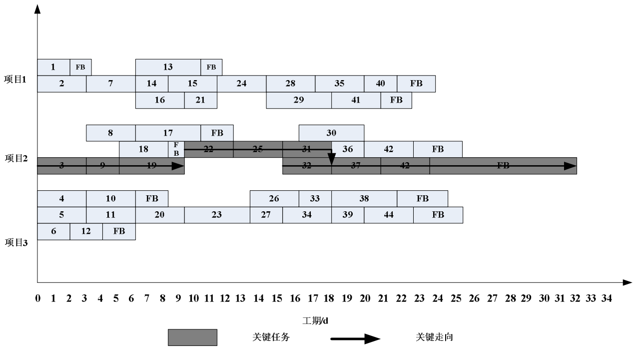

Figure 1. Gantt Chart Of Multi-Project Scheduling

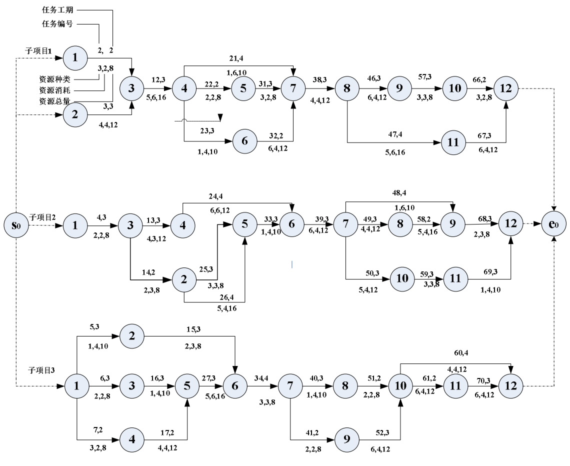

Figure 2. Network Structure Of Multi-Project

Realize CGA by using Mpple12.0 to solve the built model. Assume the population size $Popsize = 50$ and let evolution algebra $Maxgen = 50$ in order to ensure the diversity of population, then the maximum utility value of project 2 on critical chain is : $u(T, C, Q, L) = 0.893$, $T = 32$, $C = 1632$, $Q = 0.911$, $L = 0.884$.

Seen from fig.1 that the result of the optimization model, with four attributes of time, cost, quality and robustness, can help managers and operators control and supervise the process of multi-project scheduling effectively. More energy should be given to activities on critical chain according to the experimental result and operators can change decisions based on different emphasis points to achieve maximum global utility.

6. CONCLUSION

Tradeoff relationship among attributes in CCMPS was considered in this model and utility function was employed to realize optimization. The randomness and stable tendency of a normal cloud model are adopted to design the probabilities of crossover and mutation which was given a greater value in the initial stage for the purpose of generating superior individuals quickly, while in the later stage a smaller value were set up along with crossover operator of maximum retention mechanism to protect superior genes from damage. In the last, effectiveness of the model and algorithm were verified by a case study.

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