

RESEARCH ON DESIGN FIR DIGITAL FILTER USING MATLAB AND WINDOW FUNCTION METHOD

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ABSTRACT

Traditional design method of digital filter is complicated but getting no direct-vision output. The paper utilized Matlab to solve the problem because of its ability of scientific compute and advantage of figure demonstration. On the base of analyzing window function method, this paper performed simulation calculations applying Matlab and window function method to design digital filter. Observing the output of filter, the paper achieved the desired results with its high precision and velocity.

Keywords: *Window Function Method (WFM), FIR Digital Filter (FDF), Matlab*

1. INTRODUCTION

The digital filter is to complete a specific task of discrete time system; it can be realized by the finite precision algorithm. When using a causal stability of discrete linear shift invariant system function to approximate the filter performance requirements, system function has two types which includes the Infinite Impulse Response (IIR) system function and the Finite Impulse Response (FIR) system function, corresponding to the digital filter has two types which includes the Infinite Impulse Response (IIR) digital filter and the Finite Impulse Response (FIR) digital filter.

The design of Analog filter can use lots of available charts, and the design of an Infinite Impulse Response (IIR) digital filter process can be utilized in the analog filter design results, so it is more convenient. But the IIR digital filter has the shortcomings of phase nonlinear, it need to be increased by the correction network.

Finite impulse response (FIR) digital filter impulse response is finite, so it can be used for Fast Fourier Transform (FFT) algorithm to achieve the filtered signal, which can greatly improve the efficiency of operation. In addition, FIR digital filter can be designed a linear phase digital filter which is convenient for image processing and data transmission applications. The so-called a linear phase digital filters refers to the filter impulse response unit is a symmetric sequence, and the filter group delay is equal to half of the length of unit impulse response sequence.

Matlab is a very strong scientific computing and graphics software system which can be accurately mathematical filter design and convenient for the FFT spectrum analysis and spectrum display. Therefore, the realization of Matlab program is simple. The hardware requirement is not high and easy to implement which can quickly in the ordinary PC machine to accomplish the signal spectrum analysis and spectrum display. Matlab can design a digital filter rapidly and test or analysis.

2. TRAINING OF ANN PARAMETERS

2.1 Principle of Window Function Method

We can design FIR digital filter from the time domain or the frequency domain, which method starts from a frequency domain called frequency sampling design method and which methods from starts from the time domain called the window function method, also known as Fourier series method that has wide applications.

The design idea of window function method is in accordance with the requirements of the ideal filter frequency response $H_d(e^{j\omega})$ to design a FIR digital filter whose frequency response

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n)e^{-j\omega n} \text{ is approximate to } H_d(e^{j\omega}).$$

Because the design is carried out in time domain, required by the Fourier inverse transform $H_d(e^{j\omega})$ derived sequence $h_d(n)$, i.e.



$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega$$

Because $H_d(e^{j\omega})$ is a rectangular frequency characteristic and $h_d(n)$ is an infinite sequence and $h_d(n)$ is not causal. But the designed FIR digital filter impulse response sequence's length is limited sequence, so must to use finite sequences $h(n)$ to approximate the infinitely long sequences $h_d(n)$. The most effective method is to truncate $h_d(n)$, or use a limited length window function sequence $w(n)$ to intercept $h_d(n)$, i.e.

$$h(n) = w(n)h_d(n)$$

According to the complex convolution formula, in the time domain can be expressed as the product of relationships in the frequency domain of the periodic convolution relations, then the design of FIR digital filter frequency response

$$H(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(\theta)W(\omega - \theta)d\theta$$

Wherein, $W(\omega)$ as a truncated window function of frequency characteristics.

Thus, the actual FIR digital filter frequency response $H(e^{j\omega})$ is approximation to the desired filter frequency response $H_d(e^{j\omega})$ which depends entirely on the window function of frequency characteristics.

2.2 Design Steps of Window Function Method

The design steps Window function method includes $H_d(e^{j\omega})$;

1) Give the frequency response function of the FIR digital filter;

2) Compute the unit impulse response $h_d(n) = IDTFT[H_d(e^{j\omega})]$ of the FIR digital filter (i.e., inverse Fourier transform);

3) According to the requirements of the transition bandwidth and minimum stopband attenuation, selected window function shapes $w(n)$ by checking the table and calculate the size of filter order N ;

4) Computer the unit impulse response $h(n) = h_d(n)w(n)$ of the designed FIR digital filter;

5) Computer the frequency response $H(e^{j\omega}) = DTFT[h(n)]$ of the designed FIR digital filter (i.e., Fourier transform). Test whether to meet the design requirements, if not, you will need to design.

2.3 Design Processing of Window Function Method

Assume the group delay of the linear phase filter is τ , and then the ideal frequency response of the filter is

$$H_d(e^{j\omega}) = e^{-j\omega\tau} \triangleq H_d(\omega)e^{-j\omega\tau}$$

Get the sequence

$$\begin{aligned} h_d(n) &= \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j\omega(n-\tau)} d\omega \end{aligned}$$

It is the symmetric infinite non causal sequence at the central point. Now using the rectangular window $R_N(n)$ to intercept the infinite length sequence and get the finite length sequence, the window function is

$$w(n) = R_N(n)$$

And the frequency characteristic of window function is

$$\begin{aligned} W_R(e^{j\omega}) &= \sum_{n=0}^{N-1} w(n)e^{-j\omega n} = \sum_{n=0}^{N-1} e^{-j\omega n} \\ &= \frac{\sin(\omega N / 2)}{\sin(\omega / 2)} e^{-j\omega\tau} \\ &\triangleq W_R(\omega)e^{-j\omega\tau} \end{aligned}$$

In which

$$W_R(\omega) = \frac{\sin(\omega N / 2)}{\sin(\omega / 2)}$$

$$\tau = (N - 1) / 2$$

Finally we can get the frequency response of the designed FIR digital filter by complex convolution formula

$$H(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(\theta)W_R(\omega - \theta)d\theta$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\theta})e^{j(\omega-\theta)}d\theta$$

3. DESIGN EXAMPLES AND PERFORMANCE ANALYSIS OF FIR DIGITAL FILTER

The transfer function of FIR digital filter can be written as

$$H(z) = \frac{Y(z)}{X(z)}$$

$$= b(1) + b(2)z^{-1} + \dots + b(n+1)z^{-n}$$

Using the window function method to design FIR digital filter, and select respectively the FIR digital filter with order 4, order 8 or order 12 to simulate and analysis.

Select sampling points is 24 in each periodic, i.e. the sampling frequency is 1200Hz. Design a high-pass filter whose approximate magnitude is 0 in the frequency domain 0–132Hz, approximate amplitude is 1 in the frequency domain 144–600Hz. Use the test signal as follows to test the designed FIR digital filter.

$$f(t) = \sum_{k=1}^{11} 10 \sin(100kt\pi), t \in [0, 0.1]$$

3.1 FIR Digital Filter with Order 4

The transfer function of FIR digital filter with order 4 is

$$H(z) = \frac{Y(z)}{X(z)}$$

$$= b(1) + b(2)z^{-1} + \dots + b(5)z^{-4}$$

In which the parameters are:

$$b(1) \sim b(5) = -0.15782293,$$

$$-0.21173583, 0.76822177,$$

$$-0.21173583, -0.15782293.$$

Figure 1 shows the response of the designed FIR digital filter with order 4. Figure 2 shows the test results of the designed FIR digital filter with order 4 by the test signal, which (a) is the input signal of the FIR digital filter, (b) is the ideal output signal of

the FIR digital filter and (c) is the actual output signal of the FIR digital filter.

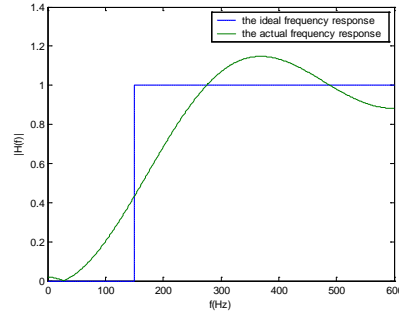


Figure 1: Response of Filter with Order 4

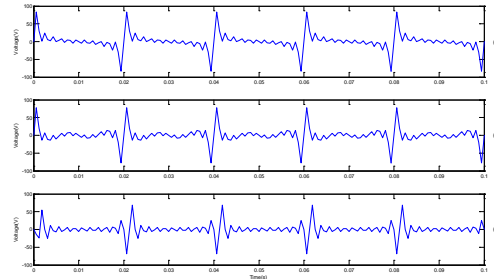


Figure 2: Test result of filter with order 4

3.2 FIR Digital Filter with Order 8

The transfer function of FIR digital filter with order 8 is

$$H(z) = \frac{Y(z)}{X(z)}$$

$$= b(1) + b(2)z^{-1} + \dots + b(9)z^{-8}$$

In which the parameters are:

$$b(1) \sim b(9) = -0.02072878,$$

$$-0.08801979, -0.15746561,$$

$$-0.20956134, 0.77112387,$$

$$-0.20956134, -0.15746561,$$

$$-0.08801979, -0.020728783.$$

Figure 3 shows the response of the designed FIR digital filter with order 8. Figure 4 shows the test results of the designed FIR digital filter with order 8 by the test signal, which (a) is the input signal of the FIR digital filter, (b) is the ideal output signal of the FIR digital filter and (c) is the actual output signal of the FIR digital filter.

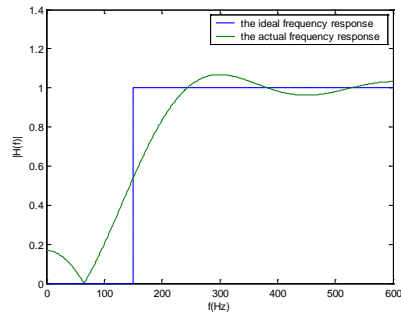


Figure 3: Response of Filter with Order 8

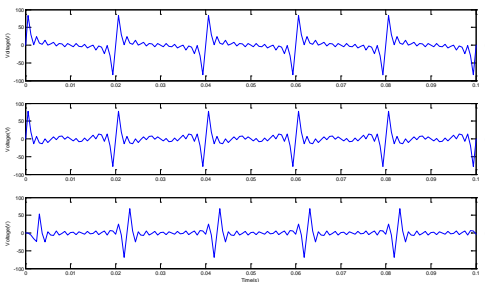


Figure 4: Test result of filter with order 8

3.3 FIR Digital Filter with Order 12

The transfer function of FIR digital filter with order 12 is

$$H(z) = \frac{Y(z)}{X(z)}$$

$$= b(1) + b(2)z^{-1} + \dots + b(13)z^{-12}$$

In which the parameters are:

$$b(1) \sim b(13) = 0.04882396,$$

$$0.02929569, -0.01886463,$$

$$-0.08694512, -0.15771661,$$

$$-0.21101489, 0.76918967,$$

$$-0.21101489, -0.15771661,$$

$$-0.08694512, -0.01886463,$$

$$0.02929569, 0.04882396.$$

Figure 5 shows the response of the designed FIR digital filter with order 12. Figure 6 shows the test results of the designed FIR digital filter with order 12 by the test signal, which (a) is the input signal of the FIR digital filter, (b) is the ideal output signal of the FIR digital filter and (c) is the actual output signal of the FIR digital filter.

3.4 Performance Analysis of FIR Digital Filter

Through the use of window function method of the design of FIR digital filter with order 4, order 8 or order 12, we can analyze the performance characteristics of FIR digital filter:

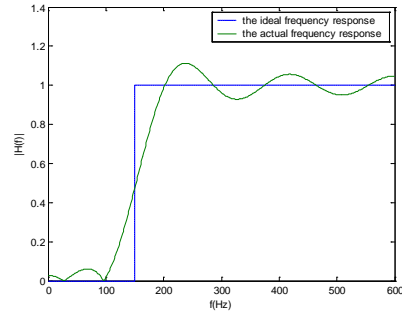


Figure 5: Response of Filter with Order 12

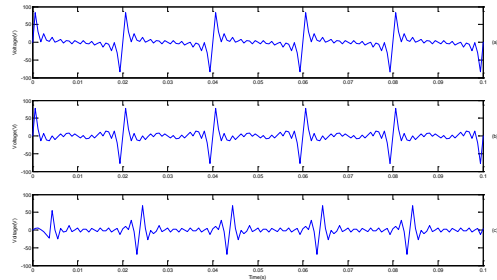


Figure 6: Test result of filter with order 12

1) Seeing from the frequency response graph of the FIR filter, the actual frequency response is much close to the desired frequency response when the order of the filter is higher.

2) Seeing from the frequency characteristic graph of the input signal and the output signal of the FIR filter, the filter has better filtering effect and stronger ability to filter the interference signal when the order of the filter is higher.

3) When the order of the filter is increased, data calculation workload is increased significantly but the improvement of the frequency characteristic of the FIR filter is not very clear.

4. CONCLUSIONS

IIR digital filters and FIR digital filters have advantages and disadvantages respectively. They also have different applications domain:

1) When meeting the same performance requirements, the order of IIR digital filter is secondary lower than the order of FIR digital filter. Therefore, IIR digital filter should be choosing to



use when calculation speed is the much requirements.

2) FIR digital filter has a precise linear phase much than IIR digital filter because IIR digital filter has a nonlinear phase. Therefore, FIR digital filter should be choosing to use when calculation accuracy is the much requirements.

In general, on the base of considering the calculation speed and calculation accuracy, the paper argues that using FIR digital filter with order 8 is the optimal solution because FIR digital filter with order 8 has capable of both accuracy and speed.

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