This paper explores the relay selection scheme for two-way relay channels with physical layer network coding (PNC), where two end nodes exchange information via a selected relay node with the minimal symbol error rate (SER) performance among multiple candidate relay nodes. To reduce the computational complexity, we also propose a suboptimal minimal SER-based relay selection. The end-to-end SER performance of the proposed suboptimal scheme is analyzed. It is proved that full diversity order can be achieved by the suboptimal scheme if the set of candidate relays includes all relay nodes in the system. Extensive simulation results show that the proposed relay selection scheme can considerably improve the SER performance of two-way relay networks with PNC.

Keywords: Physical Layer Network Coding (PNC), Relay Selection, Symbol Error Rate (SER)

1. INTRODUCTION

The two-way relay network model, where two end nodes exchange their information via a relay node, has attracted much attention from researchers [1,2], owing to its potential applications in various kinds of modern wireless networks, including relay-assisted cellular networks and multi-hop mesh networks. Compared with two separated one-way relay transmissions, two-way relay network can greatly improve spectral efficiency and throughput by providing opportunities to employ some advanced coding technologies, such as superposition coding and network coding [3, 4].

As for network coding, it was introduced in [3], since then a great deal of result has been obtained, which show that network coding is a promising technology for next generation wireless networks. As opposed to separating data in traditional approaches, network coding allows relay node to mix the received data or signals before forwarding them. Therefore, amount of transmission can be reduced in the network and the overall network throughput can be improved. In two-way relay network, if analog network coding (ANC) [5] or physical layer network coding (PNC) [6] is employed, two time slots are required to accomplish a round of information exchange between the two end nodes, while in traditional relay transmission, four time slots are needed. In the first time slot, the process of ANC and PNC are the same. The two end nodes transmit their signals to the relay node simultaneously. However, the process is different in the second time slot. In ANC scheme, the relay node simply amplifies and forwards the received signal to the end nodes. In PNC scheme, the relay node decodes the received signals, maps the decoded signals to a network-coded packet by bitwise XOR, and then broadcasts them to the end nodes. Compared with ANC scheme, PNC scheme is more complex since it needs to decode and map the received signal to a network-coded packet by bitwise XOR, and then broadcasts them to the end nodes. Compared with ANC scheme, PNC scheme is more complex since it needs to decode and map the received signal to a network-coded packet. However, in ANC scheme, relay does not remove received noise at the relay and the received noise is forwarded together with the signals to the end nodes. As a result, the performance of ANC is not as good as PNC [7]. Therefore, we focus on PNC in this paper.

In many networks, such as cellular networks and wireless mesh networks, a set of nodes can be used as relay nodes for the transmissions between two end nodes relatively far from each other. It was pointed out that by relay selection, system performance in terms of low hardware cost and high...
power efficiency can be greatly enhanced [8]. Consequently, relay selection has been considered as a promising technology for practical implementation of relay networks and has been widely investigated for one-way relay channels recently [9, 10]. However, for two-way relay networks, relay selection has not been as well studied as that for one-way relay networks. To the author’s knowledge, some researchers have begun to consider relay schemes for two-way relay networks [11-13]. The relay selection for ANC scheme was proposed in [11-13]. In [11], relay selection scheme was proposed to maximize the weighted sum-rate capacity. In [12], the authors maximized the worse signal-to-noise ratio (SNR) of the two end users. In [13], the relay which minimized the maximum SER of two source nodes was selected. The relay selection for PNC scheme was proposed in [14, 15]. In [14], a relay selection scheme with PNC was proposed by modifying the well-known selection cooperation, where the relay that can maximize the minimal mutual information of the two channels is selected. In [15], the author discussed the MaxMin criterion for modular network coding at layer link, and the frame error rate (FER) is analyzed.

It is well known that symbol error rate (SER) is an important measure to evaluate the network performance. In light of the preceding discussion, we propose a relay selection scheme based on minimizing SER. The main idea of our proposed scheme is follows. In the first time slot, the two end nodes transmit their information to the relays simultaneously. In the second time slot, the relay with the best end-to-end SER performance is selected to map the received signals to a network-coded packet and broadcast it to the two end nodes.

The main contributions of the paper are summarized as follows. (1) A new relay selection scheme is proposed to minimize the end-to-end SER for two-way relay channels with PNC. (2) A suboptimal scheme is also proposed to make the relay selection scheme more practical, and through the analysis, it is prove that the full diversity order can be achieved if the candidate relays consist of all relay nodes in the system. (3) Extensive simulations are performed and the results show that the suboptimal has very similar SER performance as optimal scheme. The simulation results also show that the SER performance of the proposed relay selection system is better than no relay selection used system.

The rest of this paper is organized as follows. Section 2 discusses the system model. Section 3 proposes the SER-minimized relay selection scheme for two-way relay PNC. The performance of the suboptimal scheme is discussed in Section 4. In Section 5, simulation results are presented to validate our proposed relay selection scheme can improve the SER performance. Section 6 concludes this paper.

2. SYSTEM MODEL

Consider a relay network shown in Fig. 1, where two end nodes A and B exchange their information with the help of L relays, noted as R_1, R_2, ..., R_L. For PNC, two time slots are needed to accomplish a round of information exchange. In the first time slot, A and B simultaneously send their information to all relays, and all the L relays received the superimposed signal via different channels. In the second time slot, a single relay which provides the minimal end-to-end SER performance is selected to map the superimposed signal into a network-code packet and forwards to two end nodes and the unselected relay nodes keep silence. It is assumed that all the nodes are equipped with one antenna and all the channels experience slow fading.

Let x_A and x_B denote the signal transmitted by A and B, and powers of A and B are denoted by P_A and P_B, respectively. Thus, the received signal at the relay R_i (i = 1, ..., L) can be written as:

$$y_{R_i} = \sqrt{P_A}h_{A,R_i}x_A + \sqrt{P_B}h_{B,R_i}x_B + n_{R_i},$$  \hspace{1cm}(1)$$

where $h_{A,R_i}$ and $h_{B,R_i}$ denote the channel coefficients from A and B to R_i, and modeled as zero mean symmetric complex Gaussian distributed with variance of $\sigma^2_A$ and $\sigma^2_B$, respectively. $n_{R_i}$ is complex Gaussian distributed noise with a noise power spectral density of $N_0/2$ per dimension.

![Fig.1 System Model](image)
Upon receiving the superimposed signal, \( R_k \) decodes the superimposed signal from A and B by maximum likelihood (ML) joint detection. The ML detection can be expressed as

\[
(x_k, x_\theta) = \arg \min_{(x_k, x_\theta)} | y_k - \sqrt{P_{\text{th},k}} b_{\theta,k} x_k - \sqrt{P_{\text{th},k}} b_{\theta,k} x_\theta |, \tag{2}
\]

where \( x_k \) and \( x_\theta \) denote the estimated results of \( x_k \) and \( x_\theta \) respectively. Note that, the decoding of the superimposed signal, \( C(x_k, x_\theta) \), such as bitwise XOR is a conventional mapping scheme.

After detection, \( R_k \) performs bitwise XOR to the decoded signal, and the estimated results are mapped into a network-coded signal which is denoted by \( C(x_k, x_\theta) \). Such as bitwise XOR is a conventional mapping scheme.

Then, \( R_k \) broadcasts the network-coded signal \( C(x_k, x_\theta) \) to A and B. Thus, the received signals at A and B can be given by

\[
y_A = \sqrt{P_{\text{th},k}} b_{\theta,k} C(x_k, x_\theta) + n_A \\
y_B = \sqrt{P_{\text{th},k}} b_{\theta,k} C(x_k, x_\theta) + n_B
\tag{3}
\]

Respectively, where \( n_A \) and \( n_B \) are complex Gaussian distributed with a noise power spectral density of \( N_0/2 \) per dimension and \( P_{\text{th},k} \) denotes the power of each relay.

Upon receiving the signals, A could detect its desired signal of B by its own signal as follows

\[
x_k = \arg \min_{x} | y_k - \sqrt{P_{\text{th},k}} b_{\theta,k} C(x_k, x) |. \tag{4}
\]

Similarly, B detects the signal of A by its own signal \( x_\theta \).

3. RELAY SELECTION SCHEME

In the proposed SER-minimization relay selection scheme, only one of the \( L \) candidate relays can be selected to process and forward the received superimposed signal. We assume that, at the beginning of each transmission, some pilot symbols are transmitted to decide the optimal relay which provides the minimized SER performance. Let \( \rho_{ee}^{\theta} \) denote the end-to-end SER when only \( R_i \) helps the transmission, and the selected relay is denoted by \( R_i \). Then, the relay selection scheme can be given by

\[
R_i = \arg \min_{R_k} \rho_{ee}^{\theta}. \tag{5}
\]

3.1 Optimal Relay Selection

In this subsection, we use BPSK modulation as an example to show the proposed scheme. By expanding \( \rho_{ee}^{\theta} \), we can derive the optimal relay selection scheme for two-way relay PNC channels.

The two-way relay system is composed by two directional transmissions. One is from A to B via \( R_i \) and the other one is from B to A via \( R_i \). For the end-to-end transmission from A to B, B acts as a receiver and may receive signals incorrectly in two cases. In the first case, the detection at \( R_i \) is correct but detection at B is incorrect, while in the second case, the detection at \( R_i \) is incorrect but detection at B is correct. Thus, the SER of the end-to-end transmission from A to B can be given by

\[
\rho_{AB}^{ee} = (1 - \rho_b^A)p_r^B + (1 - \rho_b^B)p_r^A,
\tag{6}
\]

where \( p_r^A \) and \( p_r^B \) denote the detection error probabilities at B and \( R_i \), respectively. Note that, since BPSK modulation is implied, B receives the signal correctly when the detection at \( R_i \) and B are both incorrect.

Similarly, the SER of the end-to-end transmission from B to A can be given by

\[
\rho_{BA}^{ee} = (1 - \rho_b^B)p_r^A + (1 - \rho_b^A)p_r^B,
\tag{7}
\]

where \( p_r^A \) and \( p_r^B \) denote the detection error probability at A.

As a result, in terms of (7) and (8), \( \rho_{ee}^{\theta} \) can be derived by averaging \( \rho_{AB}^{ee} \) and \( \rho_{BA}^{ee} \) as follows

\[
\rho_{ee}^{\theta} = \frac{1}{2} (\rho_{AB}^{ee} + \rho_{BA}^{ee})
\tag{8}
\]

Therefore, by substituting (8) into (5), the optimal SER-minimization relay selection for two-way relay PNC can be given by

\[
R_i = \arg \min_{R_k} \left\{ \frac{1}{2} (\rho_b^A + \rho_b^B + 2p_r^A - 2p_r^A, p_r^B, p_r^B) \right\}.
\tag{9}
\]

3.2 Suboptimal Relay Selection Scheme

Due to the complexity of (9), the optimal relay selection scheme is hard to deploy in a practical system. Therefore, we propose a suboptimal relay selection scheme which is much simpler implement, and the performance of the suboptimal scheme will be discussed in the next section.

For (8), since \( p_r^i (0 < 1) \) where \( i \in \{A, B, R_i\} \) is often much closer to 0 than to 1 in practical, it can be inferred that \( p_r^A, p_r^B \) and \( p_r^A, p_r^B \) are much smaller.
than $p_k^*$, $p_n^*$ and $p_h^*$, and then (8) can be approximated as

$$p_{k}^{2^{*}} \approx p_{h}^{*} + \frac{1}{2} p_{n}^{*} + \frac{1}{2} p_{h}^{*}. \quad (10)$$

To simplify the analysis, we assume $P_i = P_k = P_0$. In [15], it is shown that the tight upper bound of $p_k$ for BPSK modulation is

$$Q(\sqrt{2 \min(|h_{A,i}|^2, |h_{B,i}|^2) P_i / N_0}). \quad (11)$$

Moreover, as is known

$$p_{h}^{*} = Q(\sqrt{2 |h_{A,i}|^2 P_k / N_0}).$$

Substituting (11) and (12) into (10), we can derive

$$p_{k}^{2^{*}} \approx Q(\sqrt{2 \min(|h_{A,i}|^2, |h_{B,i}|^2) P_i / N_0}) + \frac{1}{2} Q(\sqrt{2 |h_{A,i}|^2 P_k / N_0})$$

$$+ \frac{1}{2} Q(\sqrt{2 |h_{B,i}|^2 P_k / N_0}). \quad (13)$$

Based on the character of $Q$-function, (13) could be further simplified to be

$$p_{k}^{2^{*}} \approx Q(\sqrt{2 \min(|h_{A,i}|^2, |h_{B,i}|^2) P_i / N_0}) + \frac{1}{2} Q(\sqrt{2 \min(|h_{A,i}|^2, |h_{B,i}|^2) P_k / N_0}). \quad (14)$$

From (14), it can be seen the end-to-end SER via $R_i$ is determined by the channel coefficients $h_{A,i}$ and $h_{B,i}$. As $Q$-function is a monotonic decreasing function, $p_{k}^{2^{*}}$ achieves the minimum if and only if $\min(|h_{A,i}|^2, |h_{B,i}|^2)$ becomes maximum. Therefore, the SER performance can be improved by selecting a relay which provides the maximum value of $\min(|h_{A,i}|^2, |h_{B,i}|^2)$. Thus, the suboptimal SER-minimization relay selection for two-way relay PNC can be given by

$$k = \arg \max_{h_{A,i}} \min(|h_{A,i}|^2, |h_{B,i}|^2) \quad (15)$$

Compared with (9), in the suboptimal scheme, it is unnecessary to calculate $p_{k}^{2^{*}}$, and only the channel state information is required. So the complexity is greatly reduced.

4. PERFORMANCE ANALYSIS

In this section, we will analyze the end-to-end SER performance of the suboptimal scheme. At first, we consider a special case, where we assume that $P_k = P_k$. Then we consider more practical and general case, where $P_k > P_k$ and $P_k < P_k$ are discussed separately.

4.2.1 Performance Analysis when $P_k = P_k$

As for $P_k = P_k$, we let $P_0$ denote $P_0$ and $P_0$. $p_{k}^{2^{*}}$ can be simplified to be

$$\frac{1}{2} Q(\sqrt{2 \min(|h_{A,k}|^2, |h_{B,k}|^2) P_k / N_0})$$

in terms of (14). Let $\gamma_{A,k}$ and $\gamma_{B,k}$ denote $|h_{A,k}|^2 P_k / N_0$ and $|h_{B,k}|^2 P_k / N_0$, respectively, which also represent the instantaneous SNR of links $A \rightarrow R_k$ (or $R_k \rightarrow A$) and $B \rightarrow R_k$ (or $R_k \rightarrow B$), respectively. It can be inferred that $\gamma_{A,k}$ and $\gamma_{B,k}$ are exponential distribution variables with mean of $\sigma_{A,k}^2 P_k / N_0$ and $\sigma_{B,k}^2 P_k / N_0$. Therefore, the probability density function (PDF) and cumulative distribution function (CDF) of $\gamma_{k}$ can be given by

$$f(\gamma_{k}) = \frac{(\sigma_{A,k} + \sigma_{B,k}) N_0}{\sigma_{A,k}^2 \sigma_{B,k}^2 P_k} \exp\left(-\frac{(\sigma_{A,k} + \sigma_{B,k}) N_0}{\sigma_{A,k}^2 \sigma_{B,k}^2 P_k} \gamma_{k}ight) \quad (16)$$

and

$$F(\gamma_{k}) = 1 - \exp\left(-\frac{(\sigma_{A,k} + \sigma_{B,k}) N_0}{\sigma_{A,k}^2 \sigma_{B,k}^2 P_k} \gamma_{k}\right). \quad (17)$$

respectively, where $\gamma_{k} = \min(|h_{A,k}|^2, |h_{B,k}|^2) P_k / N_0$. From (15), the CDF of $\gamma_{k}$ for the selected relay $R_k$ can be given by

$$P(\gamma_{k}) = p(\max(\gamma_{1}, \cdots, \gamma_{k}) \leq \gamma_{k})$$

$$= \prod_{i=1}^{k-1} F(\gamma_{k}) \quad (18)$$

Thus, when $P_k = P_k = P$, the SER of the proposed relay selection scheme can be given by

$$p_{k}^{2^{*}} = \frac{3}{2} \int \frac{1}{\sqrt{2 \min(\gamma_{A,k}, \gamma_{B,k})} d\gamma_{A,k}}$$

$$= \frac{3}{2} \int \frac{1}{\sqrt{2 \min(\gamma_{A,k}, \gamma_{B,k})} d\gamma_{A,k}} \left[1 - \exp\left(-\frac{(\sigma_{A,k}^2 + \sigma_{B,k}^2) N_0}{\sigma_{A,k}^2 \sigma_{B,k}^2 P_k} \gamma_{k}\right)\right]. \quad (19)$$

At high SNR, (19) can be further simplified to be

$$p_{k}^{2^{*}} = \frac{\sigma_{A,k}^2 + \sigma_{B,k}^2}{2 \sigma_{A,k}^2 \sigma_{B,k}^2} \gamma_{k} e^{-\frac{(\sigma_{A,k}^2 + \sigma_{B,k}^2) N_0}{\sigma_{A,k}^2 \sigma_{B,k}^2 P_k} \gamma_{k}} + o\left((P / N_0)^{-1}\right), \quad (20)$$
and it can be seen that the proposed scheme can achieve full diversity order at high SNRs when \( P_s = P_k \).

### 4.2.2 Performance Analysis when \( P_s > P_k \)

As for \( P_s > P_k \), accounting for the characteristic of Q-function, \( p_{\gamma_{k}^{SR}} \) is dominated by the second term of (14) at high SNRs. In this case, \( p_{\gamma_{k}^{SR}} \approx \frac{1}{2}Q(2\min(h_{A,s}, h_{B,s})P_k / N_0) \). Similar to the case of \( P_s = P_k \), let \( \gamma_{A,s} \) and \( \gamma_{B,s} \) denote \( |h_{A,s}|^2 P_k / N_0 \) and \( |h_{B,s}|^2 P_k / N_0 \), respectively. Then,

\[
f(\gamma_{k}^{\min}) = \frac{\sigma_A^2 + \sigma_B^2 N_0}{\sigma_A^2 \sigma_B^2 P_k} \exp \left( -\frac{\sigma_A^2 + \sigma_B^2 N_0}{\sigma_A^2 \sigma_B^2 P_k} \gamma_{k}^{\min} \right)
\]

(21)

and

\[
F(\gamma_{k}^{\min}) = 1 - \exp \left( -\frac{(\sigma_A^2 + \sigma_B^2 N_0)}{\sigma_A^2 \sigma_B^2 P_k} \gamma_{k}^{\min} \right).
\]

(22)

Respectively. In this case, the CDF of \( \gamma_{k}^{\min} \) for the selected relay \( R_k \) can be given by

\[
P(\gamma_{k}^{\min} = \max(\gamma_{k}^{\min}, \ldots, \gamma_{k}^{\min})) \leq \gamma_{k}^{\min}) \]

\[
= \prod_{i=1}^{L} p(\gamma_{k,i}^{\min} \leq \gamma_{k}^{\min})
\]

\[
= 1 - \exp \left( -\frac{\sigma_A^2 + \sigma_B^2 N_0}{\sigma_A^2 \sigma_B^2 P_k} \gamma_{k}^{\min} \right).
\]

(23)

Thus, when \( P_s > P_k \), the SER of the proposed relay selection scheme can be given by

\[
P_{\gamma_{k}^{\min}}^{SR} = \int \frac{1}{2}Q(2\gamma_{k}^{\min} / \gamma_{k}^{\min}) dP(\gamma_{k}^{\min})
\]

\[
= \sum_{r=0}^{L} \frac{1}{4} C_r^{(-1)r} \frac{1}{\sqrt{1 + \frac{1}{P_s^2 / P_k}}} \exp \left( -\frac{\sigma_A^2 + \sigma_B^2 N_0}{\sigma_A^2 \sigma_B^2 P_k} \gamma_{k}^{\min} \right).
\]

(24)

At high SNR region, (24) can be further simplified to be

\[
P_{\gamma_{k}^{\min}}^{SR} = \frac{1}{2}Q(2\gamma_{k}^{\min} / \gamma_{k}^{\min}) \left( P_s / N_0 \right)^{-\frac{1}{2}} \exp \left( -\frac{\sigma_A^2 + \sigma_B^2 N_0}{\sigma_A^2 \sigma_B^2 P_k} \gamma_{k}^{\min} \right).
\]

(25)

So we can see that the proposed scheme also can achieve full diversity order at high SNR when \( P_s > P_k \).

### 4.2.3 Performance Analysis when \( P_s < P_k \)

As for \( P_s < P_k \), \( p_{\gamma_{k}^{SR}} \) is dominated by the first term of (14) at high SNR. Thus, the end-to-end SER can be further approximated to be

\[
Q(\sqrt{2\min(h_{A,s}, h_{B,s})P_k / h_{A,s} P_k / h_{B,s} P_k / N_0}) \quad \text{and the selection criterion also can be expressed by (11). Let} \quad \gamma_{A,s} \quad \text{and} \quad \gamma_{B,s} \quad \text{denote} \quad |h_{A,s}|^2 P_k / N_0 \quad \text{and} \quad |h_{B,s}|^2 P_k / N_0.
\]

Then

\[
f(\gamma_{k}^{\min}) = \frac{(\sigma_A^2 + \sigma_B^2 N_0)}{\sigma_A^2 \sigma_B^2 P_k} \exp \left( -\frac{(\sigma_A^2 + \sigma_B^2 N_0)}{\sigma_A^2 \sigma_B^2 P_k} \gamma_{k}^{\min} \right)
\]

(26)

and

\[
F(\gamma_{k}^{\min}) = 1 - \exp \left( -\frac{(\sigma_A^2 + \sigma_B^2 N_0)}{\sigma_A^2 \sigma_B^2 P_k} \gamma_{k}^{\min} \right).
\]

(27)

respectively. In this case, the CDF of \( \gamma_{k}^{\min} \) for the selected relay \( R_k \) can be given by

\[
P(\gamma_{k}^{\min} = \max(\gamma_{k}^{\min}, \ldots, \gamma_{k}^{\min}) \leq \gamma_{k}^{\min}) \]

\[
= \prod_{i=1}^{L} p(\gamma_{k,i}^{\min} \leq \gamma_{k}^{\min})
\]

\[
= 1 - \exp \left( -\frac{(\sigma_A^2 + \sigma_B^2 N_0)}{\sigma_A^2 \sigma_B^2 P_k} \gamma_{k}^{\min} \right).
\]

(28)

Thus, when \( P_s < P_k \), the SER of the proposed relay selection scheme can be given by

\[
P_{\gamma_{k}^{\min}}^{SR} = \int \frac{1}{2}Q(2\gamma_{k}^{\min} / \gamma_{k}^{\min}) dP(\gamma_{k}^{\min})
\]

\[
= \sum_{r=0}^{L} \frac{1}{4} C_r^{(-1)r} \frac{1}{\sqrt{1 + \frac{1}{P_s^2 / P_k}}} \exp \left( -\frac{(\sigma_A^2 + \sigma_B^2 N_0)}{(P_s / P_k)^{-\frac{1}{2}}} \gamma_{k}^{\min} \right).
\]

(29)

At high SNR region, (29) can be further simplified to be

\[
P_{\gamma_{k}^{\min}}^{SR} = \frac{(\sigma_A^2 + \sigma_B^2 N_0)(2L-1)!}{2\sqrt{2\sigma_A^2 \sigma_B^2 (L-1)!}} \left( P_s / P_k \right)^{-\frac{1}{2}} \exp \left( -\frac{(\sigma_A^2 + \sigma_B^2 N_0)}{(P_s / P_k)^{-\frac{1}{2}}} \gamma_{k}^{\min} \right) + o(P_s / N_0)^{-\frac{1}{2}}.
\]

(30)

So we can see that the proposed scheme also can achieve full diversity order at high SNR when \( P_s < P_k \).

### 5. SIMULATION RESULTS

In this section, we present some simulation results and analytical results.

At first, the simulated SER of our proposed relay selection scheme is presented. The SER performance of no relay selection scheme is also included as a benchmark. In no relay selection scheme, each relay performs the PNC and forwards the network coded symbols to two source nodes at power \( P_k / L \) and the maximal ratio combining (MRC) is used to combine the received signals.
from all relays. Fig. 2-4 show the simulated SER performance under different $L$ with $R_3 = P_R$, $R_1 < P_R$ and $R_1 > P_R$, respectively. The results show that the proposed relay selection scheme always have better SER performance than the no relay selection scheme. For no relay selection scheme, with the increasement of relay’s number $L$, it is even more difficult to ensure that all the relays can derive correct network-coded signal in the first time slot. Thus, although MRC is applied in the second time slot, full diversity order cannot be achieved in no relay selection scheme.

Secondly, Fig. 5-7 shows the SER performance of optimal and suboptimal scheme with $R_3 = P_R$, $R_1 < P_R$ and $R_1 > P_R$ respectively. The relay numbers $L$ from 1 to 4 are considered. $L = 1$ means only one candidate relay helps the transmission, and also stands for the one-relay end-to-end SER of PNC. Owing to the consistency between SER of simulation and the analytical result, it can derive our approximation for the end-to-end SER is valid. When $L = 2,3,4$, it can be seen that the optimal result and suboptimal result almost have the same SER performance. Thus, the suboptimal relay selection can be considered as the near-optimal relay selection. It also can be seen that the analytical result of the suboptimal scheme is consistent with the simulation result at high SNRs, and the deviation is marginal even at low SNRs. Meanwhile, with the increase of the relay number $L$, SER decreases quickly. It can be seen from Fig. 5 that to make the SER fall to below $10^{-3}$, we should rise $P_0/N_0$ above 28dB when $L = 1$. However, for $L = 4$, $P_0/N_0$ is just needed to maintain 12dB.

Finally, the effect of relay location is also investigated, as shown in Fig. 8-10, where $d_{a,R}$ and $d_{b,R}$ denote the distances between A and R and between B and R, respectively. In the simulations, $d_{a,R} + d_{b,R}$ is fixed to be 1 and the path loss
exponent is set as four. Additionally, \( \sigma_R^2 = d_{A,R}^4 \) and \( \sigma_B^2 = (1 - d_{A,R})^4 \). It can also be seen from Fig. 5-7 that simulation results match the analytical ones very well. Moreover, it shows that the best SER performance is achieved, when R is located at the middle point between A and B.

**Fig. 7** SER Performance Against \( P_e / N_0 \) When \( P_e = P_k - 10\text{dB} \).

**Fig. 8** SER Performance Against \( d_{A,R} \) When \( R = P_k = 5\text{dB} \).

**Fig. 9** SER Performance Against \( d_{A,R} \) When \( P_k = 5\text{dB}, P_e = 15\text{dB} \).

### 6. CONCLUSIONS

In this paper, we proposed a relay selection scheme to improve the SER performance for two-way relay PNC. The proposed SER-Minimization relay selection scheme could select a best relay to minimize the end-to-end SER for two-way relay channels with PNC. To make it more practically, a suboptimal scheme has less complexity was proposed and it was proved that full diversity order could be derived if the set of candidate relays included all relay nodes in the suboptimal system.

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