



OPTIMIZATION OF DEPARTURE TRAJECTORY FOR NOISE ABATEMENT OF SENSITIVE LOCATIONS

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ABSTRACT

Adopting the noise abatement procedures(NAPs) is the common approach for abating the noise around airport for the international civil aviation. But there is only one or two procedures used in China. This paper first gives a preliminary model of optimizing the departure trajectory, which is one of the most important in the NAPs. Then it proposes two approaches with two algorithms respectively, the ant colony algorithm and the relaxed lexicographic algorithm, to solve the problem. The two approaches will be compared in the nature of themselves, and the simulation results will verify the feasibility of the both algorithms. It provides the foundation for the next further study.

Keywords: *Departure Trajectory, Ant Colony Optimization, Lexicographic Optimization*

1. INTRODUCTION

The aircraft noise around the airport is a puzzle for a long time and restricts the development of the airport badly. The impact of it can be limited by adopting the noise abatement procedures (NAPs), which redesigns the route of the aircraft to avoid the over-fly of populated areas, especially at aircraft departure, arrival and approach.

The scholars and the technicians abroad have had much more interested in NAPs since four or five decades before. Early in the year 1967, Zalovcik & Shaefer proposed increasing the final approach slope to about 6° for abating the aircraft noise [1]. In 1970, the aircraft noise became the major concern, and Paullin gave some solutions such as thrust reduction in departures, noise preferential runways and even runway redesign and reorientation [2]. In 1982, three NAPs were tested in John Wayne airport, in Santa Ana(California, USA) followed by telephonic surveys before and after the application of these new procedures [3]. At the same year, the International Civil Aviation Organization (ICAO) established the official NAPs, and revised it in 1996. In recent years, the European Community set up a special research institute to develop and estimate a new NAPs [4]. Nowadays, the most common international departure procedure is the noise abatement departure procedure (NADP) established by the ICAO in 2006 [5]. Deserve to be mentioned, the PhD thesis written by Xavier Prats gives a relatively comprehensive and efficient

research on the NAPs [6], and many results in it are included in this paper.

Compare to the international, the study of the aircraft noise in China started late and has attracted some attentions until recent years. Therefore, there were few positive results and did not form a system. In most of the papers involving the NAPs, the analyses are qualitative. [7]~[9] According to [10], besides the Hong Kong international airport, there are only Beijing capital airport and Shanghai Hongqiao airport in the mainland have adopted the NADP. Moreover, all studies on the design of flight procedure focus on avoiding obstacles. But the algorithms used in them can be references.

To optimize the trajectory is the most important part of the redesign of the procedure. It is a 4-dimensional continuous strong constraint optimal control problem and not easy to solve. So a simplified model will be constructed in the following part. Then two approaches will be proposed respectively to solve the model. And a hypothesis scenario will be used to verify the algorithms are feasible.

2. MODELING OF THE OPTIMIZATION PROBLEM

The goal of the optimization problem is finding the best trajectory \vec{j} which makes a set of $n+1$ objectives minimum. The objective function can be written as followed:

$$\min_{\vec{j} \in J} \bar{E}(\vec{j}) = \min_{\vec{j} \in J} [E_1(\vec{j}), E_2(\vec{j}), \dots, E_n(\vec{j}), E_{n+1}(\vec{j})], \quad (1)$$

Where $E_i(i \in [1, n])$ means the evaluation functions of n sensitive locations to the certain trajectory \vec{j} , and E_{n+1} represents the length of the trajectory \vec{j} . Here $E_i(i \in [1, n])$ use the LAmaz noise levels of locations. J is the set of all feasible trajectories.

The vector of trajectory \vec{j} comprises the control vector \vec{c} and the position vector \vec{x} :

$$\vec{j} = [\vec{c}, \vec{x}] \quad (2)$$

The control vector and the position vector are chosen as:

$$\vec{c} = [v, \theta, \gamma], \quad (3)$$

$$\vec{x} = [e, n, h], \quad (4)$$

where v is the true airspeed, θ is the yaw angle of aircraft and γ is the pitch angle. And \vec{x} is the position of the aircraft center of mass.

Then we can write the functions of the relationship between the vector \vec{x} and the variables of \vec{c} as:

$$\dot{e} = v \cdot \cos \theta \cdot \cos \gamma + W_e, \quad (5)$$

$$\dot{n} = v \cdot \sin \theta \cdot \cos \gamma + W_n, \quad (6)$$

$$\dot{h} = v \cdot \sin \gamma + W_h, \quad (7)$$

where $W_{e,n,h}$ represent the east, north and up velocity components of wind.

3. ALGORITHM FOR THE OPTIMIZATION PROBLEM

Scalarization Approach. For determining solutions of this multi-objective optimization problem, we use the common approach called scalarization approach. The objective of the problem can be transform to a linear weighted sum of the evaluation functions E_i :

$$\min_{\vec{j} \in J} [E_1(\vec{j}), E_2(\vec{j}), \dots, E_{n+1}(\vec{j})] = \min_{\vec{j} \in J} \sum_{i=1}^{n+1} \omega_i E_i(\vec{j}), \quad (8)$$

where $\omega_i \geq 0$ are real numbers and $\sum_{i=1}^{n+1} \omega_i = 1$.

Ant Colony Algorithm. To find the best trajectory is to find a safe and high-performance path from the start point to the goal point which two are both given, and the path should make the weakest noise impact on all sensitive locations. For

the projection of the trajectory on the horizontal plane is clear to observe the impact, it is paramount to optimize. At the same time, according to the departure procedure, the pitch angle γ and the true speed v can be set as constant values.

For compute easily, a new coordinate system $S-X'Y'$ is set up with the origin S which is the start point. The straight-line SG is the horizontal axis where G is the goal point, and the straight-line which is vertical to the SG is set as the vertical axis. Then the feasible area between S and G can be shown with gridding properly as Figure.1 below.

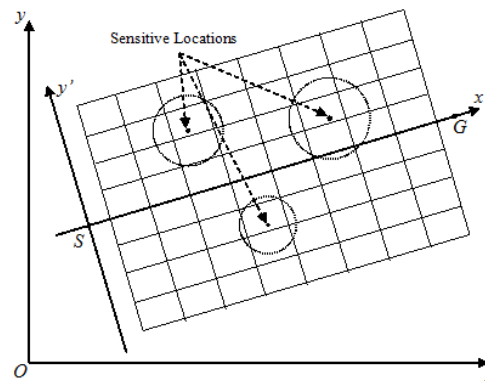


Figure.1: The Coordinate System $S-X'Y'$ And Gridding

The equation of the translation between $S-X'Y'$ and original coordinate system $O-X'Y'$ can be written as:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \varphi & \sin \varphi \\ -\sin \varphi & \cos \varphi \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} x_s \\ y_s \end{bmatrix}, \quad (9)$$

Where φ is the included angle of OX to SG .

Therefore, the trajectory can be defined with the nodes chosen at every step in the path from S to G .

For abating noise of all sensitive locations and controlling the cost of the aircraft, the ant colony algorithm in [11] which is used in path planning will be adopted to find the best trajectory. So the evaluation function of the ant k can be rewritten as followed:

$$F^k = \sum_{i=1}^n \omega_i N_i^k + \omega_{n+1} L^k. \quad (10)$$

L^k represents the total length of the path where the ant k passed, and $N_i^k (i \in [1, n])$ are the LAmaz noise levels of the n sensitive locations in the path. $\omega_i (i \in [1, n])$ are optimization coefficients. When one of them is larger, the noise impact on the sensitive location is weaker.



Following the definition above, the classical ant colony algorithm can be used to find the path which is the objective of the problem. The algorithm is so classical and popular that its detail will be no longer expatiatory.

Hierarchical Sequence Approach. The theory of the scalarization approach is intuitional, and the approach is easy to operate. However, for the weight coefficients do not have any physical meaning, it is difficult to confirm the values and compare the relative importance of each other. On the other hand, the linear weighted objective function can just find the solutions in the convex region of the feasible region. Thus, it will miss some solutions in the non-convex region, which may be more acceptable to the user.

In the real life, people will sort the problems with the weight and solve them in a certain sequence. The hierarchical sequence approach is proposed out of that thinking. It translates a multi-objective problem into some single-objective problems.

Lexicographic Algorithm. The lexicographic Algorithm is one of the hierarchical sequence approaches. It sorts all objectives from the importance to the secondary, and solves them in the sequence, where the problem is solved on the basis of the result of the former except the first one. After the sorting, the objective function (1) will be rewritten as:

$$\min_{\vec{j} \in J} \bar{E}(\vec{j}) = \min_{\vec{j} \in J} [E_{t_1}(\vec{j}), E_{t_2}(\vec{j}), \dots, E_m(\vec{j}), E_{n+1}(\vec{j})], \quad (11)$$

Where $t_i (i \in [1, n])$ is the original number of the objectives and i is the sorted sequence number. While i is smaller, the priority of the objective is higher.

The ant colony algorithm can also be used to solve the problem of every single objective. For the certain objective t_i , the best evaluation function is written as N_{ii}^* , and the solution set is $\{\vec{j}_{ii}^*\}$. So the problem follow the definition before can be described like this:

round 1:

$$\min_{\vec{j} \in J} F_{t_1}^k = \min_{\vec{j} \in J} (N_{t_1}^k + \omega_{n+1} L^k), \quad (12)$$

round $i (i \in [2, n])$:

$$\min_{\vec{j} \in J^*} F_{t_i}^k = \min_{\vec{j} \in J^*} (N_{t_i}^k + \omega_{n+1} L^k), \quad (13)$$

$$\text{s.t. } N_m^k = N_m^*, m = t_1, t_2, \dots, t_{i-1} \quad (14)$$

However, at certain round i , the solution set may get only one element. The rounds behind should have no need to continue, and the objective $m (m=i+1, \dots, n)$ will be ignored. To avoid that, the penalty function can be defined.

$$Pf_{t_i}^k = M_{t_i} (N_{t_i}^k - N_{t_i}^*) \quad (15)$$

M_{t_i} in the function is the penalty coefficient. When M_{t_i} is larger, it means the punishment is severer and the objective t_i has higher priority. Thus, the constraint condition (14) will be relaxed:

$$N_m^k \leq N_m^* + Pf_m^k, m = t_1, t_2, \dots, t_{i-1}. \quad (16)$$

And a weak efficient solution will be gotten by this relaxed lexicographic algorithm.

Then the pseudocode of the algorithm will be shown:

Algorithm: Relaxed Lexicographic

Input: Flight constraint; Sensitive locations;
Status of ants; Relevant parameter;

Output: the best trajectory \vec{j}

```

1 Initialization;
2 while round<=n do begin;
3     if round~=1 then begin
4         add the influence of locations
         before;
5     end
6     while cycle index<=maximum do begin
7         for i=1,...,step num do begin
8             for j=1,..., the amount
             of ants do begin
9                 ant j chooses
                 the next step as roulette with the probabilities;
10            end
11        end
12        if round~=1 then begin
13            compute the
            punishment functions;
14        end
15        compute the evaluation
            functions of the current location;
16        sort the evaluation functions;
17        update the Pheromone;
18        record the best trajectory;
19        if the best trajectory is
            convergent then begin break; end
20    end
    
```

The input of the algorithm includes the flight constraint (the feasible region, the start point and the goal point), the sensitive location (the location coordinates and the priorities), the status of ants (the amount of ants and the initialized pheromone matrix) and relevant parameters.

4. COMPUTER SIMULATION

The ant colony algorithm and the relaxed lexicographic algorithm will be implemented respectively to solve the problem with a simple and hypothesis scenario. As mentioned before, the LMax noise level of the sensitive location will be used as the noise objective function. It is based on the noise-thrust-distance (NTD) table in the INM software, which is very common in forecasting and computing the aircraft noise around airport. For the Boeing 737-300 is one of the most common aircraft in the civil aviation of China, it will be used as the aircraft in the experiments.

The flight constraint in the hypotheses scenario will be shown in the Figure.2 (a), and there are five sensitive locations with the coordinates in the Table.1. The start point S is (0km, 0km) and the initial height is 0.3km, while the goal point G is (10km, 20km). The climb gradient is set as 3.3%.

Table.1: Noise Sensitive Locations

Sensitive Location	East Coordinate	North Coordinate
Location 1	2km	1.5km
Location 2	6km	2.5km
Location 3	4km	5km
Location 4	7km	8km
Location 5	6km	13km

The amount of ants is set as 100. By repeated experiments, the optimization coefficients $\omega_i=5/26$ ($i \in [1, n]$) and $\omega_{n+1}=1/26$. Moreover the specific gravity coefficients $\alpha=3, \beta=6$.

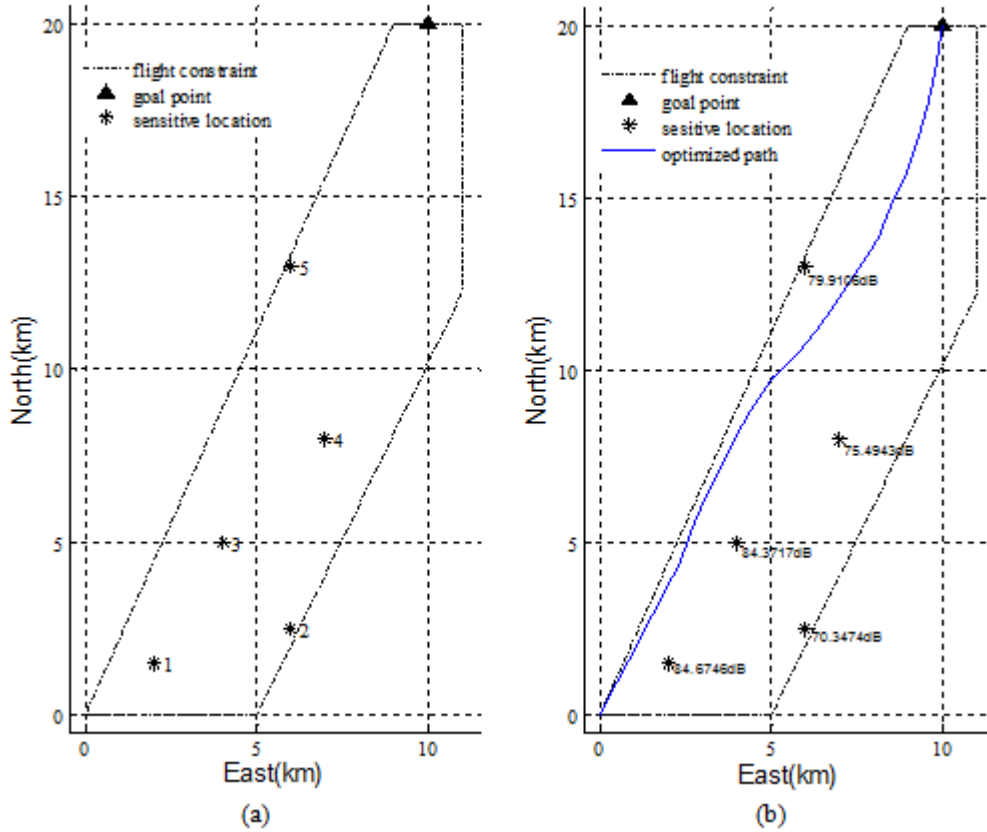


Figure.2: (A) The Flight Constraint In The Hypotheses Scenario; (B) The Result Optimized By Using The Ant Colony Algorithm

For the positional relationship between the path and the sensitive locations can be observed directly in the 2D plane, it is shown first. The Figure.2 (b)

displays the result optimized by the ant colony algorithm. The path really made the noise impact at

every location weaker and swung not too much. The 3D trajectory can be seen in the Figure.3.

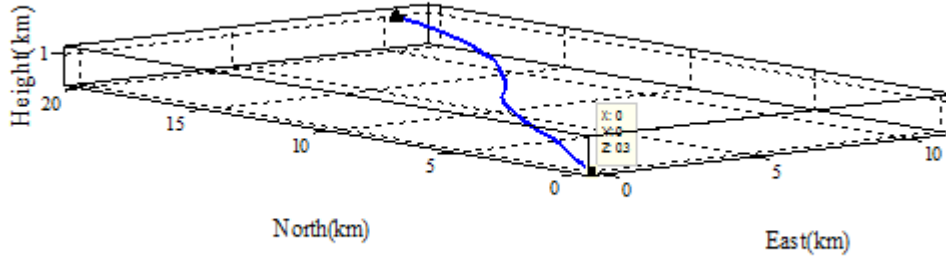


Figure.3: The 3D Trajectory By The Ant Colony Algorithm

For the relaxed lexicographic algorithm, the priorities are assumed as the sequence [2, 5, 1, 4, 3]. The Figure.4 (a) displays the optimization process. It can be seen that the path on each round is the optimization for the current objective and at the

same time with the influence of the objective before. The noise impact cannot be certain to be better than the result by the ant colony algorithm. But it has more possibilities to find the best solution.

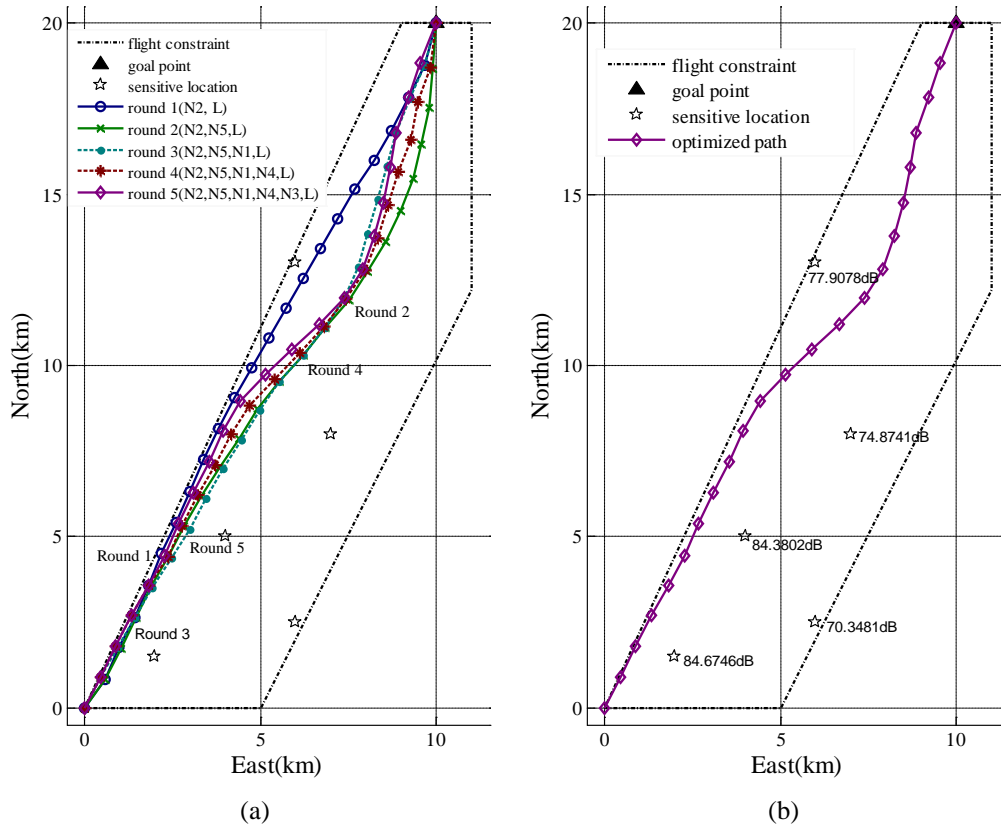


Figure.4: (A) The Optimization Process By Using The Relaxed Lexicographic Algorithm; (B) The Final Result And The Noise Impacts Of The Optimization

5. CONCLUSION

There are two approaches, the scalarization approach and the hierarchical sequence approach, presented in this paper, which are aimed at optimizing the departure trajectory for noise abatement of sensitive locations. First, the ant colony algorithm is used to solve the scalarization problem. Then according to the drawback of the scalarization approach, the relaxed lexicographic algorithm is proposed, and hoped to find the best trajectory under the more comprehensive circumstance. However, the optimization of the departure trajectory is a complex problem, and many parameters are thought as constant values just now. On the other hand, the algorithms can be optimized, such as determining the priority sequence and considering the background noise. They all could be the next study.

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