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EXPERIMENTAL CHAOS IN A TWO-DEGREE-OF FREEDOM VIBRATION SYSTEM WITH TIME-DELAY FEEDBACK

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ABSTRACT

In this paper, numerical simulations are carried out to verify the feasibility of the time-delay feedback in controlling the motion from order to chaotic, and then an experiment is implemented based on two-degree-of-freedom vibration system. Results show that time-delay feedback control is effective and simple in chaotifying practical mechanical system. This study is of great importance in applying the chaos-based theory of reducing line spectra to engineering practice.

Keywords: *Experiment, Chaos; Time-Delay Feedback, Mechanical System*

1. INTRODUCTION

Spectral lines, used to characterize the level of noise emitted from the mechanical vibration of underwater vehicles, are regarded as harmful features for acoustic stealth performance. The power spectrum of a chaotic state may present a continuous spectrum and the intensity of line spectrum could be decreased, so an important application of chaos is of improving the concealment capability of underwater vehicles [1, 2].

Current studies have shown that a system containing time delayed component is able to exhibit remarkable complex dynamical behavior. It is well-known that a nonlinear delayed dynamic system of one dimension may exhibit chaotic behavior [3， and there maybe exists Hopf bifurcation in dynamic system with time delays [4, 5]. The effects of linear and nonlinear feedbacks on the dynamical behaviors were studied by Wulf and Ford [6] and Reddy et al [7], respectively, which indicated that there exist several different types of bifurcations and chaos in the system under time-delay feedback control. Recently, the effect of time delay on a system containing external force was exactly investigated [8]. The recent development was presented by Jian Xu et al., which showed that time delay may be used as a simple but efficient switch to control motions of a system, such as from order to complex motions [9-11]. Chaos was presented in Chen circuit system by time delay control [12]. However, to our knowledge, no results have been published regarding in an actual mechanical system.

The main objective of this paper is to investigate the effect of time delay on the dynamical behaviors of a two-degree-of-freedom vibration system. Furthermore, a test on the actual mechanical system will be conducted and the effect of time delay on chaotification will be discussed.

2. THEORETICAL ANALYSIS

Consider a two-degree-of-freedom vibration system with time-delay feedback control, as shown in Figure 1. As shown in Figure1, the mass M2 is exciter, which is supported by a linear damper and an actuator Fc utilized to implement time-delay feedback control. The mass M1, the linear damper and the M3 can be seen as a flexible base.

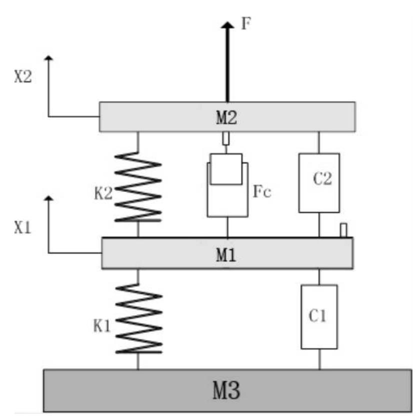

Figure1 Two-Degree-Of-Freedom Vibration Isolation Model

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2.1 Theoretical Modeling

The governing equations of the two-degree-of-freedom vibration system can be given by

$$\begin{cases} M_1\ddot{X}_1+C_1(\dot{X}_1-\dot{X}_2)+K_1(X_1-X_2)=F-F_c \\ M_2\ddot{X}_2+C_2\dot{X}_2+K_2X_2-C_1(\dot{X}_1-\dot{X}_2)-K_1(X_1-X_2)=F_c \end{cases} \quad (1)$$

The following parameters are introduced

$$w_{10}=\sqrt{\frac{K_1}{M_1}}\ ,\quad w_{20}=\sqrt{\frac{K_2}{M_2}}\ ,\quad \zeta_1=\frac{C_1}{2M_1w_{10}}\ ,$$

$$\zeta_2=\frac{C_2}{2M_2w_{20}},\ \beta=\frac{M_1}{M_2},\ F=F_0\cos(wt) \quad (2)$$

And according to Wang and Chen's method [11], the time-delay feedback controller F_c can be designed

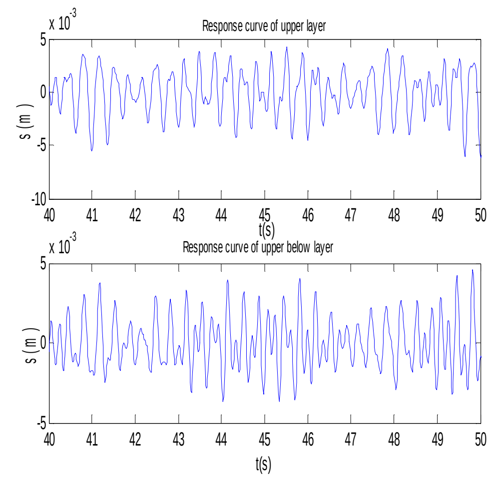
$$F_c=f\sin\{\sigma[-X_1(t-\tau)+\frac{(4\zeta^2{}_2-1)}{\beta}X_2(t-\tau)$$
$$+2\zeta_2\frac{\dot{X}_1(t-\tau)}{w_{20}}+2\frac{\zeta_2}{\beta}\frac{\dot{X}_2(t-\tau)}{w_{20}}]\} \quad (3)$$

2.2 Numerical Examples

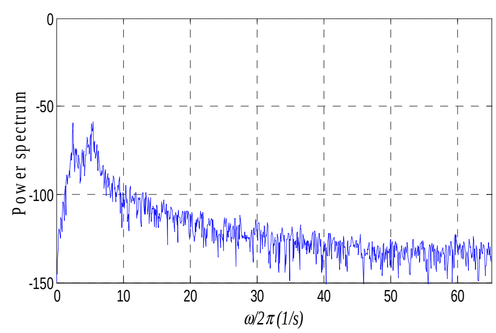
It is well known that a chaotic oscillation appears to be a broad-band characteristic in frequency domain [1, 2]. As a result, line spectrum can be converted into broad-band spectrum by driving a dynamic system chaotic. Zhou at al [14] has proposed a chaotification method based on spectrum optimization and time-delay control. In Ref. [14], a spectrum performance index was constructed as the objective, and the controller parameter τ was chosen as the design variable. The optimal control parameter is found to minimize the performance index by genetic algorithm [15], and then the chaotic oscillation was present in the system under time-delay feedback control with the optimal parameter.

The two-degree-of-freedom vibration isolation system is driven by a harmonic excitation, and the system parameters are listed in Table.1. The optimal time delay of the controller is obtained by using the chaotification method proposed by Zhou at al [14].

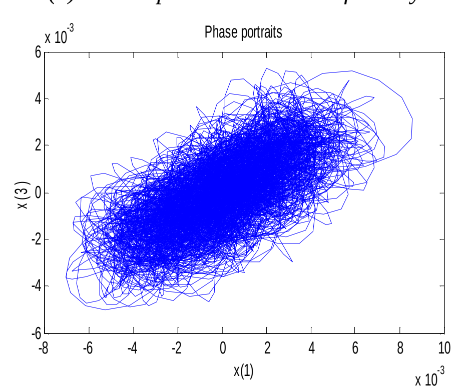
When the excited force at low frequency $w=6*\pi$, the optimal time delay $\tau=2.24$. And the response curve and the phase portrait as well as Welch power spectral density are plotted in Figure2. When the system is not controlled by the active force, it presents periodic oscillations, and the power spectrum is discrete, there is only one circle in the phase portrait. As shown in Figure3. However, when the system is controlled, the power spectrum becomes continuous (Figure 2(b)) and the trajectory of orbits fills up the section of phase space (Figure2(c)). Especially the continuous spectrum in the frequency domain is greater than the excitation frequency in Fig2 (b). More importantly, the line spectrum (Figure 3(b)) has been changed into the continuous one(Fig2(b)). According to the analysis above, the motion is more likely to be chaotic.



(a) Response Curve Of Upper/Below Layer



(b)Power Spectral Densities Of The System



(c)*Phase Portraits*

Figure2 Response Curse And Power Spectral Densities As Well As Phase Portraits Of The System With Time Delay Feedback Controller

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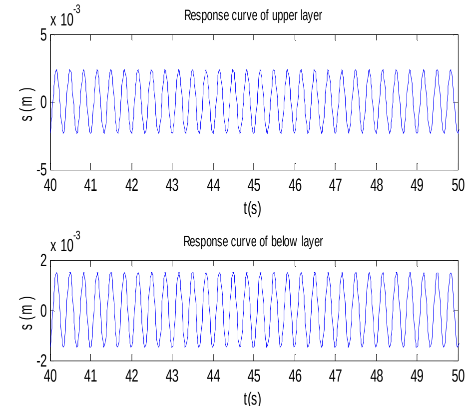
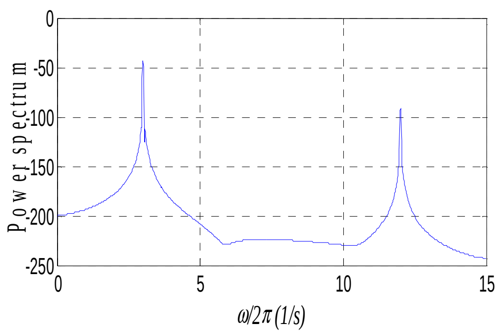
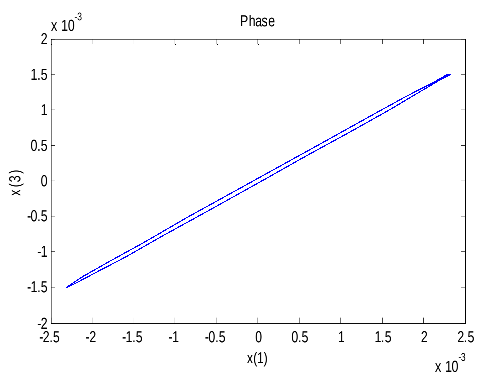


(a)Response Curse Of Upper/Below Layer



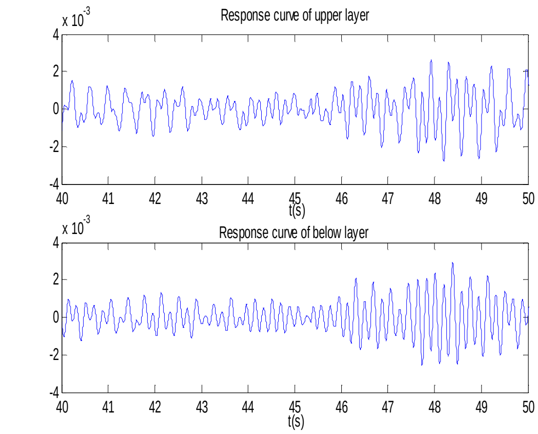
(b)Power Spectral Densities Of The System



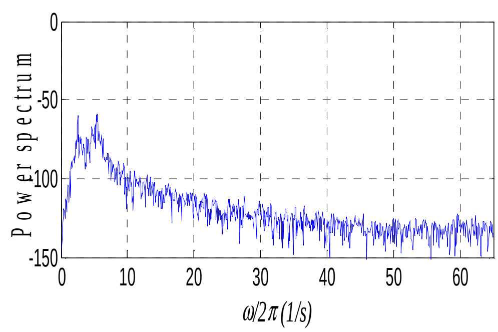
(c)Phase Portraits
Figure3 Response Curve And Power Spectral Densities As Well As Phase Portraits Of The System Without Time Delay Feedback Controller

When the excited force at frequency $w=26*\pi$, the optimal time delay is $\tau = 3.42$. The motions of the controlled system are compared with those of the system without control, as demonstrated in Figure4 and Figure5. Like the above cases, without control, the state of the system is periodic (Figure5), and the motion of the system with control is obviously chaotic, as observed by power spectrum (Figure 4(b)) and phase portrait (Figure4(c)). It is also seen from Figure 4(b) that there is a region of continuous spectrum below the excitation frequency, and a number of sub-harmonics are excited.

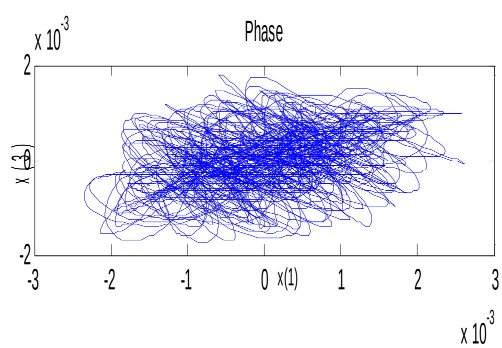
The same results are obtained for many cases as above through numerical simulations. For the sake of brevity, other numerical examples are not mentioned. Consequently, it can be concluded that time delay feedback is efficient in anti-control of chaos. In order to prove its validity, an experimental will be conducted as shown in section 3.



(A)Response Curse Of Upper/Below Layer



(B) Power Spectral Densities Of The System



(C)Phase Portraits
Figure4 Response Curve And Power Spectral Densities As Well As Phase Portraits Of The System With Time Delay Feedback Controller

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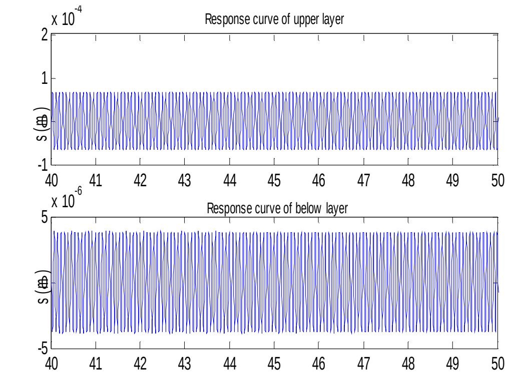
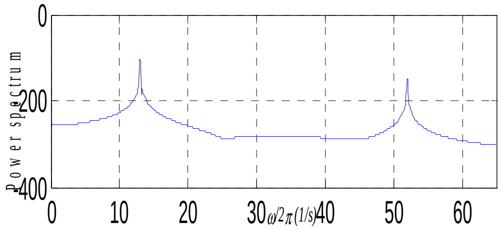
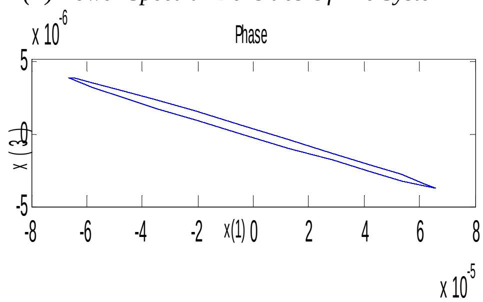


(A)Response Curse Of Upper/Below Layer



(B) Power Spectral Densities Of The System



(C)Phase Portraits

Figure5 Response Curve And Power Spectral Densities As Well As Phase Portraits Of The System Without Time Delay Feedback Controller

3. EXPERIMENTAL RESULTS

In order to verify the efficiency of the time-delay feedback in anti-control of chaos, an experiment of a two-degree-of-freedom vibration isolation system was conducted. In despite of the validity of this method is testified in circuit system, to our knowledge, no results have been published regarding an actual mechanical system. The experimental setup is shown in Figure6.

As shown in Figure6, the external force is provided by a hanging exciter, while the feedback control is carried out by the exciter between two mass. The displacement and acceleration sensors are used to detect the motion of layers, and the signals are received by acquisition system. According to the controller principle Eq.(3), the control signals are exported to the below exciter by control system based on FPGA.

Figure6 Experimental Setup And Data Acquisition System

When the excited frequency at $w = 6 * \pi$ (Figure 7(a)), the response curve of the below layer is shown in Figure 7(b). When the time delay controller is absent, the corresponding curve is shown Figure 8 (b). Observed from these figures, we find that there are more harmonics in Figure 7(b) than in Figure 8 (b). There are only several circles in phase space (Figure8(c)); however, when the time delay controller becomes active, the phase space is full of the trajectory of orbits. From those observations, the motion is more likely to be chaotic. To further diagnose whether the system is chaotic or not, the largest Lyapunov exponent (LE) is calculated by a method proposed by Andrzej et al [16-17], and the largest LE is about 0.8512. The positive exponent implies chaotic oscillation [18].

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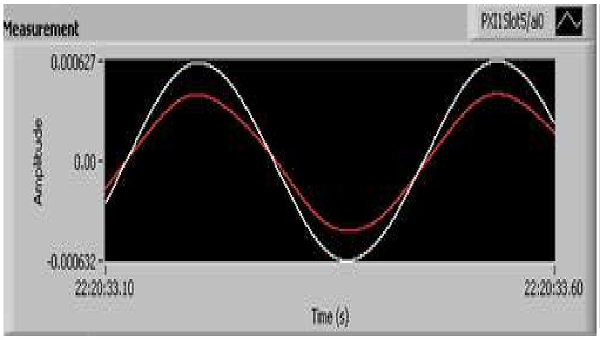
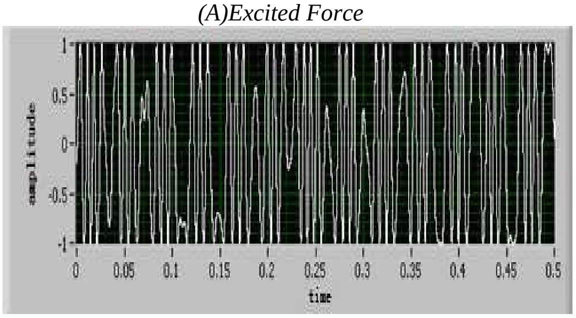
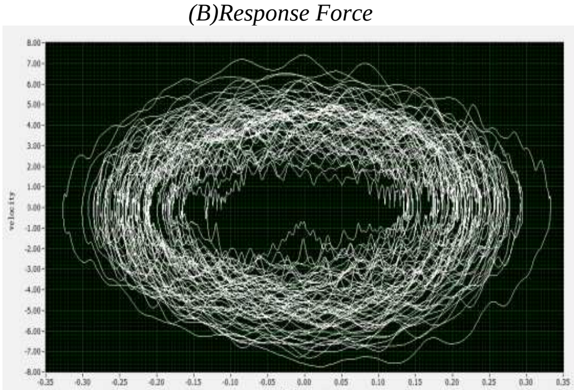


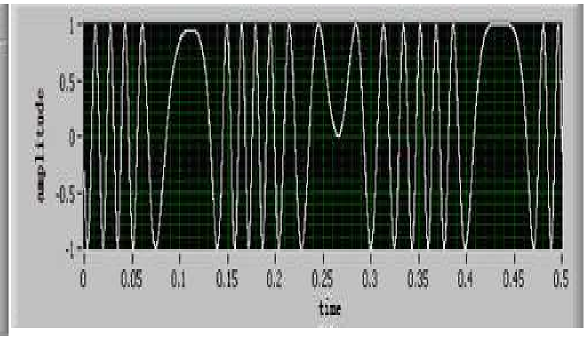
(A)Excited Force



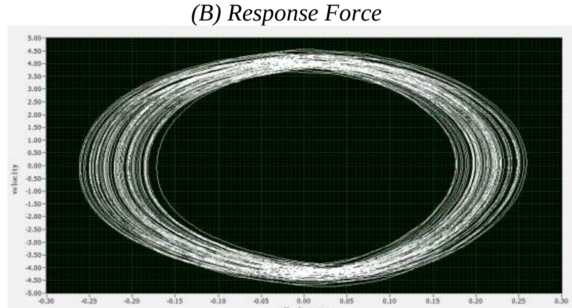
(B)Response Force



(C)Phase Portraits
*Figure7 Excited Force, Response Force As Well As Phase Portraits Of The System With Time Delay Controller Driven By The Excitation Frequency At 6*Pi.*

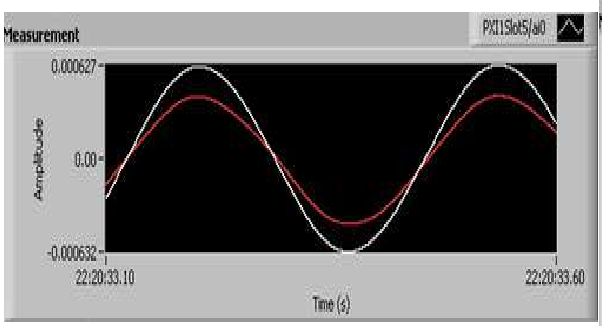


(B) Response Force

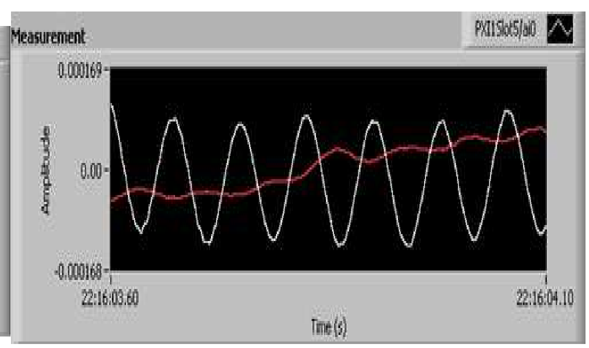


(C) Phase Portraits
*Figure8 Excited Force, Response Force As Well As Phase Portraits Of The System Without Time Delay Controller Driven By The Excitation Frequency At 6*Pi.*

When the excited frequency at $w = 26*\pi$ (Figure 9(a)), the motion of the controlled system is obviously chaotic, which is detected by response curve, phase portrait (Fig9.b， Fig9.c), and the largest LE (1.7643). It is seen from Fig10.b that a number of sub-harmonics are excited when there is no controller, and a number of circles are demonstrated in the phase space (Fig10.c), as well as the calculation of the largest LE is about 0.1381, the behavior can be diagnosed to be weak chaotic. Obviously, the time-delay feedback method makes the extent of previous chaotic state more deeply.



(A)Excited Force



(A)Excited Force

Journal of Theoretical and Applied Information Technology
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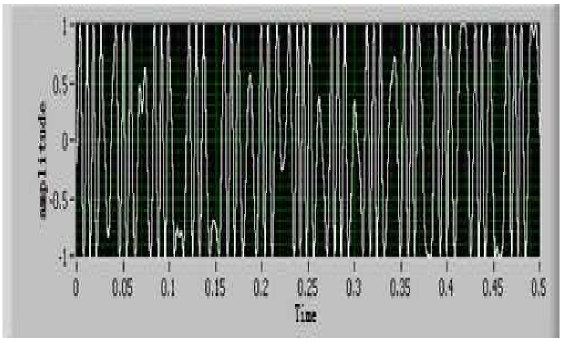
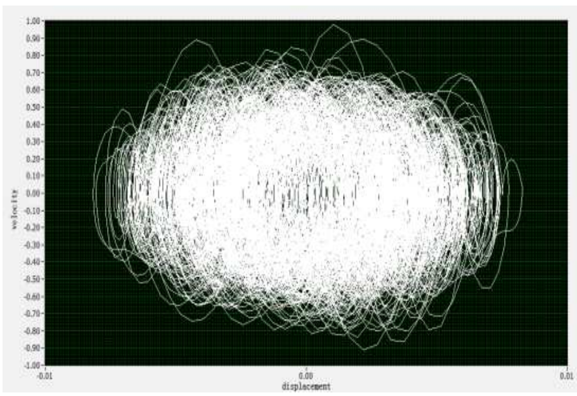
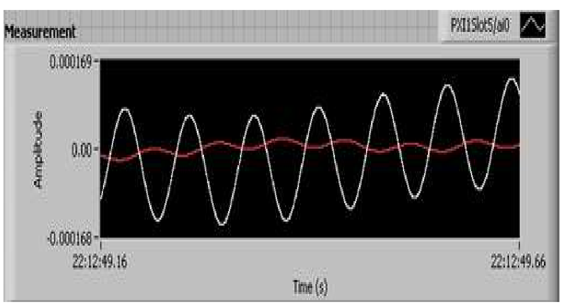


(B) Response Force

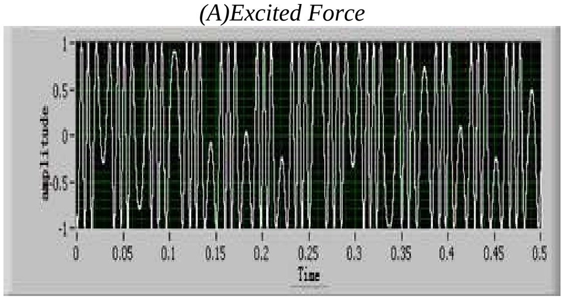


(C) Phase Portraits

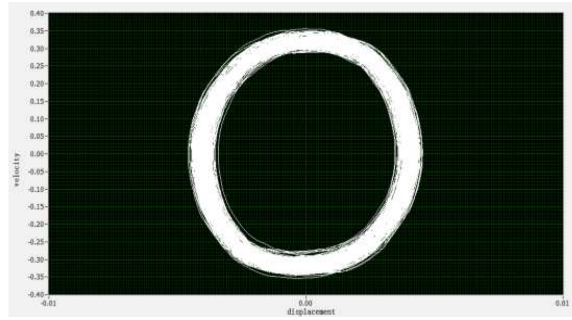
*Figure9 Excited Force, Response Force As Well As Phase Portraits Of The System With Time Delay Controller Driven By The Excitation Frequency At 26*Pi.*



(A)Excited Force



(B) Response Force



(C) Phase Portraits

*Figure10 Excited Force, Response Force As Well As Phase Portraits Of The System Without Time Delay Controller Driven By The Excitation Frequency At 26*Pi.*

4. CONCLUSIONS

In this work, a nonlinear time-delay feedback controller is applied to make a two-degree-of-freedom vibration system chaotic. Numerical simulations are carried out to verify the efficiency of this method, which shows that the motion of the two-degree-of-freedom vibration system excited by harmonic force can be driven to be chaotic by feedback control with proper time delay. Furthermore, an experiment was conducted on the two-degree-of-freedom vibration system with the FPGA. Results show that the time-delay feedback controller was an efficiency switch to control the motion of a system from order to chaotic, and it is also can make the extent of previous chaotic state more deeply. Noted that this method is easily conducted to chaotify a system with known parameters; however, parameters of the system are not easily derived in practical engineering. The combination of adaptive control and time delay feedback is our future work.

5. ACKNOWLEDGEMENT

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6. TRAINING OF ANN PARAMETERS

Table.1

Mass m_1 , m_2	123kg，190kg
Stiffness k_1 , k_2	6.4×10^4N/m，1.0×10^5N/m
Damping factor ς_1 , ς_2	0.009 , 0.016

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