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POSITION ANALYSIS OF A 3-SPR PARALLEL MECHANISM

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ABSTRACT

This paper deals with closed form solutions for the inverse and forward position analyses of a 3-SPR parallel mechanism, which can be used to form the main body of a 5-DOF hybrid robot manipulator. In the inverse position analysis, a fourth order polynomial equation has been formulated, leading to four set of solutions of limb lengths for a given position of the reference point on the platform. In the forward position analysis, a sixteenth order polynomial equation has been derived, leading to four sets of mirror poses given a set of limb lengths. Consequently, the complete sets of solutions for the inverse and forward position analyses of the mechanism have been achieved. An example is given to illustrate the effectiveness of this approach.

Keywords: *3-SPR Parallel Mechanism, Forward and Inverse Position Analyses*

1. INTRODUCTION

Inverse and forward position analyses are fundamentally important in the development of parallel mechanisms for workspace analysis, dimensional synthesis and control purposes, etc. Approaches for solving these problems can be classified into two categories, i.e., analytical method and numerical algorithm. The analytical approach is focused on finding the complete set of solutions using the procedure that can usually be implemented by two steps: (1) formulate the kinematic constraints into a set of nonlinear equations, and (2) generate a polynomial equation having a single unknown by means of certain elimination methods. As a result, all configurations can be found by solving the end polynomial equation [1-12]. The complexity of the polynomial approach depends upon the geometry of the object and proper choice of elimination techniques. Numerical approach can be used to find one solution using iterative algorithms or optimization techniques [13-15].

It is well recognized that for most parallel mechanisms, inverse position analysis is much easier than the forward one. For the lower mobility parallel mechanisms having coupled degrees of freedom (DOF) in terms of both translations and rotations, if the dependent coordinates can explicitly be expressed in terms of the independent ones, the inverse position analysis is still simple, 3-PRS and 3-RPS parallel mechanisms for instance [16,17]. However, it would be not the case if the dependent coordinates can not be explicitly expressed in terms of the independent ones, leading to both inverse and forward position analysis problem being complicated. The 3-SPR parallel mechanism having one translational and two rotational movement capabilities would be a typical example.

The above mentioned problem has been notified by only a few researchers. By reversing the base and moving platforms, Lu et al [18] dealt with the inverse position analysis problem of a 3-SPR parallel mechanism, which involves 15 unknowns in terms of 9 direction cosines of the orientation matrix and 6 translational coordinates of two reference points on the base and moving platforms, leading to a nonlinear equation containing a single unknown derived by the elimination method. Unfortunately, the mathematic model for the forward position analysis was failed to give all the solutions of the problem. Lukanin [19] dealt with the closed-form solution for the inverse position analysis problem of the same mechanism. Although eight real solutions were obtained from a fourth order polynomial equation, four of them did not satisfy the constraint equations. In addition, the numerical algorithm was merely presented for the forward kinematics.

This paper revisits the inverse and forward position analyses of a 3-SPR parallel mechanism with a goal to find the complete set of closed-form solutions by using elimination method.

2. SYSTEM DESCRIPTIOT

As shown in Fig.1, the 3-SPR parallel mechanism under consideration consists of a base and a moving platform connected by three identical SPR limbs. Here, S and R represent the spherical and revolute joints, and the underlined P denotes

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the actuated prismatic joint. Place the reference frame *B-xyz* attached to the base and the moving frame *A-uvw* attached to the moving platform with *B* and *A* being the origins located at the centers of equilateral triangles $\Delta A_1A_2A_3$ and $\Delta B_1B_2B_3$ with the *x* and *u* axes being normal to B_1B_2 and A_1A_2, and the *z* and *w* axes being normal to $\Delta B_1B_2B_3$ and $\Delta A_1A_2A_3$. Here, A_i (i=1,2,3) are the centers of the spherical joints and B_i (i=1,2,3) are the intersection of the axes of the revolute joints and actuated prismatic joints, respectively.

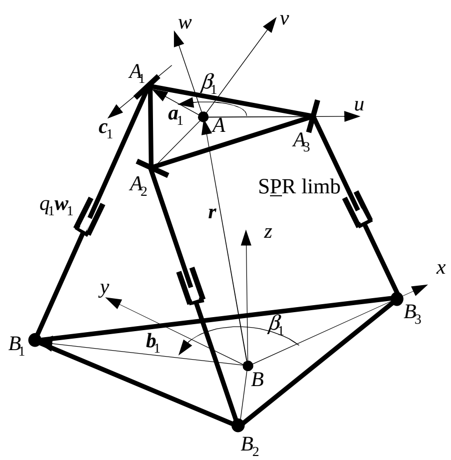

Figure.1 Diagram Of A 3-SPR Parallel Mechanism

The orientation matrix of the *A-uvw* with respect to the *B-xyz* can be formulated by three Euler angles ψ, θ and ϕ satisfying *z-x-z* conventions:

$$\boldsymbol{R}=\begin{bmatrix} c\psi c\phi - s\psi c\theta s\phi & -c\psi s\phi - s\psi c\theta c\phi & s\psi s\theta \\ s\psi c\phi + c\psi c\theta s\phi & -s\psi s\phi + c\psi c\theta c\phi & -c\psi s\theta \\ s\theta s\phi & s\theta c\phi & c\theta \end{bmatrix} = \begin{bmatrix} u_x & v_x & w_x \\ u_y & v_y & w_y \\ u_z & v_z & w_z \end{bmatrix} = [\boldsymbol{u} \quad \boldsymbol{v} \quad \boldsymbol{w}] \tag{1}$$

where $\boldsymbol{u}$, $\boldsymbol{v}$ and $\boldsymbol{w}$ are the unit vectors of the frame *A-uvw* with respect to the frame *B-xyz*, and ψ, θ and ϕ are three Euler angles of precession, nutation and body rotation, respectively; "s" and "c" here denote sin and cos functions.

3. INVERSE POSITION ANALYSIS

Inverse position analysis of the 3-SPR parallel mechanism involves the determination of the limb lengths given the pose of the platform. The problem can be solved by two kernel steps: 1) formulation of the constraints that relate three Euler angles with the coordinates of the reference point *A*, and 2) generation of an end polynomial equation in terms of procession angle ψ as the single unknown by elimination method.

In the *B-xyz*, the position vector $\boldsymbol{r}=(x\ y\ z)^{\mathrm{T}}$ of *A* can be expressed as

$$\boldsymbol{r}=\boldsymbol{b}_i+q_i\boldsymbol{w}_i-\boldsymbol{a}_i,\ i=1,2,3 \tag{2}$$

where q_i and $\boldsymbol{w}_i$ are the length and unit vector of limb i, $\boldsymbol{a}_{i0}=a(\cos\beta_i\ \ \sin\beta_i\ \ 0)^{\mathrm{T}}$ and $\boldsymbol{b}_i=b(\cos\beta_i\ \ \sin\beta_i\ \ 0)^{\mathrm{T}}$ are the position vectors of A_i and B_i measured in *A-uvw* and *B-xyz* with *a* and *b* being the radii of the platform and the base, and $\beta_i=2\pi i/3$ the position angles to the R joints. Note that the constraint imposed by the R joint restricts both $\boldsymbol{w}_i$ and $\boldsymbol{a}_i$ to be normal to the unit vector $\boldsymbol{c}_i$ of the R joint axis. Thus, taking the dot product with $\boldsymbol{c}_i$ on both sides of Eq.(2), leads to

$$(\boldsymbol{r}-\boldsymbol{b}_i)^{\mathrm{T}}\boldsymbol{c}_i=0,\ i=1,2,3 \tag{3}$$

where $\boldsymbol{c}_i=\boldsymbol{R}\boldsymbol{c}_{i0}$, $\boldsymbol{c}_{i0}=(-\sin\beta_i\ \ \cos\beta_i\ \ 0)^{\mathrm{T}}$.

Substituting Eq.(1) into (3) and implementing subtraction and addition, yields

$$\boldsymbol{u}^{\mathrm{T}}\boldsymbol{r}=\frac{b}{2}(v_y-u_x) \tag{4}$$

$$\boldsymbol{v}^{\mathrm{T}}\boldsymbol{r}=\frac{b}{2}(3u_y-v_x) \tag{5}$$

$$\boldsymbol{v}^{\mathrm{T}}\boldsymbol{r}=bv_x \tag{6}$$

Equating Eqs.(4) and (6) gives to $u_y=v_x$. This leads to $\phi=-\psi$ as $\theta<\pi$. Then, Eqs.(4) and (5) can be rewritten as

$$\boldsymbol{u}^{\mathrm{T}}\boldsymbol{r}=-\frac{1}{2}b\cos 2\psi(1-\cos\theta) \tag{7}$$

$$\boldsymbol{v}^{\mathrm{T}}\boldsymbol{r}=\frac{1}{2}b\sin 2\psi(1-\cos\theta) \tag{8}$$

At this stage, multiplying Eq.(7) by $\cos\psi$ and (8) by $\sin\psi$, and implementing addition; while multiplying Eq.(7) by $\sin\psi$ and (8) by $\cos\psi$, and implementing subtraction, yields

$$x\cos\psi+y\sin\psi=-\frac{1}{2}b\cos 3\psi(1-\cos\theta) \tag{9}$$

$$-x\sin\psi\cos\theta+y\cos\psi\cos\theta+z\sin\theta=\frac{1}{2}b\sin 3\psi(1-\cos\theta) \tag{10}$$

Thus, substituting $t_\theta=\tan(\theta/2)$ into Eqs.(9) and (10), gives

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$$a_1 t_\theta^2 + a_2 = 0 \quad (11)$$

$$b_1 t_\theta^2 + b_2 t_\theta + b_3 = 0 \quad (12)$$

where, $a_1 = x\cos\psi + y\sin\psi + b\cos 3\psi$
$a_2 = x\cos\psi + y\sin\psi$
$b_1 = -x\sin\psi + y\cos\psi + b\sin 3\psi$
$b_2 = -2z$, $b_3 = x\sin\psi - y\cos\psi$.

Multiplying t_θ on both sides of Eqs.(11) and (12) results in two complementary equations which, together with Eqs. (10) and (11), can be written in a matrix form

$$\boldsymbol{Kt} = \mathbf{0} \quad (13)$$

where $\boldsymbol{K} = \begin{bmatrix} 0 & a_1 & 0 & a_2 \\ a_1 & 0 & a_2 & 0 \\ 0 & b_1 & b_2 & b_3 \\ b_1 & b_2 & b_3 & 0 \end{bmatrix}$, $\boldsymbol{t} = \begin{pmatrix} t_\theta^3 \\ t_\theta^2 \\ t_\theta \\ 1 \end{pmatrix}$.

The necessary condition for Eq.(13) to have nontrivial solutions yields the following triangular equation

$$\begin{aligned} &g_1\cos^2 2\psi + g_2\sin^2 2\psi + g_3\cos 2\psi\sin 2\psi + \\ &g_4\cos 2\psi + g_5\sin 2\psi + g_6 = 0 \end{aligned} \quad (14)$$

where $g_1 = 4bxz^2 + (2xy + by)^2$
$g_2 = (x^2 - y^2 - bx)^2$
$g_3 = 4byz^2 - 2(x^2 - y^2 - bx)(2xy + by)$
$g_4 = 2z^2(x^2 - y^2 + bx)$, $g_5 = 2z^2(2xy - by)$
$g_6 = 2z^2(x^2 + y^2 - bx)$.

Substituting $t_\psi = \tan(\psi)$ into Eq.(14) finally results in a fourth order polynomial equation in terms of t_ψ as the single unknown.

$$\sum_{i=0}^{4} h_i t_\psi^i = 0 \quad (15)$$

where $h_4 = g_1 - g_4 + g_6$, $h_3 = 2(-g_3 + g_5)$
$h_2 = 2(-g_1 + g_2 + g_6)$, $h_1 = 2(g_3 + g_5)$
$h_0 = g_1 + g_4 + g_6$.

Solving Eq.(15) for ψ , θ can then be determined by Eqs.(9) and (10). This allows the orientation matrix $\boldsymbol{R}$ to be generated such that

$$q_i = \left|\boldsymbol{r} + \boldsymbol{b}_i - \boldsymbol{R}\boldsymbol{a}_{0i}\right|, \quad i=1,2,3 \quad (16)$$

Note that:

1) If $x=y=0$, then $h_4 = h_3 = h_2 = h_1 = h_0 = 0$, leading to $\psi = \theta = 0$. In this case, $q_1 = q_2 = q_3 = \sqrt{z^2 + (b-a)^2}$.

2) If $y=0$, then $h_4=h_3=0$, Eq.(15) degenerates to $h_2 t_\psi^2 + h_1 t_\psi + h_0 = 0$, meaning that t_ψ has at most two solutions.

4. FORWARD POSITION ANALYSIS

The forward position analysis of the mechanism is concerned with the determination of the moving platform pose given the limb lengths, q_i (i=1,2,3). The skills to solve this problem can be divided into two kernel steps: 1) expression of x and y in terms of ψ , θ and z as $\phi = -\psi$; and 2) generation of an end polynomial equation in terms of procession angle ψ as the single unknown by the elimination method.

Firstly, taking Euclidean norm on both sides of Eq.(2), and implementing necessary addition and subtraction, yields

$$x = \frac{1}{6b}\left(q_1^2 + q_2^2 - 2q_3^2 - 6ab(1-\cos\theta)\cos 2\psi\right) \quad (17)$$

$$y = \frac{1}{6b}\left(6ab(1-\cos\theta)\sin 2\psi - \sqrt{3}\left(q_1^2 - q_2^2\right)\right) \quad (18)$$

$$\begin{aligned} &x^2 + y^2 + z^2 = \frac{1}{3}\left(q_1^2 + q_2^2 + q_3^2\right) - a^2 - \\ &b^2 + ab(1+\cos\theta) \end{aligned} \quad (19)$$

Note that

$$\boldsymbol{w}^{\mathrm{T}}\boldsymbol{r} = x\sin\psi\sin\theta - y\cos\psi\sin\theta + z\cos\theta \quad (20)$$

In comparison with Eq.(10), Eq.(20) can also be rewritten as

$$\boldsymbol{w}^{\mathrm{T}}\boldsymbol{r} = \frac{1}{\cos\theta}\left(z - \frac{1}{2}b\sin 3\psi(1-\cos\theta)\sin\theta\right) \quad (21)$$

Substituting Eqs.(7),(8) and (21) into the following identity

$$\boldsymbol{r} = \left(\boldsymbol{u}^{\mathrm{T}}\boldsymbol{r}\right)\boldsymbol{u} + \left(\boldsymbol{v}^{\mathrm{T}}\boldsymbol{r}\right)\boldsymbol{v} + \left(\boldsymbol{w}^{\mathrm{T}}\boldsymbol{r}\right)\boldsymbol{w} \quad (22)$$

leads to another two expressions of x and y in terms of ψ , θ and z

$$\begin{aligned} &x = z\sin\psi\tan\theta + \frac{1}{2}b(1-\cos\theta)\cdot \\ &(-\cos\psi\cos 3\psi - \sin\psi\sin 3\psi\sec\theta) \end{aligned} \quad (23)$$

$$\begin{aligned} &y = z\cos\psi\tan\theta - \frac{1}{2}b(1-\cos\theta)\cdot \\ &(-\sin\psi\cos 3\psi + \cos\psi\sin 3\psi\sec\theta) \end{aligned} \quad (24)$$

Secondly, equating Eq.(24) with (18) on the one hand, and substituting Eq.(17) and (18) into (19) on the other, gives

$$c_1 z + c_2 = 0 \quad (25)$$

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$$z^2 + d_1 = 0 \quad (26)$$

Then, substituting Eq.(25) into (26), yields

$$c_1^2 + c_2^2 d_1 = 0 \quad (27)$$

where $c_1 = \cos\psi \sin\theta$, $c_2 = f_1 \cos^2\theta + f_2 \cos\theta + f_3$

$$d_1 = f_4 \cos^2\theta + f_5 \cos\theta + f_6$$

$$f_1 = (1/4b - a)\sin 2\psi - 1/2 b \sin 2\psi \cos 2\psi$$

$$f_2 = a \sin 2\psi + b \sin 2\psi \cos 2\psi - \sqrt{3}\left(q_1^2 - q_2^2\right)/6b$$

$$f_3 = -1/4 b \sin 2\psi - 1/2 b \sin 2\psi \cos 2\psi, \; f_4 = a^2$$

$$f_5 = a\left(\left(q_1^2 + q_2^2 - 2q_3^2\right)/3b\right)\cos 2\psi + a\left(\sqrt{3}\left(q_1^2 - q_2^2\right)/3b\right)\sin 2\psi - ab - 2a^2$$

$$f_6 = b^2 - 2ab + \left(\left(q_1^2 + q_2^2 - 2q_3^2\right)/6b\right)^2 + \left(\sqrt{3}\left(q_1^2 - q_2^2\right)/6b\right)^2 - \left(q_1^2 + q_2^2 + q_3^2\right)/3 - f_5 .$$

Expansion of Eq.(27) leads to

$$\sum_{i=0}^{4} k_i \cos^i\theta = 0 \quad (28)$$

Thirdly, equating Eqs.(23)-(24) with Eqs.(17)-(18), respectively to eliminate z, leads to

$$\left(q_1^2 + q_2^2 - 2q_3^2\right)\cos\psi - \sqrt{3}\left(q_1^2 - q_2^2\right)\sin\psi = 3b(2a - b)(1 - \cos\theta)\cos 3\psi \quad (29)$$

Here, we assume $2a \neq b$, Eq.(29) can be rewritten as

1) In case of $q_1 = q_2$

$$\cos\theta = 1 - \frac{4\left(q_1^2 - q_3^2\right)}{3b(2a - b)(3\cos 2\psi - 5)} \quad (30a)$$

2) In case of $q_1 = q_3$

$$\cos\theta = 1 - \frac{2\left(q_2^2 - q_1^2\right)}{3b(2a - b)} \cdot \frac{\left(2\sin 2\psi - 2\sqrt{3}\cos 2\psi + 2\sqrt{3}\right)}{\left(3\sin 2\psi \cos 2\psi - 5\sin 2\psi\right)} \quad (30b)$$

3) In case of $q_2 = q_3$

$$\cos\theta = 1 - \frac{2\left(q_1^2 - q_2^2\right)}{3b(2a - b)} \cdot \frac{\left(2\sin 2\psi + 2\sqrt{3}\cos 2\psi - 2\sqrt{3}\right)}{\left(3\sin 2\psi \cos 2\psi - 5\sin 2\psi\right)} \quad (30c)$$

4) In case of $q_1 \neq q_2 \neq q_3$

$$\cos\theta = \frac{2\cos^2 2\psi + s_1 \cos 2\psi + s_2 \sin 2\psi + s_3}{2\cos^2 2\psi + \cos 2\psi - 1} \quad (30d)$$

where $s_1 = 1 - \frac{q_1^2 + q_2^2 - 2q_3^2}{3b(2a - b)}$, $s_2 = \frac{\sqrt{3}\left(q_1^2 - q_2^2\right)}{3b(2a - b)}$

$$s_3 = -1 - \frac{q_1^2 + q_2^2 - 2q_3^2}{3b(2a - b)} .$$

Lastly, substituting the adequate equation in Eq.(30) into (28) and letting $t_\psi = \tan(\psi)$ finally results in a sixteenth polynomial equation with t_ψ being the unknown

$$\sum_{i=0}^{16} p_i t_\psi^i = 0 \quad (31)$$

where, p_i (i=0,1, ⋯ 16) are functions of the dimensions of the mechanism and the the limb lengths.

Given a set of limb lengths, sixteen solutions of ψ can be determined. For each real solution of ψ, we can solve for θ by using Eq.(30). Thus, the orientation matrix $\boldsymbol{R}$ can be generated because of $\phi = -\psi$, leading to the solution of x, y and z by using Eqs.(17), (18) and (26).

5. EXAMPLE

In this section, the procedure for the inverse and forward position analyses of a 3-S<u>P</u>R parallel mechanism is presented to illustrate effectiveness of the proposed methods. The dimensional parameters and coordinates of the reference point A in the B-xyz frame are assigned in Table 1 for the inverse position analysis. Then, one set of limb lengths is chosen as the joint variables for the forward position analysis.

In the inverse position analysis, a fourth order polynomial equation (see Eq.(15)) is formulated using the data given in Table 1.

Table 1: The Dimensional Parameters And Position Vector Of A

a(mm)	b(mm)	$(x,y,z)^T$ (mm)
300	400	$(200, 100, 900)^T$

$$t_\psi^4 - 7.0928 t_\psi^3 - 15.9381 t_\psi^2 + 7.0928 t_\psi + 10.1856 = 0 \quad (32)$$

With the aid of root search routine in the Matlab, four real solutions of t_ψ can be determined as follows:

$$t_\psi = \{8.7978 \quad -1.8759 \quad 0.8757 \quad -0.7048\}$$

Consequently, ψ, θ and $q_i\,(i = 1,2,3)$ can be obtained as shown in Table 2. Fig.2 depicts the configurations associated with these solutions. It is

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easy to see from Fig.2 that given the coordinates of *A* there is only one set of limb lengths (see Fig.2(b)) that makes the mechanism to be free of mechanical interference.

Table 2: Solutions Of ψ And θ For The Inverse Position Analysis

No.	ψ (rad)	θ (rad)	$(q_1\ q_2\ q_3)$ (mm)
(a)	1.4576	-2.5509	(985.6939,969.2243, 1165.3375)
(b)	-1.0810	-0.2430	(936.5959,1012.9202, 846.9695)
(c)	0.7192	2.8358	(1244.3039, 939.2895, 939.5435)
(d)	0.6139	-2.9325	(1127.61912,1010.642, 1014.5123)

In the forward position analysis, given the set of limb lengths associated with Fig.2(b), i.e. q_1=936.5959 mm, q_2=1012.9202 mm, and q_3=846.9695 mm, a sixteenth order polynomial equation can be formulated

$$\begin{aligned} & t_\psi^{16} + 7.6830 t_\psi^{15} + 14.8714 t_\psi^{14} + 26.8046 t_\psi^{13} + \\ & 41.8795 t_\psi^{12} + 30.4458 t_\psi^{11} + 40.1028 t_\psi^{10} + \\ & 7.5296 t_\psi^{9} + 4.0414 t_\psi^{8} - 7.1520 t_\psi^{7} - \\ & 11.7715 t_\psi^{6} - 2.9020 t_\psi^{5} - 2.4277 t_\psi^{4} + \\ & 0.5882 t_\psi^{3} + 1.2876 t_\psi^{2} + 1.1327 t_\psi - 0.0028 = 0 \end{aligned} \quad (33)$$

Solving Eq.(33) using Matlab function "roots" results in four real and six pairs of conjugate complex solutions as follows

$$t_\psi \in \begin{Bmatrix} \text{-5.6934, 0.0179,-0.5774} \pm \text{i ,-0.097} \pm \text{i , 0.5774} \pm \text{i} \\ \text{-1.8761,-0.1314, 0.0231} \pm \text{i, -0.0231} \pm \text{i, 0.097} \pm \text{i} \end{Bmatrix}$$

Consequently, the nutation angle θ and the coordinates of the reference point *A*corresponding to the four real solutions of ψ can be determined as shown in Table 3. It can be seen from Table 3 and Fig. 3 that for a given ψ, θ has a pair of solutions, one is positive and the other is negative, leading to a pair of mirror image configurations with respect to the base. Note that the mirror images merely make sense in mathematics and they thereby should be regarded as the extraneous solutions.

Table 3: Solutions Of ψ And θ For The Forward Position Analysis

No.	ψ (rad)	θ (rad)	$(x\ y\ z)^{T}$ (mm)
(a)	-1.3969	2.0309	$(602.4655, -40.2937, 570.6485)^{T}$
		-2.0309	$(602.4655, -40.2937, -570.6485)^{T}$
(b)	-1.0811	0.2427	$(200.1141, 100.0270, -900.0057)^{T}$
		-0.2427	$(200.1141, 100.0270, 900.0057)^{T}$
(c)	-0.1307	2.8018	$(-367.8511, -43.2719, -702.4174)^{T}$
		-2.8018	$(-367.8511, -43.2719, 702.4174)^{T}$
(d)	0.0179	2.9117	$(-396.5313, 128.5471, 672.9851)^{T}$
		-2.9117	$(-396.5313, 128.5471, -672.9851)^{T}$

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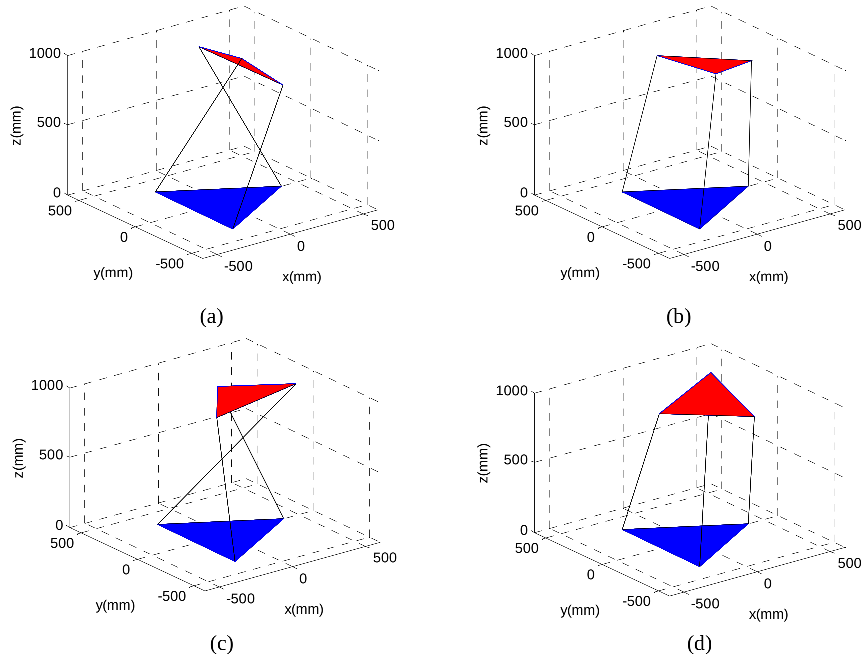

Figure 2 The Results Of Inverse Position Analysis Given:
$x = 200$ mm, $y = 100$ mm, $z = 900$ mm
(a) $\psi = 1.4576,\ \theta = -2.5509$, (b) $\psi = -1.0810,\ \theta = -0.2430$

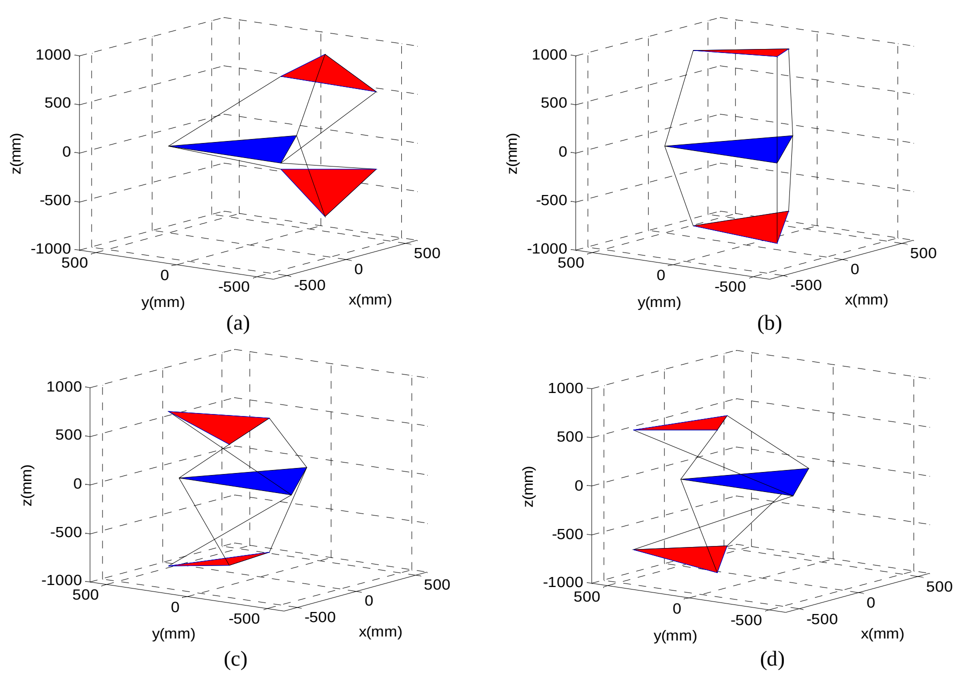

Figure 3 The Results Of Forward Position Analysis Given:
$q_1 = 963.5959$ mm, $q_2 = 1012.9202$ mm, $q_3 = 846.9695$ mm . (a) $\psi = -1.3969,\ \pm\theta = 2.0309$,
(b) $\psi = -1.0811,\ \theta = \pm 0.2427$, (c) $\psi = -0.1307,\ \theta = \pm 2.8018$, (d) $\psi = 0.0179,\ \theta = \pm 2.9117$

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6. CONCLUSIONS

This paper deals with the closed-form solutions for the inverse and the forward position analyses of a 3-SPR parallel mechanism. The conclusions are drawn as follows.

1）For the inverse position analysis, there exist at most four sets of limb lengths given the coordinates of the reference point on the platform, and only one of them makes the mechanism to be free of mechanical interference. If x=y=0, then $q_1 = q_2 = q_3 = \sqrt{z^2 + (b-a)^2}$; and if y=0, there are at most two sets of limb lengths.

2） In the forward position analysis, there exist at most four sets of mirror image poses given a set of limb lengths. If a/b=1/2, ψ can directly be obtained, and θ can then be determined by a second/third/fourth order polynomial equation according to the specific arrangements of q_1, q_2 and q_3.

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