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OPTIMAL APPROXIMATE ALGORITHM TO SOLVE LOCATION-SELECTION OF RECYCLE WATER STATION PROBLEM

[1]XIAOXU LU, [2]JING LI

[1]Department of Mathematics and Physics, Zhengzhou Institute of Aeronautical Industry Management, Zhengzhou 450015, China

[2]Department of Public Teaching, Zhengzhou Railway Vocational & Technical College, Zhengzhou 450052, China

ABSTRACT

The minimum Steiner tree problem has wide application background, such as transportation system, communication network, pipeline design and VISL, etc. It is unfortunately that the computational complexity of the problem is NP-hard. People are common to find some special problems to consider or find the approximate algorithm to solve different problems. Since the complexity of the Steiner tree problem, the almost of papers are relate to the object of small data problem, i.e., the number of involved objects is small. Those conclusions are useful to the theoretical research from which some algorithms are originated. For the practical problems, there are large number of objects are need to be considered. In paper [10], [11] and [12], the authors consider the similar approximate algorithms to solve the Steiner problem, such as minimal Steiner tree and networks. But how to find the more optimal approximate algorithm, and even more the algorithm can be adjusted by different cases, is the reason of paper. We want to find the more optimal approximate approach by the analysis of the different cases such that we can choose the more optimal approximate approach.

Keywords: *Steiner Tree, Computational Complexity, Location-Selection, Wireless Network.*

1. INTRODUCTION

With the advancement in networking and multimedia technologies enables the distribution and sharing of building communicating network problem widely. People should be considered in the construction of some cities (site) how to lay cable, to make these cities become a connected networks and at the same time to consider how to minimize the cost of the total attachment in traffic. It has the similar problem in transmission system. Since the 1950s, information science has the vigorous development, such that promoting the shortest connection of this kind of problem, formed the research field of a very active combination optimization's topic. There are too much models on this area, roughly divided into two categories: the attachment problem in the plane and the network connection problems. For the former, it is considered the connection is the straight line of the plane and the definition of distance is Euclidean distance; For the latter the connection is considered the shortest path in the network, where the distance of two vertices is equals to the distance of the shortest path between the vertices.

On location-selection of recycle water station, it is expensive to build such station to provide the use of waste water. There are some units to consider laying pine to connect the station. Then it is necessary to consider the cost of transmission of the water, i.e., we need to consider the sum length of the connected pipes. The problem is equitable to the famous Steiner tree problem. The computer complexity is NP-complete, i.e., we cannot find a polynomial time algorithm unless P=NP. In this paper, we consider an approximate algorithm to solve the large number of units Steiner tree problem and give an example with seven objects to consider the location-selection of recycle water station problem. In paper [10], [11] and [12], the authors consider the similar approximate algorithms to solve the Steiner problem, such as minimal Steiner tree and networks. But we want to find the more optimal approximate approach by the analysis of the different cases to choose the different curve by comparing the value of object function.

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The minimal spanning tree problem is originated from the comprehensive work of Kruskal and Prim. In the plane of the minimum spanning tree problem can be stated as follows: Given a set N which has some fixed vertices and |N|=n in a plane, where the distance of each line (or edge) e= (u, v) is defined the Euclidean distance between the two vertices u and v. Then we can construct a weight graph G according to the model: $V(G) = N \cup P$, $w(e) = d(u,v)$, where P is the Steiner vertex. Now for a spanning trees of G, namely a connection graph with the vertex set is N to make the sum weight of edges minimize. Kruskal and Prim has established by the famous Greedy algorithm, even applied to mastoid optimization problem. By using the algorithm, the spanning tree problem can be briefly described as follows:

(1) for the graph G, we can sort the distance of the edges;

(2) we choose a smallest edge to construct an edge set;

(3) choose one the one the shortest edge e which is not selected, if e and selected edges does not constitute a circle, then we choose the edge to the edge set, otherwise, we abandon the edge;

(4) the algorithm does not end until we choose $n-1$ edges to the edge set, such that we have a smallest spanning tree T of the graph G.

The algorithm time bound of the algorithm is $O(n^2 \log n)$, now there exists more effective polynomial algorithm. Minimum Steiner tree problem is originated from three-point sitting Fermat problem, where there are three villages whose position fitly constitute a triangle, and assumed the ground can be used anywhere for all road's construction. The problem is how to design a graph G such that the villages are connected and the distance of roads reached the shortest.

The answer to this question is very simple. We can choose one point inside the triangle such that the point attachment angle to the three points is 120^0 . This is an optimal plan (if the maximum angle among the triangle is at least 120^0 , then the optimal plan is the two shorter edges.)

There are many papers to consider the spanning tree problems. In [4], Ming-yi Yue gives some new and simple proofs for the cases n=3, 4 and 5, and a proof of the Steiner Ratio Conjecture. Some researchers consider some application on Steiner tree problem, such as in [8], the authors considered intelligent optimization algorithms for wireless network problem. In the paper, we give an approximate method to solve the large number of units, which the distance between the units and the recycle water station is minimum.

2. APPROXIMATE ALGORITHM

The problem is to find a suitable launcher station. The model is

there are n consumers, the plane coordinates $(x_i, y_i),\ i = 1, 2, \cdots, n,$ it is needed to build one station to connect the consumers such that

$$\min_{(x,y)} f = \sum_{i=1}^{n} \sqrt{(x-x_i)^2 + (y-y_i)^2} \quad (1).$$

The distance between the units and the station is Euclidean distance. The problem is to select a suitable location such that the sum distance between the units and the station is minimal. It is easy to see that the problem is equitable to the minimum Steiner tree problem. Since the computer complexity is Np-complete, it is difficult to find the accurate Steiner vertex to minimize the distance. Therefore, there are lots of papers are considered the small data Steiner tree problem such that $|N| = 2,\ 3,\ 4,\ 5$ and some special cases such as the restricted condition, like the Steiner vertex on the line, some curve etc. For the practical problem, like we proposed the location-selection of wireless network problem, maybe there are many consumers to connect the station, the previous achievements cannot to solve the model. We construct an approximate algorithm to solve the model.

Least square method is a kind of optimal technique. It can determine the unknown data and make deviation between these values of data and the practical data minimal. The method can also be used to curving fitting. There are n units, denoted the plane coordinates $(x_i, y_i),\ i = 1, 2, \cdots, n.$ We use the Least square method to find a quadratic curve L. Nextly, we find a vertex P on the line L such that the distance between the vertex and all of units is minimum. Therefore, P is an approximate Steiner vertex of the problem.

Approximate Algorithm

(1) for units $(x_i, y_i),\ i = 1, 2, \cdots, n,$ we use the least square method to fine the different power of curve , i.e., the power is one , two or more, directly attained by the matlab computation;

(2) we find one vertex $P \in L$ such that the distance between P and the all of consumers is minimum, this is conditional extreme value

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problem, and we can use the Lagrange multiplier method to find the vertex.

We will consider different method directly by the analysis of cases:

Application of the approximate Algorithm

We use the approximate algorithm to solve an example. There are seven units and the according coordinates are x=[15,17,19,21,23,25,27], y=[20 25 26 30 26 22 18].

Case 1 we use the power is one to find the fixed line L using the Least square method by matlab

Input data x0=15:2:27;

x0=[x0];

y0=[20 25 26 30 26 22 18];

[p,s]=polyfit(x0,y0,1);

p

[y,delta]=polyconf(p,x0,s);

y

polytool(x0,y0,1)

the computation result of matlab is

p = -0.2143 28.3571

y = 25.1429 24.7143 24.2857 23.8571 23.4286 23.0000 22.5714

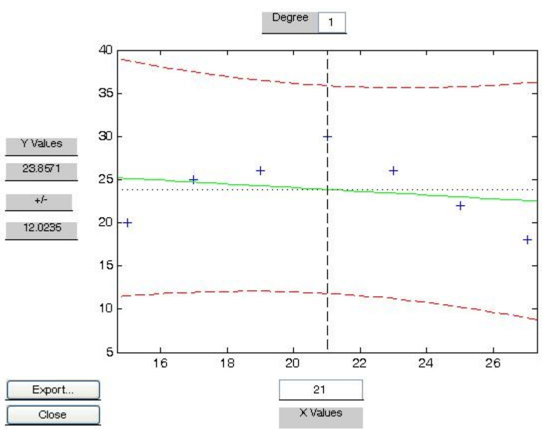

Figure 1: The Linear Function

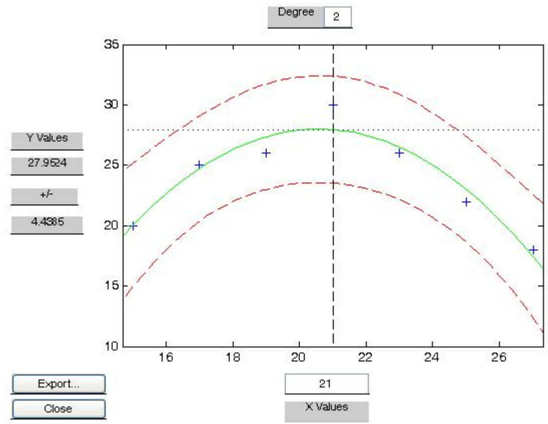

Figure 2: The Quadratic Function

Although Therefore we have the function of L $y = -0.2143x + 28.3571$, see figure 1.

Step 2 find a suit vertex P on L such that the distance between P and vertices is minimum using the Lagrange multiplier method.

To simplified calculation, we consider the square of distance instead of the distance.

$$\min\ f = \sum_{i=1}^{7}((x-x_i)^2 + (y-y_i)^2,$$

$$where,\ y = -0.2143x + 28.3571.$$

We can have using the matlab computation

$$x \approx 21.37,\ y \approx 23.87,$$

$$\sum_{i=1}^{7}\sqrt{(x-x_i)^2+(y-y_i)^2} \approx 36.16,$$

and the distance between the each consumer and the station is approximate 7.4534, 4.5137, 3.1865, 6.1412, 2.6821, 4.0834, and 8.1335, respectively.

Case 2 we use the power of curve is two by the same data, the result is see figure 2

p = -0.2560 10.5357 -80.4226

y = 20.0238 24.7143 27.3571 27.9524 26.5000 23.0000 17.4524

To simplified calculation, we consider the square of distance instead of the distance, such that the model has been modified to the following problem:

$$\min\ f = \sum_{i=1}^{7}((x-x_i)^2 + (y-y_i)^2),$$

$$y = -0.2560x^2 + 10.5357x - 80.4226.$$

By the similar computation we can have

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$x \approx 22.84,\ y \approx 24.68$

$$\sum_{i=1}^{7}\sqrt{(x-x_i)^2+(y-y_i)^2} \approx 37.31$$

and the distance between the each unit and the station is approximate 9.1306 5.8488 4.0605 5.6292 1.3297 3.4421 7.8694 37.3103. respectively.

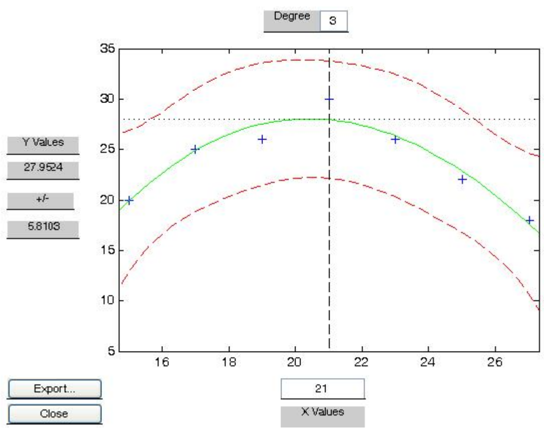

Figure 3: The Cubic Function

Case 3 we use the power of curve is three by the same data, the result is see figure 3

p = 0.0035 -0.4747 15.0322 -110.5372

y = 19.8571 24.8810 27.5238 27.9524 26.3333 22.8333 17.6190

$$\min\ f = \sum_{i=1}^{7}((x-x_i)^2+(y-y_i)^2)$$

$$y = 0.0035x^3 - 0.4747x^2 + 15.0322x - 110.5372$$

By the similar computation we can have

$x \approx 17.56,\ y \approx 26.01$

$$and\ \sum_{i=1}^{7}\sqrt{(x-x_i)^2+(y-y_i)^2} \approx 40.67$$

and the distance between the each unit and the station is approximate 6.5325, 1.1549, 1.4400, 5.2682, 5.4400, 8.4518, 12.3804, and 40.6678, respectively.

Case 4 we use the power of curve is four by the same data, the result is see figure 4

p = 0.0040 -0.3346 10.0204 -127.5842 604.8858

y =20.1883 24.1082 27.6342 28.6147 26.4437 22.0606 17.9502

$$\min\ f = \sum_{i=1}^{7}((x-x_i)^2+(y-y_i)^2)$$

$$y = 0.004x^4 - 0.3346x^3 + 10.0204x^2 - 127.5842x + 604.8858$$

By the similar computation we can have

$x \approx 13.72,\ y \approx 52.82$

$$\sum_{i=1}^{7}\sqrt{(x-x_i)^2+(y-y_i)^2} \approx 210.61$$

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and the distance between the each unit and the station is approximate 32.8450, 28.0127, 27.3348, 23.9531, 28.3801, 32.8194, and 37.2665, respectively.

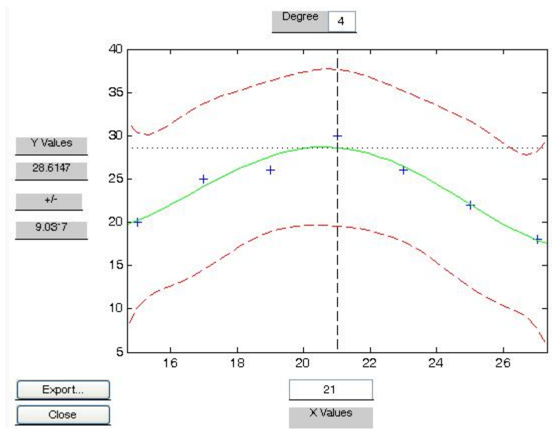

Figure 4: The Quartic Function

By the contrast, we attained the following conclusion:

(1)if we add the power of the curve, the curve fitting is well by the definition and the figures;

(2)but the approximate result is not more optimal than the higher power.

We consider the restricted condition is an obstacle for finding the more optimal approximate method. If we want to find the more optimal approximate result, we can use the approximate algorithm to find the approximate result by testing the power, i.e., power is one or two, maybe we can examine the power is three or four , and compare the results to find the more optimal approximate result.

3. CONCLUSION

Since the computation complexity of the famous Steiner tree problem is Np-complete we consider an approximate algorithm to solve it. We find a suitable vertex on a special line L to approximate the Steiner vertex using LLQ-algorithm. Nextly, we

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can use the least square method to find some special curve, where locate the suitable vertex on the curve such that it approximate the Steiner vertex. Although the approximated vertex is not the accurate Steiner vertex, it provides a method to solve the location-selection of the Steiner tree problem. Even to find more optimal approximate result we can use the comparing method by the above conclusion. Comparing with the former approximate method, we give the LLQ-algorithm having the following properties:

(1) it can be aided by computer to give the figure, such that it is more intuition;

(2) it is easy to be used, i.e., it can deal with a huge of information data;

(3) we can compare different curve to choose the more optimal approximate method.

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