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IMAGE SEGMENTATION ON IMPLICIT SURFACE BASED ON CHAN-VESE MODEL

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ABSTRACT

A new model to segment the images on implicit surface based on the Chan-Vese model and variational level set method was proposed. The classic Chan-Vese model for two phase's segmentation of planar images is extended to image segmentation on implicit surfaces. The spatial contour for image segmentation is defined by means of intersection of the above implicit surface and the zero set of another dynamic level set function. The novel energy functional includes two parts, the first one is the sum of square of the difference between pixel intensity and mean values in two regions on the surface, and the second part is the length of spatial contour. The evolution equation of level set function is obtained using variation technique and is discretized by simple explicit finite difference method. Some numerical examples are given finally to validate the model presented in this paper. Experimental results show that the model has promising effect on image segmentation on implicit surface and preserves salient features of the surface.

Keywords: *Implicit Surface, Image Segmentation, Chan-Vese Model, Variational Level Set*

1. INTRODUCTION

Image segmentation is a traditional field of image processing and has been achieved fruitful results in the plane image segmentation. However, in many cases, the images on the surface reflect more accurately than the images on the plane. Such as different regions of the cerebral cortex and three dimensional terrain surface mountains. As the image on the surface couple with the characteristics of geometry, the mathematical model of image segmentation cannot be directly applied to the surface of the image segmentation. In this paper, we change Chan-Vese image segmentation model [1] into implicit surface reconstruction image segmentation model with implicit surface expression and variational level set expression.

The Chan-Vese image segmentation model is a simplified Mumford-Shah model [2] the variational level set [3] implementation. The result is a piecewise constant image segmentation of the image expression, the dividing line is the zero level set function contours. On the two-phase image segmentation, Chan-Vese energy functional models including the use of zero-level set by the regional model energy term and the minimum contour model energy term. For two-phase on the surface image segmentation, contour lines will need to use space is divided into two different surface areas, when the curved surface of a level set by the zero surface expression of the function, the space can be defined as Cain-type contour surfaces and Another function of the dynamic level set surface intersection of the zero line.

Purpose of this study is to extend the Chan-Vese model in plane image segmentation to the implicit surface on the image segmentation. To this end, the first space-based level set segmentation contour expression. On the establishment of two-phase implicit surface energy functional for image segmentation, and then obtained using variational level set function evolution equation of the gradient drop, and the evolution equation obtained by a simple explicit difference scheme for the discrete solution and numerical example .

2. CHAN-VESE MODEL

Chan-Vese model is not dependent on the edge. Assuming the image constituted by the two regions, in each region of each pixel in gray is the same. The edge of the two regions is , the average gray level of the image is , the curve internal , the curve outside , with a piecewise constant value to approximate the area of these two parts, get the energy functional shows (1):

$$F(C,u_1,u_2) = \mu \cdot \text{length}(C) + \nu \cdot \text{Area}(\text{inside}(C)) + \lambda_1 \int_{\text{inside}(C)} |u_0(x,y) - u_1|^2 \, dxdy + \lambda_2 \int_{\text{outside}(C)} |u_0(x,y) - u_2|^2 \, dxdy \quad (1)$$

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Where, is an arbitrary initial contour, is parameters, the first two curves to maintain a certain degree of regularity, the latter two curves get closer to the edge of the object.

Set function Heaviside (2) and Dirac (3):

$$H(\alpha)=\begin{cases}1 & \alpha\ge 0\\ 0 & \alpha<0\end{cases} \tag{2}$$

$$\delta(\alpha)=\mathrm{d}H(\alpha)/\mathrm{d}\alpha \tag{3}$$

Thus calculate the contour area and the area surrounded by contour length:

$$\mathrm{Area}(\phi\ge 0)=\int_\Omega H(\phi(x,y))\mathrm{d}x\mathrm{d}y \tag{4}$$

$$\mathrm{Length}(\phi=0)=\int_\Omega|\nabla H(\phi(x,y))|\mathrm{d}x\mathrm{d}y=\int_\Omega\delta(\phi(x,y))|\nabla\phi(x,y)|\mathrm{d}x\mathrm{d}y \tag{5}$$

With variational principle, get Chan-Vese model by lowering the gradient equation (6) by the functional (1):

$$\partial\phi/\partial t=\delta_\varepsilon(\phi)\begin{pmatrix}\mu\mathrm{div}(\nabla\phi/|\nabla\phi|)-\lambda_1|u_0(x,y)-u_1|^2\\ +\lambda_2|u_0(x,y)-u_2|^2-\nu\end{pmatrix} \tag{6}$$

3. IMAGE SEGMENTATION OF IMPLICIT SURFACE BASED ON CHAN-VESE MODEL

3.1 Equations of Space Curve

Suppose ϕ and ψ are two four-dimensional level set function, $\phi=0$ and $\psi=0$ are zero level set function of ϕ and ψ, the two zero-level set function, respectively, a three-dimensional closed surface, the two closed surfaces in space Intersection will have a space in the intersection. Partial differential equations for ϕ get $\phi_t+v\Box\nabla\phi=0$, similarly, the partial differential equations for ψ get $\psi_t+v\Box\nabla\psi=0$. Together these two partial differential equations are space curve together with speed movement partial differential equations. [4].

$$\begin{cases}\phi_t+v\Box\nabla\phi=0\\ \psi_t+v\Box\nabla\psi=0\end{cases} \tag{7}$$

3.2 Image Segmentation Model on Implicit Surface

Assumptions implicit surface S set by level set function ψ with the zero level, u_0 is the intensity of image surface, the surface of the image is divided into two regions, each region of the image intensity of each point is the same, with the u_1 and. ϕ is a four-dimensional level set function, with $\psi=0$ represents a closed surface, it generate closed intersection curve with $\phi=0$, and set it as the split line.

This article on the design of implicit surfaces for image segmentation energy functional $E(\phi)$, including zero level set contour length of the expression of $E_1(\phi)$, $E_2(\phi)$ and the evolution of regional models for the signed distance level set function to keep the penalty term $E_3(\phi)$. That is

$$E(\phi)=E_1(\phi)+E_2(\phi)+E_3(\phi) \tag{8}$$

$$E_1(\phi)=\gamma\int_\Omega|P_{\nabla\psi}\nabla\phi||\nabla\psi|\delta(\psi)\delta(\phi)\mathrm{d}x \tag{9}$$

$$E_2(\phi)=\alpha_1\int_\Omega(u_0-u_1)^2|\nabla\psi|\delta(\psi)H(\phi)\mathrm{d}x+\alpha_2\int_\Omega(u_0-u_2)^2|\nabla\psi|\delta(\psi)(1-H(\phi))\mathrm{d}x \tag{10}$$

$$E_3(\phi)=(\mu/2)\int_\Omega(|P_{\nabla\psi}\nabla\phi|-1)^2|\nabla\psi|\delta(\psi)\mathrm{d}x \tag{11}$$

$\gamma\ge 0,\alpha_1>0,\alpha_2>0,\mu>0$ are parameters , $P_{\nabla\psi}\nabla\phi$ is intrinsic gradient [6]，the below (12) is the full：

$$P_{\nabla\psi}\nabla\phi=\nabla\phi-\left(\left(\sum_{m=1}^{3}\nabla\psi[m]\Box\nabla\phi[m]\right)\Big/\|\nabla\psi\|^2\right)\Box\nabla\psi \tag{12}$$

Also get:

$$\partial E/\partial u_1=0\Rightarrow\int_\Omega(u_1-u_0)|\nabla\psi|\delta(\psi)H(\phi)\mathrm{d}x=0 \tag{13}$$

$$u_1=\frac{\int_\Omega u_0|\nabla\psi|\delta(\psi)H(\phi)\mathrm{d}x}{\int_\Omega|\nabla\psi|\delta(\psi)H(\phi)\mathrm{d}x} \tag{14}$$

Similarly：

$$u_2=\frac{\int_\Omega u_0|\nabla\psi|\delta(\psi)(1-H(\phi))\mathrm{d}x}{\int_\Omega|\nabla\psi|\delta(\psi)(1-H(\phi))\mathrm{d}x} \tag{15}$$

(16) get from (8):

$$\left.\frac{\partial E(\phi+\varepsilon\eta)}{\partial\varepsilon}\right|_{\varepsilon=0}=\left.\frac{\partial E_1(\phi+\varepsilon\eta)}{\partial\varepsilon}\right|_{\varepsilon=0}+\left.\frac{\partial E_2(\phi+\varepsilon\eta)}{\partial\varepsilon}\right|_{\varepsilon=0}+\left.\frac{\partial E_3(\phi+\varepsilon\eta)}{\partial\varepsilon}\right|_{\varepsilon=0} \tag{16}$$

$$\left.\frac{\partial E_1(\phi+\varepsilon\eta)}{\partial\varepsilon}\right|_{\varepsilon=0}=-\int_\Omega\nabla\Box\left(\frac{P_{\nabla\psi}\nabla\phi|\nabla\psi|}{|P_{\nabla\psi}\nabla\phi|}\right)\delta(\psi)\delta(\phi)\eta\mathrm{d}x+\int_{\partial\Omega}\frac{P_{\nabla\psi}\nabla\phi\Box\vec{n}}{|P_{\nabla\psi}\nabla\phi|}|\nabla\psi|\delta(\psi)\delta(\phi)\eta\mathrm{d}s \tag{17}$$

$$\left.\frac{\partial E_2(\phi+\varepsilon\eta)}{\partial\varepsilon}\right|_{\varepsilon=0}=\delta(\psi)\delta(\phi)\left(\alpha_1(u_1-u_0)^2|\nabla\psi|-\alpha_2(u_2-u_0)^2|\nabla\psi|\right) \tag{18}$$

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$$\left.\frac{\partial E_3(\phi+\varepsilon\eta)}{\partial\varepsilon}\right|_{\varepsilon=0} = -\mu\int_{\Omega}\frac{1}{|\nabla\psi|}\nabla\cdot\left(\left(|P_{\nabla\psi}\nabla\phi|-1\right)\frac{P_{\nabla\psi}\nabla\phi}{|P_{\nabla\psi}\nabla\phi|}|\nabla\psi|\right)|\nabla\psi|\delta(\psi)\eta\,\mathrm{d}x$$
$$+\mu\int_{\partial\Omega}\frac{\left(|P_{\nabla\psi}\nabla\phi|-1\right)|\nabla\psi|\delta(\psi)}{|P_{\nabla\psi}\nabla\phi|}P_{\nabla\psi}\nabla\phi\cdot\vec{n}\eta\,\mathrm{d}s \tag{19}$$

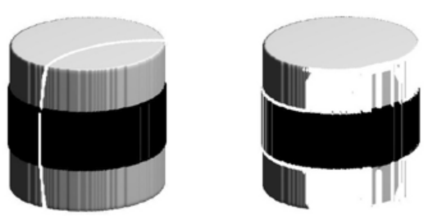
Obtained the gradient steepest descent equation of (8) by (17), (18) and (19) is as follows:

$$\begin{cases} \dfrac{\partial\phi}{\partial t} = \delta(\psi)\delta(\phi)\left[\begin{array}{l}\gamma\nabla\cdot\left(\dfrac{P_{\nabla\psi}\nabla\phi}{|P_{\nabla\psi}\nabla\phi|}|\nabla\psi|\right)\\ +\alpha_2(u_2-u_0)^2|\nabla\psi|-\alpha_1(u_1-u_0)^2|\nabla\psi|\end{array}\right]+\mu\nabla\cdot\left(\left(|P_{\nabla\psi}\nabla\phi|-1\right)\dfrac{P_{\nabla\psi}\nabla\phi}{|P_{\nabla\psi}\nabla\phi|}|\nabla\psi|\right)\delta(\psi) & \text{in } (0,\infty)\times\Omega \\ \dfrac{|\nabla\psi|\delta(\psi)\delta(\phi)}{|P_{\nabla\psi}\nabla\phi|}P_{\nabla\psi}\nabla\phi\cdot\vec{n}+\dfrac{\left(|P_{\nabla\psi}\nabla\phi|-1\right)|\nabla\psi|\delta(\psi)}{|P_{\nabla\psi}\nabla\phi|}P_{\nabla\psi}\nabla\phi\cdot\vec{n}=0 & \text{on } \partial\Omega \\ \phi(0)=\phi_0 & \text{in } \Omega\cup\partial\Omega \end{cases} \tag{20}$$

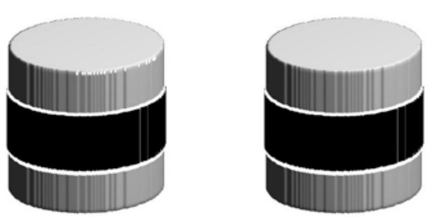
4. NUMERICAL RESULTS AND ANALYSIS

In the present results are in the PC, (Intel (R), CPU 2.33GHz, RAM 2.00GB) on the use of Matlab7.0 as a programming tool to achieve. Surface experiments were obtained with the CT image reconstruction.

Figure 1 shows the cylindrical surface of the present model two-phase image segmentation. Figure (a) is the result of the level set initialization, Figure (b), (c), (d), respectively; the level set function iterations 1, 10, 50 times the results. The experiments take parameters $\alpha_1=1,\alpha_2=1,\gamma=1,\mu=1$, time step $\Delta t=0.1$, space step $h=1$.



（a） Initialization level （b） Iteration 1 time

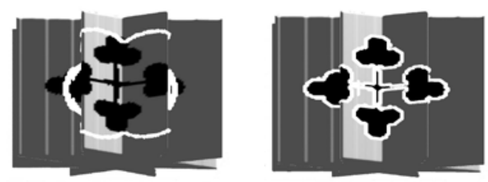


（c） Iteration 10 times （d） Iteration 50 times

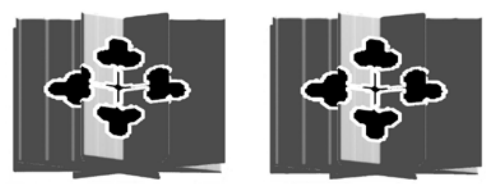
Figure 1. Two-phase Image Segmentation of Cylindrical Surface

It can be seen in Figure 1, for relatively simple on the surface two-phase images, this model can achieve good segmentation results, but in the iterative process of dividing lines cannot remain closed curve, because the iterative process of cylindrical Zero level set surface and a spherical function that overlapped too much with the increase in the number of iterations will eventually become the dividing line closed curve.

Figure 2 shows the shape of the surface on the rough image segmentation. Figure (a) is the result of the level set initialization, Figure (b), (c), (d), respectively, is the level set function iterations 1, 10, 50 times. The experiments take parameters $\alpha_1=1,\alpha_2=1,\gamma=1,\mu=1$, time step $\Delta t=0.1$, and space step $h=1$.



（A） Initialization Level （B） Iteration 1 Time



（c） Iteration 10 Times （D） Iteration 50 Times

Figure 2. Segmentation Of Rough Image Segmentation

It can be seen from Figure 2, this model for the shape of the uneven surface can still achieve better image segmentation results. In the first iteration of the experiment, only the broad contours can be segmented, but some small details in the division when the speed is slower.

5. CONCLUSION

In this paper, implicit surface segmentation on the two-phase study were analyzed and discussed the Chan-Vese model in image segmentation of the advantages of flat and extends to the surface application of image segmentation, level set functions are implicitly expressed Surface, generated by two level set intersection space curve, proposed variational level set method based on surface segmentation model. Will find the optimal solution to the problem of energy model is converted to a steady state solution of the partial differential equation problems. Experimental

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Comparison of the different images on different shapes of image segmentation results confirm the proposed model can be divided in the image while maintaining the inherent geometric surface features, and the complexity of complex shape can achieve good image segmentation.

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