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SYNCHRONIZATION OF COMPLEX NONLINEAR SYSTEMS AT A PRESET TIME

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ABSTRACT

In this paper, a novel finite-time synchronization framework for complex nonlinear systems is established. In such a framework, the problem of synchronization at any preset time of complex nonlinear systems is solved by using linear continuous state feedback control laws with time-varying gains. The proposed preset time dependent time-varying control laws can solve synchronization of chaotic systems problems at the preset time. Numerical simulations are presented to illustrate the effectiveness of the obtained theoretical results, not only for integer order chaos, but also for fractional order chaos.

Keywords: *Complex Nonlinear Systems, Chaotic Systems, Finite-time Synchronization, Fractional Order*

1. INTRODUCTION

Complex nonlinear systems synchronization has attracted increasing attention of scientists from various research fields for its advantages in practical applications has been of broad interest in recent years [1-11]. The main idea of synchronization is to use the states of the master system to control the slave system so that the states of the slave system follow the states of the master system asymptotically [5-9]. The master system and the slave system may have identical or completely different structures. The convergence of the synchronization procedure in is exponential with infinite settling time.

Using finite time control techniques to achieve faster convergence in control systems is one of the effective methods. Finite time stability means the optimality in settling time [12]. Moreover, the finite time control techniques have demonstrated better robustness and disturbance rejection properties. The problem we want to discuss here is that can these complex nonlinear systems be synchronized in finite settling time? The answer is certain, which we will demonstrate in the following sections.

So, our goal in this paper is to develop feedback control laws to make the synchronization procedure converging in finite time, i.e., the states of the slave system follow the states of the master system in finite time. The finite time control technique we discussed here is based on continuous state feedbacks. The rest of this paper is organized as follows. In Section 2, some propositions and control laws are introduced. In Section 3, finite-time synchronization of two Lorenz systems is presented. In Section 4, fractional Chaos is introduced, and finite-time synchronization of two fractional order Chen systems is presented. Finally, conclusions are given out in Section 5.

2. THE THEORY OF FINITE TIME SYNCHRONIZATION OF CHAOTIC SYSTEMS

Some results of finite time control techniques using continuous feedbacks are given in [12], and now we will propose the following three propositions.

Proposition 1. *The system*

$$\frac{dx(t)}{dt} = u(t) \tag{1}$$

can be globally stabilized in finite time under the feedback control law

$$u(t) = \frac{-k}{t_f - t} x(t) \ , \quad t \in [0, t_f), k > 0 \tag{2}$$

It follows that

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$$x(t)=(\frac{t_f-t}{t_f})^k x(0) \quad (3)$$

Since k is positive, we have

$$x(t)\to 0 \ , \ as \ \ t\to t_f^- \quad (4)$$

For any initial value of state $x(t)$ at $t=0$, i.e. , $x(0)$ it is easily computed that the solution trajectory of Eq.(1), Eq.(2) will reach $x=0$ in finite time.

Proposition 2. *Given any finite time* t_f *, the following time-varying feedback control law*

$$u(t)=\frac{-k}{(t_f-t)^m}x(t) \ , \ t\in[0,t_f), k>0 \quad (5)$$

solves the finite-time globally stability problem at time t_f *, for system Eq.(1) , where k is a positive constant scalar and* $m\geq 1$ *is a positive integer.*

If $m=1$, we come back to Propositon 1. If $m>1$, we have

$$x(t)=\exp(\frac{k}{(1-m)(t_f-t)^{m-1}}-\frac{k}{(1-m)t_f^{m-1}})x(0) \quad (6)$$

It is obvious that

$$x(t)=\frac{k}{(1-m)(t_f-t)^{m-1}}\to-\infty \ , as \ \ t\to t_f^- \quad (7)$$

It means that

$$x(t)\to 0 \ , \ as \ \ t\to t_f^- \quad (8)$$

Proposition 3. *The system*

$$\begin{aligned}\dot{x}(t)&=v(t)\\ \dot{v}(t)&=u(t)\end{aligned} \quad (9)$$

can be globally stabilized in finite time under the feedback control law

$$\dot{u}(t)=\frac{2k}{(t_f-t)^3}x(t)-\frac{k}{(t_f-t)^2}v(t) \quad (10)$$

Now, let us first consider the synchronization problem of second order chaotic systems which can be written in the following form

$$\begin{aligned}\frac{dx_1(t)}{dt}&=v_1(t)\\ \frac{dv_1(t)}{dt}&=f_1(x_1,v_1,t)\end{aligned} \quad (11)$$

Where $f_1(x_1,v_1,t)$ is a nonlinear function. This system is called the master system. The equations that describe the slave system are

$$\begin{aligned}\frac{dx_2(t)}{dt}&=v_2(t)+u_1(t)\\ \frac{dv_2(t)}{dt}&=f_2(x_2,v_2,t)+u_2(t)\end{aligned} \quad (12)$$

where $f_2(x_2,v_2,t)$ is a nonlinear function, and $u_1(t)$, $u_2(t)$ are the control signals to be designed.

The difference between the two distances and the two velocities is described by

$$\begin{aligned}x_3(t)&=x_2(t)-x_1(t)\\ v_3(t)&=v_2(t)-v_1(t)\end{aligned} \quad (13)$$

Thus, we get

$$\begin{aligned}\frac{dx_3(t)}{dt}&=v_3(t)+u_1(t)\\ \frac{dv_3(t)}{dt}&=f_2(x_2,v_2,t)-f_1(x_1,v_1,t)+u_2(t)\end{aligned} \quad (14)$$

Several possible choices for the control signals $u_1(t)$, $u_2(t)$ can be taken to synchronize the slave system to the master system.

2.1 Control Strategy 1

One possible choice is given by

$$\begin{aligned}u_1(t)&=-v_3(t)-\frac{k}{t_f-t}x_3(t)\\ u_2(t)&=-f_2(x_2,v_2,t)+f_1(x_1,v_1,t)-\frac{k}{t_f-t}v_3(t)\end{aligned} \quad (15)$$

Both signals $u_1(t)$, $u_2(t)$ are available in the synchronization procedure. With Eq.(14), Eq.(15) can be rewritten in the following notation:

$$\begin{aligned}\dot{x}_3(t)&=-\frac{k}{t_f-t}x_3(t)\\ \dot{v}_3(t)&=-\frac{k}{t_f-t}v_3(t)\end{aligned} \quad (16)$$

Due to Propositon 1, Eq.(16) implies that the two chaotic systems are synchronized with continuous state feedbacks in finite time.

2.2 Control Strategy 2

Another possible choice is given by

$$\begin{aligned}u_1(t)&=0\\ u_2(t)&=-f_2(x_2,v_2,t)+f_1(x_1,v_1,t)\\ &\quad+\frac{2k}{(t_f-t)^3}x_3(t)-\frac{k}{(t_f-t)^2}v_3(t)\end{aligned} \quad (17)$$

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Here, only control signal $u_2(t)$ is made available while $u_1(t) \equiv 0$.

With Eq.(17), the system Eq. (14) can be rewritten in the following notation:

$$\dot{x}_3(t) = v_3(t)$$
$$\dot{v}_3(t) = \frac{2k}{(t_f - t)^3} x_3(t) - \frac{k}{(t_f - t)^2} v_3(t) \tag{18}$$

According to Propositon 3, the differences will converge to zero in finite time.

3. THE SIMULATION OF FINITE TIME SYNCHRONIZATION OF LORENZ SYSTEMS

Now, let us consider the synchronization of two Lorenz systems. The master Lorenz system is given by

$$\begin{aligned} \frac{dx_1(t)}{dt} &= a(y_1(t) - x_1(t)) \\ \frac{dy_1(t)}{dt} &= bx_1(t) - y_1(t) - x_1(t)z_1(t) \\ \frac{dz_1(t)}{dt} &= x_1(t)z_1(t) - cz_1(t) \end{aligned} \tag{19}$$

The slave Lorenz system is given as follows:

$$\begin{aligned} \frac{dx_2(t)}{dt} &= a(y_2(t) - x_2(t)) + u_1 \\ \frac{dy_2(t)}{dt} &= bx_2(t) - y_2(t) - x_2(t)z_2(t) + u_2 \\ \frac{dz_2(t)}{dt} &= x_2(t)z_2(t) - cz_2(t) + u_3 \end{aligned} \tag{20}$$

Letting the error state be

$$\begin{aligned} x_3(t) &= x_2(t) - x_1(t) \\ y_3(t) &= y_2(t) - y_1(t) \\ z_3(t) &= z_2(t) - z_1(t) \end{aligned} \tag{21}$$

Then the error state dynamic equations satisfy

$$\begin{aligned} \dot{x}_3(t) &= a(y_3(t) - x_3(t)) + u_1(t) \\ \dot{y}_3(t) &= bx_3(t) - y_3(t) - x_2(t)z_2(t) + x_1(t)z_1(t) + u_2(t) \\ \dot{z}_3(t) &= x_2(t)y_2(t) - x_1(t)y_1(t) - cz_3(t) + u_3(t) \end{aligned} \tag{22}$$

One possible choice of control signals is

$$\begin{aligned} u_1(t) &= -a(y_3(t) - x_3(t)) - \frac{k_1}{t_f - t} x_3(t) \\ u_2(t) &= -bx_3(t) + y_3(t) + x_2(t)z_2(t) \\ &\quad - x_1(t)z_1(t) - \frac{k_2}{t_f - t} y_3(t) \\ u_3(t) &= -x_2(t)y_2(t) + x_1(t)y_1(t) \\ &\quad + cz_3(t) - \frac{k_3}{t_f - t} z_3(t) \end{aligned} \tag{23}$$

Where $k_1, k_2, k_3 > 0$ .The equations implies that the error state described by the following equations

$$\begin{aligned} \dot{x}_3(t) &= -\frac{k_1}{t_f - t} x_3(t) \\ \dot{y}_3(t) &= -\frac{k_2}{t_f - t} y_3(t) \\ \dot{z}_3(t) &= -\frac{k_3}{t_f - t} z_3(t) \end{aligned} \tag{24}$$

Thus, the three error states will converge to zero in finite time. For the two Lorenz systems, the parameters are $a = 10, b = 28, c = 8/3$. We choose values for the constants $k_1 = 6, k_2 = 5, k_3 = 4$. After a time $t_f = 100$, the slave Lorenz system Eq.(20) is to be controlled with the master system (19). The simulation results are shown in Fig.1.

Another choice is only taking $u_2(t), u_3(t)$ as available control signals while $u_1(t) \equiv 0$. The control signals are designed as follows:

$$\begin{aligned} u_1(t) &= 0 \\ u_2(t) &= -(b+a)x_3(t) + (a+1)y_3(t) + x_2(t)z_2(t) \\ &\quad - x_1(t)z_1(t) + \frac{2k_1}{a(t_f - t)^3} x_3(t) - \frac{k_2}{a(t_f - t)^2} y_3(t) \\ u_3(t) &= -x_2(t)y_2(t) + x_1(t)y_1(t) + cz_3(t) - \frac{k_3}{t_f - t} z_3(t) \end{aligned} \tag{25}$$

which implies that

$$\begin{aligned} \frac{d^2 x_3(t)}{dt^2} &= \frac{2k_1}{(t_f - t)^3} x_3(t) - \frac{k_2}{(t_f - t)^2} y_3(t) \\ \frac{dz_3(t)}{dt} &= -\frac{k_3}{t_f - t} z_3(t) \end{aligned} \tag{26}$$

Hence, the closed-loop system Eq.(24) is stabilized in finite time.

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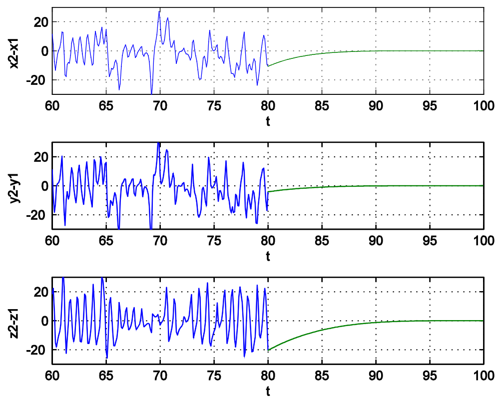

Fig.1. Finite time synchronization of a slave system to a master system with the first law

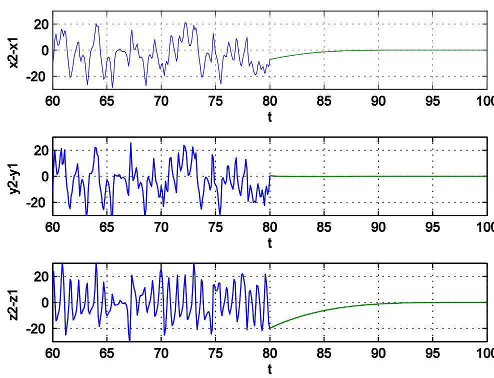

Fig.2. Finite time synchronization of a slave system to a master system with the second law

For the two Lorenz systems, we choose values for the constants $k_1 = 100, k_2 = 200, k_3 = 4$. Set time $t_f = 100$, the slave Lorenz system Eq.(20) is to be controlled with the master Lorenz system Eq.(19). The simulation result is shown in Fig.2.

4. THE SIMULATION OF FINITE TIME SYNCHRONIZATION OF FRACTIONAL ORDER CHAOS

4.1 Dynamic Simulation of Fractional Order Chaos System

There are several definitions of fractional derivatives. In this paper, we use the best known Riemann-Liouville definition [13] to solve the fractional derivatives.

$$\frac{d^\alpha f}{d^\alpha t} = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha+1-n}} d\tau \quad (27)$$

Where $\Gamma(\cdot)$ is the gamma function and $n\text{-}1 \le \alpha < n$.

Now considering zeros initial conditions, one then has the Laplace transform of Eq.(27)

$$L\left\{\frac{d^\alpha f(t)}{dt^\alpha}\right\} = s^\alpha L\{f(t)\} \quad (28)$$

So, the fractional integral operator of order "α" can be represented by the transfer function $F(s) = 1/s^\alpha$, a general method to solve the fractional differential is to approximate the fractional operators by using the standard integer order operators [13],the authors introduced the approximations for with in steps 0.1 with errors of approximately 2dB, to this paper, will be studied in the following. When,

$$\frac{1}{s^{0.9}} = \frac{c_1 s^2 + c_2 s + c_3}{s^3 + c_4 s^2 + c_5 s + c_6} \quad (29)$$

where c_1 = 1.766, c_2 = 38.27, c_3 = 4.914, c_4 = 36.15, c_5 = 7.789, c_6 = 0.01. From above, one can get the following fractional-order Chen-system easily. Consider a fractional order Chen-system mention in refer [13],

$$\begin{cases} d^\alpha x / dt^\alpha &= a(y-x) \\ d^\alpha y / dt^\alpha &= (c-a)x - xy + cy \\ d^\alpha z / dt^\alpha &= xy - bz \end{cases} \quad (30)$$

When $\alpha = 0.9$, Chen-system which can be realized by simulation. Fig.3 is depicted the simulation result obtained by Matlab/Simulink for fractional order Chen-system.

4.2 Finite Time Synchronization of Fractional Order Chen Systems

The slave Chen system is given as

$$\begin{cases} d^\alpha x_2 / dt^\alpha &= a(y_2 - x_2) + u_1(t) \\ d^\alpha y_2 / dt^\alpha &= (c-a)x_2 - x_2 y_2 + c y_2 + u_2(t) \\ d^\alpha z_2 / dt^\alpha &= x_2 y_2 - b z_2 + u_3(t) \end{cases} \quad (31)$$

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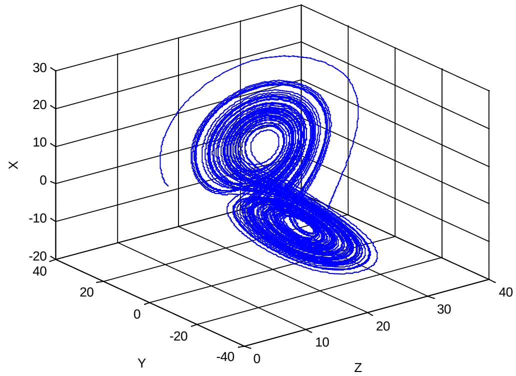

Fig.3. Simulation result of 2.7 order Chen system,with a = 35, b = 3, c = 28

Selecting the appropriate parameter of of control signals, we can get the error state described by the following equations

$$
\begin{aligned}
d^{\alpha} x_3 / dt^{\alpha} &= -\frac{k_1}{t_f - t} x_3(t) \\
d^{\alpha} y_3 / dt^{\alpha} &= -\frac{k_2}{t_f - t} y_3(t) \qquad (32) \\
d^{\alpha} z_3 / dt^{\alpha} &= -\frac{k_3}{t_f - t} z_3(t)
\end{aligned}
$$

Thus, the three error states will converge to zero in finite time. We choose values for the constants $k_1 = 6, k_2 = 5, k_3 = 4$. After a time $t_f = 100$, the slave Chen system is to be controlled with the master system. The simulation results are shown in Fig.4.

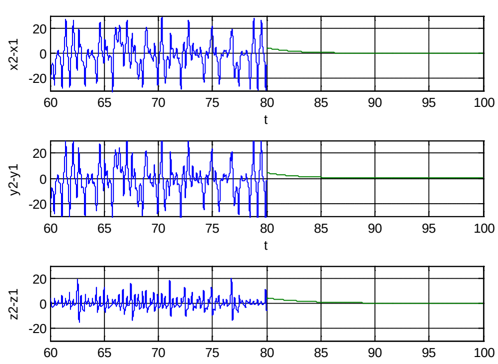

Fig.4. Finite Time Synchronization of Fractional Order Chen Systems

5. CONCLUSION

In this paper, the synchronization problem of complex nonlinear systems is studied. Based on finite time control techniques, linear continuous state feedback control laws are designed for the synchronization of chaotic systems such as integer order Lorenz systems and fractional order Chen systems. The convergence of the synchronization procedure is in finite time. So, the finite-time synchronization framework established in this paper is effective not only for integer order complex nonlinear systems, but also for fractional order complex nonlinear systems.

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