Journal of Theoretical and Applied Information Technology
10th February 2013. Vol. 48 No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

# ROBUST STABILIZATION OF A CLASS OF SWITCHED SYSTEMS

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**ABSTRACT**

In this paper, it considers the stability problem and stabilization problem of switched systems; it has been referred to guarantee feedback system stable. We transform a switched continuous disturbed norm bounded linear system into a stable switched automatic system through arbitrary pole assign approach and we transform switched systems into diagonal matrix with perturbation of $\Delta A$. At the last, a numerical result is given to illustrate our derived results.

**Keywords:** *Switched Systems, Feedback System, Stabilization*

## 1. INTRODUCTION

During the last decade, modeling, analysis and design of switched systems have been investigated. It has been investigated in a great measure on stability analysis and design for switched systems. Problem of stability and stabilization centralizes switched systems. It deserves plentiful investigation not only theoretical development but also practical applications. A switched system is a dynamical system that is composed of a family of continuous-time and discrete-time subsystems and a rule orchestrating the switching between the subsystems. Properties of this complex control model have been investigated for about several decades in practical engineer application and it is important in theory.

The switched system has been attracted considerable attention [1, 2, 3, and 4]. Switching control techniques among different subsystem controllers have been used extensively recent years [8, 9]. Stability robustness is one of the critical design objectives as such existence of some uncertainties is sources of instability and poor characteristic. Often it has many uncertainties in the factory actual control plant. In order the active control system, a complex system with a simple model description has produced uncertainties between the simplification model and the actual objective disparity such that this is called the model uncertainty. It possibly brings the uncertainty except the simplification, it may be but also lack of full understanding certain characteristics not only the link of systems which produces uncertainties, at the same time, it may come from exterior disturbance. it has been analyzed in [2].it presented continuous-time state feedback control problem with arbitrary switching rules[5].design a switching law and a state feedback control law to asymptotically stable closed-loop system[10]. An analysis of switched systems possesses stability which is very pivotal for systems [11, 12, and 13]. it has multitude choices of switching rules to make the whole system stable if and only if all subsystems have a common Lyapunov function, In [14,15]. It is well known that the transient responses of the system are determined primarily by its poles locations. So if we can configure the system poles to some pre-specified values, then favorable transient responses can be guaranteed[16]. It presented norm-bounded timed-invariant uncertainties with the state and input matrices in [17].

Notations. Throughout this note, the following notations are used. denotes compatible dimensional identity matrix. is the transpose of the matrix. is the n-dimensional Euclidean space. is the set of all n×m dimensional matrices. means that is a symmetric positive definite(negative definite) matrix.

## 2. MATHEMATICAL PRELIMINARIES

In this paper, a switched continuous linear system is described as follows:

$$\dot{x}(t) = A_{i(t)}x(t) + B_{i(t)}u(t) + D_{i(t)}w(t)\,,\; x(0) = x_0 \tag{1a}$$

$$y(t) = C_{i(t)}x(t) \tag{1b}$$

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

Where $x(t) \in R^n$ is the continuous state, $u(t) \in R^m$ is the control input of subsystem, $w(t) \in R^n$ is an exogenous input, $y(t) \in R^q$ is the measurement of the system. $i(t): R^+ \to I = \{1, 2, ..., N\}$ is a piecewise constant function of continuous time, called switching law or switching signal, which takes its values in finite set $I$ . $N > 0$ is the number of subsystems. For simplicity, at any arbitrary time t, the switching signal $i(t)$ is denoted by $i$ . $A_i$ , $B_i$ , $C_i$ and $D_i$ are real known constant matrices with appropriate dimensions.

The nominal systems of the systems (1) is

$$\dot{x}(t) = A_{i(t)}x(t) + B_{i(t)}u(t) \text{ , } x(0) = x_0 \quad (2a)$$

$$y(t) = C_{i(t)}x(t) \quad (2b)$$

We will assume that all quantities have compatible dimensions.

Theorem 2.1 Let $x = 0$ be an equilibrium point for the autonomous system

$$\dot{x} = Ax \quad (3)$$

Let $V : R^n \times R_+ \to R$ be a continuously differentiable function such that :

$$V(0) = 0 \quad (4)$$

and

$$V(x,t) > 0, \forall x \neq 0 \text{ ; } \lim_{t\to\infty} V(x,t) \to 0 \text{ ; }$$
$$\dot{V}(x) < 0, \forall x \neq 0 \quad (5)$$

Such that $x = 0$ is globally asymptotically stable.

Function $V(x,t)$ in theorem 2.2 is called the Lyapunov function. It provides sufficient conditions for the origin to be stable.

Given $A \in R^{n\times n}$ and $B \in R^{n\times n}$ , there exists $k \in R^{m\times n}$ such that $A + Bk$ 's eigenvalues may be assigned to arbitrary complex value if and only if $(A, B)$ is controllable. The nominal system of the systems (2) becomes the autonomous system.

The Bounded-Real Lemma:[6] (1) Let the transfer function $G(s) = C(sI - A)^{-1}B + D$ with $A$ being stable. Then $\|G(s)\|_\infty < 1$ hold if and only it exists $P > 0 (P < 0)$ and it satisfies the following Linear matrix inequality:

$$\begin{bmatrix} A^T P + PA & PB \\ B^T P & -I \end{bmatrix} + \begin{bmatrix} C^T \\ D^T \end{bmatrix} \begin{bmatrix} C & D \end{bmatrix} < 0 \quad (6)$$

Let us use the tilde conventions to denote perturbations. Explicitly, $A$ and $\tilde{A} = A + E$ will denote a complex matrix of order n and a perturbation of $A$ , respectively. The eigenvalues of $A$ and $\tilde{A} = A + E$ are $L(A) = \{\lambda_1, \cdots, \lambda_n\}$ and $L(\tilde{A}) = \{\tilde{\lambda}_1, \cdots, \tilde{\lambda}_n\}$ , respectively.

Definition 1.2[18] Let $A$ have eigenvalues $\lambda_1, \cdots, \lambda_n$ and $\tilde{A}$ have eigenvalues $\tilde{\lambda}_1, \cdots, \tilde{\lambda}_n$ .Then the spectral variations of $\tilde{A}$ with respect to $A$ is $\tilde{A} = \max_i \min_j |\tilde{\lambda}_i - \lambda_j|$ .

## 3. STABILITY ANALYSIS FOR SWITCHED SYSTEMS

In the first instance, we consider diagonalizable matrices of perturbation.

Lemma 3.1[18] Suppose that $A$ is diagonalizable; i.e., $X^{-1}AX = \Lambda$ ,where $\Lambda$ is diagonal. let $\|\cdot\|$ be a consistent matrix norm such that $\|diag(\delta_1, \cdots, \delta_n)\| = \max|\delta_i|$ ,then $sv_A(\tilde{A}) \leq \|X^{-1}EX\|$ and $sv_A(\tilde{A}) \leq k(X)\|E\|$ , where as usual $k(X) \leq \|X\|\|X^{-1}\|$ .

Now we analyze system (1) with $w(t) = 0$ or consider system (2),at the same time let switching state feedback control law $u(t) = k_{i(t)}x(t)$ such that the closed-loop subsystem $\dot{x}(t) = (A_{i(t)} + B_{i(t)}k_{i(t)})x(t)$ , $(A_{i(t)}, B_{i(t)})$ is controllable, it is obviously asymptotically stable if eigenvalues of $A_{i(t)} + B_{i(t)}k_{i(t)}$ may be open left half complex plane. A sufficient condition is made certainly by the following result.

Theorem 3.2 if it exists a positive definite $P = P^T \in R^{n\times n}$ and matrices $R_{i(t)} \in R^{l\times n}, l = 1, \cdots, N$ ,such that

$$A_{i(t)}P + PA_{i(t)}^T + B_{i(t)}R_{i(t)} + R_{i(t)}^T B_{i(t)}^T < 0$$
$$i(t) = 1, \cdots, N \quad (7)$$

It has the switched feedback control matrix $k_{i(t)} = R_{i(t)}P^{-1}, i(t) = 1, \cdots, N$ which makes the closed-loop subsystem stability.

Proof: let $k_{i(t)} = R_{i(t)}P^{-1}, i(t) = 1, \cdots, N$ substitute for (7),it may be rewritten as

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

$$A_{i(t)}P + PA_{i(t)}^T + B_{i(t)}k_{i(t)}P + Pk_{i(t)}^T B_{i(t)}^T < 0$$
$$\Leftrightarrow (A_{i(t)} + B_{i(t)}k_{i(t)})P + P(A_{i(t)}^T + k_{i(t)}^T B_{i(t)}^T) < 0, \quad (8)$$
$$i(t) = 1, \cdots, N$$

It exists a common Lyapunov function $V(x) = x^T Px$ whose stability of the closed-loop is guaranteed by the above definition.

Now let us analyze continually system (1) with switching state feedback control law $u(t) = k_{i(t)}x(t)$ such that the closed-loop subsystem

$$\dot{x}(t) = (A_{i(t)} + B_{i(t)}k_{i(t)})x(t) + D_{i(t)}w(t) \quad (9)$$

Hence the state feedback $H_\infty$ control problem is formulated as follows: find a $k_{i(t)}$ which satisfies the following two conditions (3.2):

(3.2I)The eigenvalues of ( $A_{i(t)} + B_{i(t)}k_{i(t)}$ ) are located in open left half complex plane.

(3.2II) $\left\| C_{i(t)} \left( sI - A_{i(t)} - B_{i(t)}k_{i(t)} \right)^{-1} D_{i(t)} \right\|_\infty < 1$

The gain $k_{i(t)}$ satisfies the above two conditions if and only if there exists a $P > 0$ ,

$$\left( A_{i(t)} + B_{i(t)}k_{i(t)} \right) P + P\left( A_{i(t)} + B_{i(t)}k_{i(t)} \right)^T + D_{i(t)}D_{i(t)}^T + PC_{i(t)}^T C_{i(t)} P < 0 \quad (10)$$

If conditions (3.2) satisfies, let us apply theorem 2.1 to infer that (10) has a symmetric solution $P > 0$ . as a result of $\left( A_{i(t)} + B_{i(t)}k_{i(t)} \right) P + P\left( A_{i(t)} + B_{i(t)}k_{i(t)} \right)^T < 0$

and $A_{i(t)} + B_{i(t)}k_{i(t)}$ is stable, such that it can infer that $P > 0$ .conversely, if $P$ satisfies (10) ,then

$$\left( A_{i(t)} + B_{i(t)}k_{i(t)} \right) P + P\left( A_{i(t)} + B_{i(t)}k_{i(t)} \right)^T < 0 \quad (11)$$

It implies that $A_{i(t)} + B_{i(t)}k_{i(t)}$ is stable. Again by theorem 2.1 it can obtain Theorem (3.2)[7].

Theorem 3.3[7] For any $k_{i(t)}$ ,

$$\left( A_{i(t)} + B_{i(t)}k_{i(t)} \right) P + P\left( A_{i(t)} + B_{i(t)}k_{i(t)} \right)^T + D_{i(t)}D_{i(t)}^T + PC_{i(t)}^T C_{i(t)} P \geq A_{i(t)}P + A_{i(t)}^T P + D_{i(t)}D_{i(t)}^T + PC_{i(t)}C_{i(t)}^T P - B_{i(t)}B_{i(t)}^T \quad (12)$$

Equality holds if and only if $k_{i(t)}P + B_{i(t)} = 0$ .

Now we analyze system (1) with $w(t) \neq 0$ again, at the same time let $w(t) = W(t)x$ and $\|W(t)\| < e$

Theorem 3.4 if it exists positive definite matrix

$$\left( A_{i(t)} + DW + B_{i(t)}k_{i(t)} \right)^T P + P\left( A_{i(t)} + DW + B_{i(t)}k_{i(t)} \right) < 0$$

switched systems (1) exist a common Lyapunov function $V(x) = x^T Px$

Proof: we can deduct

$$\dot{V}(x) = x^T \left( \left( A_{i(t)} + B_{i(t)}k_{i(t)} \right)^T P + P\left( A_{i(t)} + B_{i(t)}k_{i(t)} \right) \right) x + (Dw)^T Px + x^T PDw$$

Let $w(t) = W(t)x$

$$\dot{V}(x) = x^T \left( \left( A_{i(t)} + B_{i(t)}k_{i(t)} \right)^T P + P\left( A_{i(t)} + B_{i(t)}k_{i(t)} \right) + W^T D^T P + PDW \right) x$$
$$= x^T \left( \left( A_{i(t)} + DW + B_{i(t)}k_{i(t)} \right)^T P + P\left( A_{i(t)} + DW + B_{i(t)}k_{i(t)} \right) \right) x$$
$$< 0$$

$$\left( A_{i(t)} + DW + B_{i(t)}k_{i(t)} \right)^T P + P\left( A_{i(t)} + DW + B_{i(t)}k_{i(t)} \right) < 0$$

, it exists a common Lyapunov function and switched system which is stable.

Theorem 3.5 Let the transfer function $G(s) = C(sI - A)^{-1}B$ with $A$ being stable. From the bounded-real lemma, we have $\|G(s)\|_\infty = \left\| C(sI - A)^{-1}B \right\| < r$ if and only if there exists $P \geq 0$ such that

$$\begin{bmatrix} A_{i(t)}^T P + PA_{i(t)} + C_{i(t)}^T C_{i(t)} & PB_{i(t)} \\ B_{i(t)}^T P & -\gamma^2 I \end{bmatrix} < 0 \quad (13)$$

Now we analyze system (1) with $w(t) \neq 0$ again, at the same time let $w(t) = \Delta A(t)_{i(t)} x$ , such that the perturbation systems may be transformed into

$$\dot{x}(t) = (A_{i(t)} + B_{i(t)}k_{i(t)} + \Delta A(t)_{i(t)})x(t)$$

Theorem 3.6 if the eigenvalues of matrices $A_{i(t)} + B_{i(t)}k_{i(t)} + \Delta A(t)_{i(t)}$ are located in open left half complex plane, switched systems (1) is asymptotically stable.

Remark: it may be saw that $\|W(t)\| < e$ is restricted region, such that $\Delta A(t)$ may be treated disturbance as the switching system. We can assure

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

eigenvalues of matrices $A_{i(t)}+B_{i(t)}k_{i(t)}+\Delta A(t)_{i(t)}$ that is open left half complex plane if and only if $\left(A_{i(t)}+B_{i(t)}k_{i(t)},\Delta A(t)_{i(t)}\right)$ is controllable.

At last, let us consider a well conditioned matrix through the following deduct.

$$1\le\|I\|=\|AA^{-1}\|\le\|A\|\|A^{-1}\|=\kappa(A) \quad (14)$$

It is difficult to estimate the discernment that one obtains from the above bound.

The inverse of a matrix will be well conditioned which is insensitive to perturbation of sufficiently small. The bounded (14) makes it clear that a well conditioned matrix is one with a small condition number of $\kappa(A)$ .

Theorem 3.7[18] Let

$$\kappa(A)=\|A\|\|A^{-1}\| \quad (15)$$

be the condition number of $A$ .if $\tilde{A}$ is nonsingular, then

$$\frac{\|\tilde{A}^{-1}-A^{-1}\|}{\|\tilde{A}^{-1}\|}\le\kappa(A)\frac{\|E\|}{\|A\|} \quad (16)$$

If in addition

$$\kappa(A)\frac{\|E\|}{\|A\|}<1 \quad (17)$$

Then $\tilde{A}$ is perforce nonsingular and

$$\|\tilde{A}^{-1}\|\le\frac{\|A^{-1}\|}{1-\kappa(A)\frac{\|E\|}{\|A\|}} \quad (18)$$

Moreover

$$\frac{\|\tilde{A}^{-1}-A^{-1}\|}{\|\tilde{A}^{-1}\|}\le\frac{\kappa(A)\frac{\|E\|}{\|A\|}}{1-\kappa(A)\frac{\|E\|}{\|A\|}} \quad (19)$$

## 4. APPLICATIONS

Let us consider the switching linear systems as follows:

$$\dot{x}(t)=A_{i(t)}x(t)+B_{i(t)}u(t)+W(t)x(t),x(0)=x_0,i(t)\in\{1,2,3\}$$
$$y(t)=C_{i(t)}x(t)$$

$$A_1=\begin{pmatrix}2&3&7\\3&5&2\\3&4&3\end{pmatrix},\quad A_2=\begin{pmatrix}-2&3&1\\0&0&1\\3&2&3\end{pmatrix},$$
$$A_3=\begin{pmatrix}-2&3&1\\6&0&3\\3&0&-3\end{pmatrix}$$
$$B_1=\begin{pmatrix}1\\2\\3\end{pmatrix},B_2=\begin{pmatrix}-2&1\\0&0\\2&5\end{pmatrix},B_3=\begin{pmatrix}1&0&1\\2&1&3\\1&0&-2\end{pmatrix}$$
$$C_1=\begin{pmatrix}2&3&1\\3&5&2\\3&4&3\end{pmatrix},\quad C_2=\begin{pmatrix}-2&3&1\\0&5&2\\3&0&3\end{pmatrix},$$
$$C_3=\begin{pmatrix}-2&3&1\\6&0&2\\3&0&-3\end{pmatrix},\|W(t)\|<1$$

Suppose that a state feedback control law $u=kx$ is used to locate desired poles $\lambda_i=-7,i=1,2,3$, Then, the closed loop matrix $A+bk$ has the following real normal formal.

$$A+bk=\begin{pmatrix}0&1&0\\0&0&1\\-343&-147&-21\end{pmatrix}$$

Since

Rank$\left(B_1\ A_1B_1\ A_1^2B_1\right)$ =3; Rank$\left(B_2\ A_2B_2\ A_2^2B_2\right)$ =3; Rank$\left(B_3\ A_3B_3\ A_3^2B_3\right)$=3, such that, we may choose

$$k_1=\begin{pmatrix}4.1580 & 29.0244 & -10.4023\end{pmatrix},$$
$$k_1=\begin{pmatrix}29.6667&11.5833&1.5833\\57.3333&25.1667&4.1667\end{pmatrix},$$
$$k_1=\begin{pmatrix}114.0000&50.3333&6.6667\\126.0000&44.3333&5.6667\\-116.0000&-48.3333&-5.6667\end{pmatrix}.$$

It is obviously possessed eigenvalues of matrices $A_{i(t)}+B_{i(t)}k_{i(t)}$ in open left half complex plane $\lambda_i=-7,i=1,2,3$ . And it can obtain a desired output feedback controller.

The switching linear systems transform the following formal.

$$\dot{x}(t)=\begin{pmatrix}0&1&0\\0&0&1\\-343&-147&-21\end{pmatrix}x(t)+W(t)x(t),x(0)=x_0$$

or

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

$$\dot{x}(t)=\begin{pmatrix}-7 & 0 & 0\\ 0 & -7 & 0\\ 0 & 0 & -7\end{pmatrix}x(t)+W(t)x(t),x(0)=x_0$$

We take the above formal into account perturbation bound. In the light of the theorem 1, we may conclude that the above dynamics is equivalent to $\dot{x}(t)=(A+\Delta A)x(t),x(0)=x_0$ which is obviously stable.

## 5. CONCLUSION

In this note we have derived stabilization of a class of switched linear systems whose subsystem is controllable and we may assign pre-specified eigenvalues. The controller is design based on feedback stabilization, then the results are completing stabilization. In the end, numerical example shows that the approach in this note is very effective.

## ACKNOWLEDGEMENTS

This work was supported by the Computer center Mianyang Normal University and it also supported by science and technology department of a question for study of switched nonlinear control(2011A11). at the same time,the authors gratefully acknowledge several discussions with partner in lab.

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