

RELIABILITY EVALUATION OF MULTIPLE PERFORMANCE PARAMETERS SYSTEM ADT BASED ON MULTIDIMENSIONAL TIME SERIES MODEL

¹LI WANG, ²ZAIWEN LIU, ²CHONGCHONG YU

¹Dr., School of Computer & Information Engineering, Beijing Technology & Business University, China

²Prof., School of Computer & Information Engineering, Beijing Technology & Business University, China

E-mail: ¹xiaolizi1983@hotmail.com

ABSTRACT

This paper proposes a new Accelerated Degradation Testing (ADT) reliability evaluation method utilizing a multidimensional composite time series modeling procedure to take into account the integrated effect of system's multiple performance parameters along with the random effect of environmental variables for equivalent damage in ADT. In this paper, system performance parameter ADT data are treated as a multidimensional composite time series model to predict system failure time. First, this paper decomposes these multiple performance parameters useful for ADT into three classes as trend, cyclical or random components, and describes them with a combined multi-dependent variable regressive model, hidden periodic model and multivariate auto-regression model. Second, according to standard practice, this paper assumes that the failure of such a system obeys a competing failure rule, that is, for an individual unit there is one primary controlling variable that will indicate failure even though others degrade they do not meet any failure criterion. Failure time at each test-stress level is predicted by using the best linear unbiased prediction of the multidimensional composite time series model. Finally, the reliability at use-stress level is estimated from a failure time distribution evaluation based on the failure time predictions at each test-stress level providing a relationship between failure time and test-stress levels.

Keywords: ADT, Multidimensional Time Series, Reliability Evaluation, Multiple Performance Parameters

1. INTRODUCTION

For long lifetime and high reliability products or systems, it is difficult to obtain failure data in a short time period. Hence, Accelerated Degradation Testing (ADT) is presented to deal with the cases where no failure time data could be obtained but degradation data of parameters of the system are available. At present, the ADT reliability evaluation method is utilized primarily with feedback from a single performance parameter system ADT dataset. However, for most systems, multiple performance parameters of these systems will degrade with time, leading to failure. It is important to note that often the systems various performance parameters will interact with each other as the performance degrades. Hence, a correct reliability evaluation based on ADT data must take into account the integrated effect of a system's multiple performance parameters and the random effect of environmental variables.

In the literature, such as in the noted references [1-5], ADT reliability evaluation is studied using time series methods due to its excellent capability of stochastic and periodic information mining. However, reliability evaluations using the time series method in present literature are all based upon a one-dimensional time series analysis. To take into account multiple dimensions of system performance degradation, it is important to study these parameters using an ADT reliability evaluation based on a multidimensional time series analysis method.

2. MULTIDIMENSIONAL TIME SERIES MODELING

The stochastic analysis for multiple performance parameters degradation data with multidimensional time series analysis is based on the following hypotheses:

- (1) All performance parameters of a system degrade monotonically;
- (2) The failure mechanism for the system remains

the same during the degradation process;
 (3) The data collection for all parameters is concurrent.

In ADT, performance degradation data is usually collected at defined consistent intervals producing a homogeneous variance. Typically, degradation data is not stationary per hypothesis (1) above.

Let \mathbf{Y}_t denotes the multiple performance parameters degradation measurement at time t . Based on the Cramer Decomposition Theorem, any multidimensional time series $\{\mathbf{Y}_t\}$ can be decomposed into three components: a deterministic component, a cyclical component and a stationary random component. Hence, \mathbf{Y}_t could be expressed as,

$$\mathbf{Y}_t = \mathbf{D}_t + \mathbf{C}_t + \mathbf{R}_t, \quad t = 1, 2, \dots, N$$

$$\mathbf{Y}_t = \begin{pmatrix} y_{1t} \\ y_{2t} \\ \vdots \\ y_{nt} \end{pmatrix}, \mathbf{D}_t = \begin{pmatrix} d_{1t} \\ d_{2t} \\ \vdots \\ d_{nt} \end{pmatrix}, \mathbf{C}_t = \begin{pmatrix} c_{1t} \\ c_{2t} \\ \vdots \\ c_{nt} \end{pmatrix}, \mathbf{R}_t = \begin{pmatrix} r_{1t} \\ r_{2t} \\ \vdots \\ r_{nt} \end{pmatrix} \quad (1)$$

Here \mathbf{D}_t is multidimensional deterministic component. \mathbf{C}_t is multidimensional cyclical component. \mathbf{R}_t is the multidimensional stationary random component, y_{it} is a degradation measurement of i^{th} performance parameter, d_{it} is the deterministic component of i^{th} performance parameter, c_{it} is a the cyclical component of i^{th} performance parameter, r_{it} is the stationary random component of i^{th} performance parameter, $i=1, 2, \dots, n$, where n is the total number of performance parameters and N is total sampling time.

2.1 Multidimensional deterministic component modeling

The multidimensional deterministic component \mathbf{D}_t is extracted from the performance degradation data using a multi-dependent variable regression model,

$$\mathbf{D}_t = \begin{pmatrix} b_1 g(t) + y_{01} \\ b_2 g(t) + y_{02} \\ \vdots \\ b_n g(t) + y_{0n} \end{pmatrix} = \begin{pmatrix} y_{01} & b_1 \\ y_{02} & b_2 \\ \vdots & \vdots \\ y_{0n} & b_n \end{pmatrix} \begin{pmatrix} 1 \\ g(t) \end{pmatrix} \quad (2)$$

Here $\mathbf{D}(t)$ is a n -dependent variable regression function which effectively fits the degradation trend of the data, b_i is the degradation rate of i^{th}

performance parameter, $g(t)$ is a monotonic regression function, y_{0i} is the initial value of the i^{th} performance parameter, $i=1, 2, \dots, n$.

2.2 Multidimensional cyclical component modeling

The multidimensional deterministic component \mathbf{D}_t is extracted from multiple performance parameters. Then the multidimensional cyclical component \mathbf{C}_t , is modeled using Hidden Periodic model,

$$\mathbf{C}_t = \begin{pmatrix} c_{1t} \\ c_{2t} \\ \vdots \\ c_{nt} \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^{k_1} a_{1j} \cos(\omega_{1j}t + \phi_{1j}) \\ \sum_{j=1}^{k_2} a_{2j} \cos(\omega_{2j}t + \phi_{2j}) \\ \vdots \\ \sum_{j=1}^{k_n} a_{nj} \cos(\omega_{nj}t + \phi_{nj}) \end{pmatrix} \quad (3)$$

Here a_{ij} is the amplitude of i^{th} performance parameter, k_i is the i^{th} total number of angular frequency, ω_{ij} is the j^{th} angular frequency, ϕ_{ij} is the j^{th} phase, $i=1, 2, \dots, n$.

2.3 Multidimensional random component modeling

The multidimensional cyclical component \mathbf{C}_t is extracted from multiple performance parameters. Then the multidimensional random component \mathbf{R}_t , is modeled using a multidimensional autoregressive model,

$$\mathbf{R}_t = \sum_{j=1}^p \mathbf{H}_j \mathbf{R}_{t-j} + \mathbf{E}_t,$$

$$\mathbf{H}_j = \begin{pmatrix} \eta_{11j} & \eta_{12j} & \dots & \eta_{1nj} \\ \eta_{21j} & \eta_{22j} & \dots & \eta_{2nj} \\ \vdots & \vdots & \ddots & \vdots \\ \eta_{n1j} & \eta_{n2j} & \dots & \eta_{nnj} \end{pmatrix}, \mathbf{E}_t = \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{nt} \end{pmatrix} \quad (4)$$

Here p is the order of multidimensional autoregressive model; \mathbf{H}_j is a $n \times n$ multidimensional autoregressive coefficient matrix. ζ_{ikj} is the i^{th} performance parameter multidimensional autoregressive coefficient from k^{th} performance parameter, \mathbf{E}_t is a n -dimensional white noise vector which obeys $N[0, \mathbf{Q}]$, ε_{it} is the white noise of i^{th} performance parameter, $i=1, 2, \dots, n, k=1, 2, \dots, n$.

2.4 Multiple performance parameters degradation modeling

The multi-dependent variable regression model for the deterministic component D_t , the hidden periodic model for the cyclical component C_t and the AR model of the stationary random component R_t are combined into Y_t . Hence, the performance degradation measurement Y , is obtained as,

$$Y_t = D_t + C_t + R_t = D_t + C_t + \sum_{j=1}^p H_j R_{t-j} + E_t \quad (5)$$

It can also be expressed as,

$$\begin{pmatrix} y_{1t} \\ y_{2t} \\ \vdots \\ y_{nt} \end{pmatrix} = \begin{pmatrix} y_{01} & b_1 \\ y_{02} & b_2 \\ \vdots & \vdots \\ y_{0n} & b_n \end{pmatrix} \begin{pmatrix} 1 \\ g(t) \end{pmatrix} + \begin{pmatrix} \sum_{j=1}^{k_1} a_{1j} \cos(\omega_{1j}t + \phi_{1j}) \\ \sum_{j=1}^{k_2} a_{2j} \cos(\omega_{2j}t + \phi_{2j}) \\ \vdots \\ \sum_{j=1}^{k_n} a_{nj} \cos(\omega_{nj}t + \phi_{nj}) \end{pmatrix} +$$

$$\sum_{j=1}^p \begin{pmatrix} \eta_{11j} & \eta_{12j} & \cdots & \eta_{1nj} \\ \eta_{21j} & \eta_{22j} & \cdots & \eta_{2nj} \\ \vdots & \vdots & \ddots & \vdots \\ \eta_{n1j} & \eta_{n2j} & \cdots & \eta_{nj} \end{pmatrix} \begin{pmatrix} r_{1(t-j)} \\ r_{2(t-j)} \\ \vdots \\ r_{n(t-j)} \end{pmatrix} + \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \\ \vdots \\ \epsilon_{nt} \end{pmatrix} \quad (6)$$

Eq.5 is called multidimensional Regression-Auto Regression (RAR) model in this paper.

3. MULTIDIMENSIONAL RAR MODEL PARAMETERS ESTIMATION

3.1 Deterministic component model parameters estimation

Parameters for the multi-dependent variable regression model are estimated using a Least-Square estimation method. Its principle is to minimize the sum of quadratic sum of R_t , which is

$$Q_R = \sum_{t=1}^N \sum_{i=1}^n r_{it}^2$$

Let

$$Y = \begin{pmatrix} y_{11} & \cdots & y_{1N} \\ \vdots & \ddots & \vdots \\ y_{n1} & \cdots & y_{nN} \end{pmatrix}, \quad g = (g(1), g(2), \dots, g(N))$$

$$\hat{y}_0 = (\hat{y}_{01}, \hat{y}_{02}, \dots, \hat{y}_{0n})^T, \quad \hat{b} = (\hat{b}_1, \hat{b}_2, \dots, \hat{b}_n)^T$$

The regression coefficient is estimated by using the matrix inversion formula. That is

$$\begin{bmatrix} \hat{y}_0^T \\ \hat{b}^T \end{bmatrix} = \begin{bmatrix} \bar{Y}^T - \bar{g}L_{gg}^{-1}L_{gy} \\ L_{gg}^{-1}L_{gy} \end{bmatrix}$$

Here

$$L_{gg} = g \left(I - \frac{1}{n} \mathbf{1} \mathbf{1}^T \right) g^T, \quad L_{gy} = g \left(I - \frac{1}{n} \mathbf{1} \mathbf{1}^T \right) Y^T,$$

$$\bar{g} = \frac{1}{n} g, \quad \bar{Y} = \frac{1}{n} Y \mathbf{1}^T, \quad \mathbf{1} = (1, 1, \dots, 1), \quad I = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$$

3.2 Random component model parameters estimation

Parameters of the multidimensional autoregressive model are estimated by a Yule-Walker estimation method.

Point estimation of mean value of R_t , which is $\mu = ER_t$, is

$$\hat{\mu}_N = (\hat{\mu}_1, \hat{\mu}_2, \dots, \hat{\mu}_n)^T = \frac{1}{N} \sum_{t=1}^N R_t \quad (7)$$

The estimate of the autocovariance function $\Gamma(h)$ is

$$\begin{cases} \hat{\Gamma}(h) = \frac{1}{N} \sum_{t=1}^{N-h} (R_{t+h} - \hat{\mu}_N)(R_t - \hat{\mu}_N)^T, & 0 \leq h \leq N-1 \\ \hat{\Gamma}(-h) = \hat{\Gamma}^T(h), & 1 \leq h \leq N-1 \end{cases} \quad (8)$$

Thus the Yule-Walker equation is

$$\begin{cases} \hat{\Gamma}(0) = \sum_{j=1}^p \hat{H}_j \hat{\Gamma}(-j) + \hat{Q} \\ \hat{\Gamma}(-h) = \sum_{j=1}^p \hat{H}_j \hat{\Gamma}(h-j), h \geq 1 \end{cases} \quad (9)$$

Then

$$\begin{bmatrix} \hat{\Gamma}^T(1) \\ \hat{\Gamma}^T(2) \\ \vdots \\ \hat{\Gamma}^T(p) \end{bmatrix} = \begin{bmatrix} \hat{\Gamma}(0) & \hat{\Gamma}(1) & \cdots & \hat{\Gamma}(p-1) \\ \hat{\Gamma}(-1) & \hat{\Gamma}(0) & \cdots & \hat{\Gamma}(p-2) \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\Gamma}(-p+1) & \hat{\Gamma}(-p+2) & \cdots & \hat{\Gamma}(0) \end{bmatrix} \begin{bmatrix} \hat{H}_1^T \\ \hat{H}_2^T \\ \vdots \\ \hat{H}_p^T \end{bmatrix} \quad (10)$$

The autoregressive coefficient,

$$(\hat{H}_1, \hat{H}_2, \dots, \hat{H}_p, \hat{Q}),$$

is estimated by solving Yule-Walker equation.

4. RELIABILITY EVALUATION & ACCELERATED MODELING

In this paper, reliability evaluation for the system at each test-stress level is obtained by modeling the failure time of each performance parameter of the system at each test-stress level. These are predicted using the best linear unbiased model of the multidimensional composite time series. The

reliability evaluation for the system at use-stress level is obtained by failure time distribution of systems, reliability evaluation at each test-stress level and accelerated modeling.

4.1 Multiple performance parameters degradation prediction

The l^{th} step prediction of Y_t is obtained from the best linear unbiased prediction of Eq.4. The prediction formula is

$$Y_{N+l} = D_{N+l} + C_{N+l} + R_{N+l}$$

$$= D_{N+l} + C_{N+l} + \sum_{j=1}^p H_j R_{N+l-j} \quad (11)$$

4.2 Failure time prediction

In practice, the failure of systems having multiple performance parameters usually obeys the competing failure rule. That is, for most of these systems, they will degrade with time where the specific performance parameter that first passes a specified failure threshold for an individual unit will lead to the failure of that unit.

Hence, according to the competing failure rule, when total number of performance parameters is n , this paper sets failure threshold respectively, which is (D_1, D_2, \dots, D_n) , for each performance parameter of the system. The failure time of each performance parameter, which is $(t_{D_1}, t_{D_2}, \dots, t_{D_n})$, is the time that each performance parameter passes its own failure threshold (D_1, D_2, \dots, D_n) . This is predicted by Eq.10. The reliability evaluation of the system then is the minimum failure time prediction of all performance parameters,

$$t_f = \min(t_{D_1}, t_{D_2}, \dots, t_{D_n}) \quad (12)$$

4.3 Failure time distribution

The reliability evaluation is assumed to obey a certain location-scale distribution as determined by a Pearson chi-square Goodness of Fit Test. The estimate of the location and scale parameters of the failure time distribution are obtained by MLE. This paper denotes reliability evaluation of i^{th} system as $t_{f(i)}$, when total number of systems is m , and then the prediction of the maximum likelihood function for the distribution of failure time is

$$L(\beta) = \prod_{i=1}^m f(t_{f(i)}, \beta) \quad (13)$$

Here, $\beta = (\mu, \sigma)^T$, T means transpose of matrix.

4.4 Accelerated modeling

To obtain the ADT reliability evaluation for the at use-stress level, it is necessary to convert the reliability evaluation at each test-stress level to the equivalent reliability evaluation at the use-stress level. This paper converts the reliability evaluation of systems from each test-stress level into a reliability evaluation for the system at its' use-stress level based on the stress level-median failure time relationship and accelerated model

$$\mu = a + b\phi(S) \quad (14)$$

Here, μ is median failure time at each test-stress level; S is test-stress level; a, b are parameters estimated from degradation data. $\phi(S)$ is a known function of S .

5. ADT DATA VERIFICATION

The four temperature stress levels ADT were processed for a certain microwave electronic system to verify the multidimensional time series analysis method. The personal computer records 3 different performance parameters of each test unit every 8 hours. Table I shows the temperature test parameters. The multiple performance parameters degradation path for each system is shown in Fig.1.

Table I Parameters Of ADT

Temperature	Sample size	Num. of Sys.
60°C	247	3
70°C	174	3
80°C	155	3
85°C	114	3

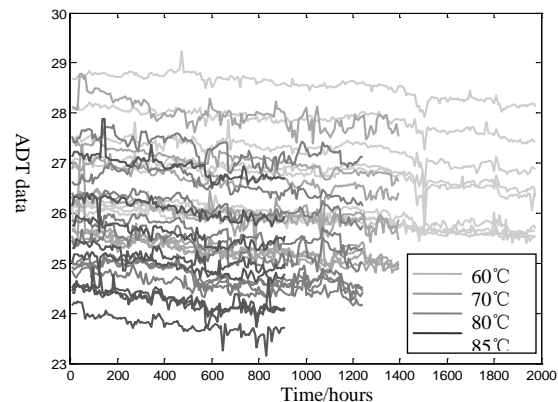


Fig.1 3-Performance Parameters ADT Data

The ADT data of each unit is preprocessed to normalize for the initial performance value to minimize sample bias. Fig.2 shows them.

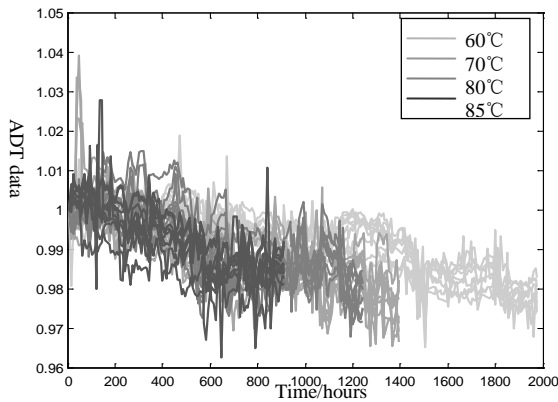


Fig.2 Preprocessed ADT Data

The D_i of each system is set as a linear form. The model parameters are estimated utilizing all performance parameters degradation data. R_i is shown in Fig.3.

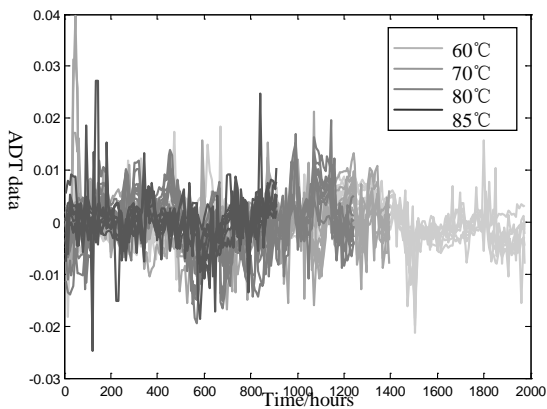


Fig.3 R_i Of ADT Data

The prediction of Y_i of each system at 60°C, 70°C, 80°C and 85°C by the multidimensional time series model is shown in Fig.4.

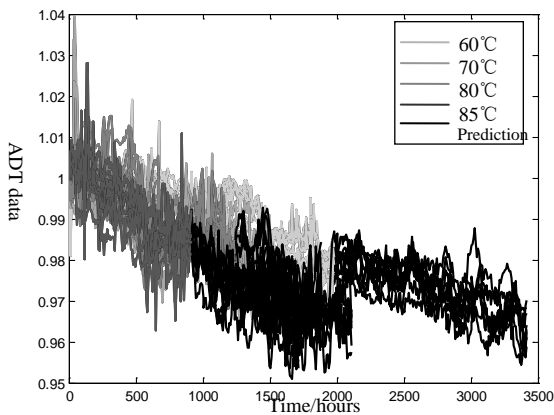


Fig.4 Multidimensional Prediction Of Y_i

The prediction of the same ADT data is also processed based on the one-dimensional time series model respectively for comparison. This is shown in Fig.5.

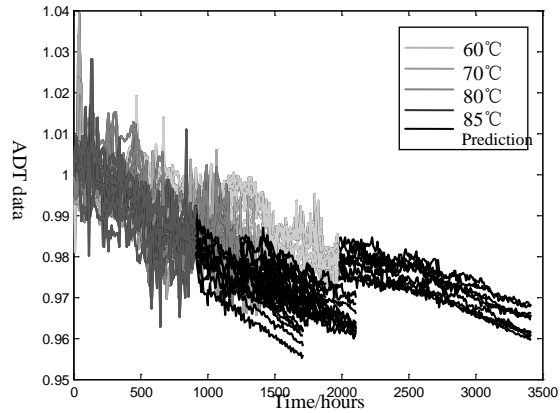


Fig.5 One-Dimensional Prediction Of Y_i

According to Fig. 4 and Fig. 5, it is obvious that compared with the prediction curves, the multidimensional time series prediction of the amplitude Y_i more closely models the original performance than the one dimensional time series method.

Given the failure threshold of each performance parameter as 96% of the initial value, the failure time of each of the performance parameters are shown in Table II.

Table II Failure Time (Hours)

Temperature		60°C	70°C	80°C	85°C
Sys. 1	Perf. 1	3080	>2112	1688	>1712
	Perf. 2	>3416	2072	1688	1672
	Perf. 3	2968	>2112	>1880	1672
Sys. 2	Perf. 1	>3416	1720	1488	1608
	Perf. 2	3184	1792	1608	1328
	Perf. 3	>3416	2040	1808	1632
Sys. 3	Perf. 1	3400	>2112	1736	>1712
	Perf. 2	>3416	1704	1720	>1712
	Perf. 3	>3416	1896	1648	1528

The reliability evaluation of each unit is the minimum failure time prediction across all performance parameters for each unit. The predicted failure time distribution is determined by the Pearson chi-square Goodness of Fit Test. Table III shows the results of the Pearson chi-square test.

Table III Pearson Chi-Square Test Of Failure Time Distribution

Temp.	60°C	70°C	80°C	85°C	Ave.
Lognorm	0.158	1.362	1.362	0.158	0.960
Weibull	0.455	0.833	2.779	0.455	1.356

According to Table III, Lognormal distribution is the best fit for failure time distribution.

The reliability evaluation for test-stress levels is converted into the use-stress level by establishing relationship between median failure time and test-stress levels, which is established assuming an Arrhenius accelerated model. That is

$$\ln \mu = 3454.04 / S - 2.4015$$

Fig.6 shows median failure time and temperature stress level relationship based on a multidimensional time series analysis.

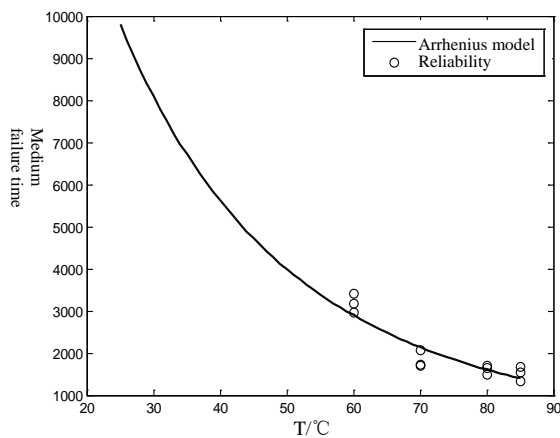


Fig.6 Median Failure Time & Temperature Relationship Based On Multidimensional Time Series

Fig.7 shows median failure time and temperature stress level relationship based on a one-dimensional time series analysis for comparison.

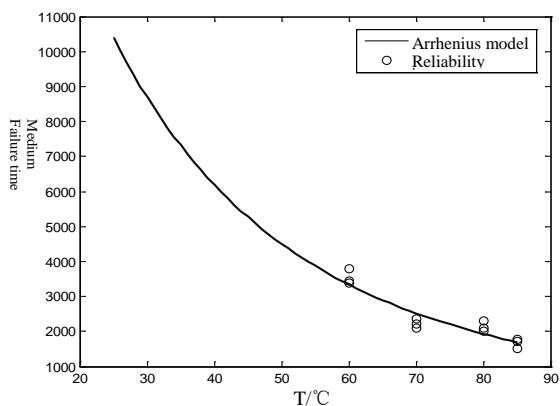


Fig.7 Median Failure Time & Temperature Relationship Based On One-Dimensional Time Series

The error of the prediction is defined as the mean square error for all parameters of each system degradation measurement across all prediction points between the start point and last time point

before system failure. The predicted error is shown in Table IV.

Table IV Error Of Predictions

Model	Multidimensional time series			
	60°C	70°C	80°C	85°C
Sys. 1	0.00114	0.00094	0.00142	0.00120
Sys. 2	0.00078	0.00111	0.00141	0.00116
Sys. 3	0.00132	0.00149	0.00091	0.00105
Model	One-dimensional time series			
	60°C	70°C	80°C	85°C
Sys. 1	0.00231	0.00158	0.00234	0.00193
Sys. 2	0.00273	0.00143	0.00156	0.00251
Sys. 3	0.00156	0.00203	0.00148	0.00253

Fig.8 shows the reliability prediction at the use-stress level 25 °C, based on the multidimensional time series analysis and the one-dimensional time series analysis is shown for comparison.

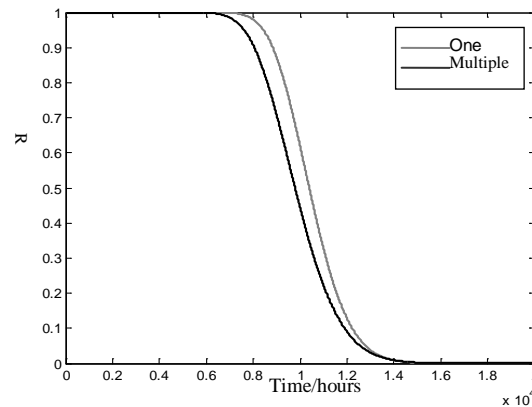


Fig.8 Reliability Prediction By Two Methods

Table V. shows failure time distribution parameters at the use-stress level based on the multidimensional time series analysis with the one-dimensional time series analysis for comparison.

Table V Failure Time Distribution Parameters

Model	Multidimensional time series	One-dimensional time series
Lognormal mean	9.1893	9.2493
Lognormal variance	0.1525	0.1268
Median failure time	9791.8 hours	10397.3 hours

According to Fig.8 and Table V, it could be concluded that reliability evaluations by a multidimensional model are more conservative than one-dimensional model. And according to Table IV, it could be concluded that reliability evaluations by a multidimensional model are more accurate

than one-dimensional model. Hence, the former is more credible than the latter.

6. CONCLUSIONS

This paper proposes a reliability evaluation method utilizing a multiple performance parameter system ADT based on a multidimensional time series modeling procedure.

- (1) Compared with one-dimensional time series analysis, the multidimensional time series analysis takes into account the interaction of multiple performance parameters on the performance degradation process.
- (2) Based on the practice, the failure of systems with multiple performance parameters is assumed to obey a competing failure rule that enables a solution for the failure determination problem given multiple system performance failure measures.
- (3) To obtain a reliability evaluation for systems with multiple performance parameters at the use-stress level, this paper proposed a conversion method based on a reliability evaluation across multiple system performance measures at test-stress level based upon a test acceleration model.
- (4) According to the ADT data verification process compared to a one-dimensional time series analysis, the prediction based on a multidimensional time series analysis of the amplitude observed in ADT more closely model the original curve. Thus any failure time or reliability prediction based on multidimensional time series method was demonstrated to be more conservative, accurate and credible.

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