ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

COMPRESSED SENSING USING ADAPTIVE WAVELET TRANSFORM AND OVERCOMPLETE DICTIONARY

[1]GUANGCHUN GAO, [2]LINA SHANG, [3]KAI XIONG

[1]Assoc Prof., Information Science & Electronic Engineering Dept.Zhejiang University City College
[2]Lecturer, Information Science & Electronic Engineering Dept. Zhejiang University City College
[3]Lecturer, Information Science & Electronic Engineering Dept. Zhejiang University City College
E-mail: [1]seesky88@126.com, [2]shangln@zucc.edu.cn , [3]xiongk@zucc.edu.cn

ABSTRACT

In this paper, we present a new compressed sensing implementation process for one dimension signal reconstruction. Firstly, one level wavelet decomposition of the one dimensional signal was finished. For using the adaptive wavelet transform based on lifting wavelet transforms, we can achieve the detail signals being zero (or almost zero) at big probability, so the signal has the better linear approximation. Secondly, the signal can be reconstructed using compressed sensing method. Because the length of the low frequency coefficients is half of the original signal length, the measurement matrix can be reduced. The redundancy of overcomplete dictionary can make it effectively capture the characteristics of the signals. The overcomplete dictionary which combined the DCT base with the unit matrix can be used for the compressed sensing. Thirdly, using the inverse adaptive wavelet transform, the signal can be reconstructed with the low frequency coefficients. Finally experimental results demonstrate the application effectiveness for this scheme in compressed sensing fields.

Keywords: *Compressed Sensing, Overcomplete Dictionary, Wavelet Transform, Measurement Matrix*

1. INTRODUCTION

A theory called compressed sensing (CS) was proposed by Candes, Tao and Romberg [1]-[3], and Donoho[4]. The theory showed that a signal having a sparse or compressible representation in one basis can be recovered from projections onto a small set of measurement vectors that are incoherent with the sparsity basis. The number of measurements is smaller in compressed sensing, the computational complex is lower. The measurement matrix size usually was decided by the length of the signal, so we hope that the signal can be shorten with some methods when the signal precision meet the demand of some application. It is well known that wavelet linear approximation (i.e. truncating the high frequencies) can approximate smooth functions very efficiently: it can achieve arbitrary high accuracy by selecting appropriate wavelet basis, it can concentrate the large wavelet coefficients in the low frequencies, and it has a multiresolution framework and associated fast transform algorithms. Considering the signal approximation reconstruction in some applications, adaptive wavelet transform based on lifting scheme was used in this paper. After finishing the one level wavelet decomposition, the length of the signal can be shorten the half of original signal length. Because the adaptive wavelet transform have better linear approximation, the high frequency coefficients can be discarded in some application. Then signal can be recovered with the low frequency coefficients. At the same time, measurement matrix size can be reduced for only using the low frequency coefficients in compressed sensing.

The signal sparse representation is the fundamental premise of CS implementation, so the optimal sparse bases of the signals were widely researched [5]-[9].H. Rauhut et al. investigated the recovery performance of a signal, which is sparse with a redundant dictionary, from CS observation [7]. However, in most cases, it is hard to find a general complete basis dictionary that supplies the sparsest representation for any signal, and it also not easy to find such an overcomplete dictionary. In this paper, the overcomplete dictionary which combined the DCT base with the unit matrix can be used for the compressed sensing in this paper.

According to above analyses, for reducing the computational complex, we proposed the new compressed sensing implementation processes of one dimension signal reconstruction. Firstly, one level wavelet decomposition of the one

Journal of Theoretical and Applied Information Technology
10th February 2013. Vol. 48 No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

dimensional signal was finished. Secondly, the low frequency coefficients were reconstructed using compressed sensing. Finally, using the inverse adaptive wavelet transform, the signal was reconstructed with the low frequency coefficients.

2. COMPRESSED SENSING

During last three decades, the assessment of potential of the sustainable eco-friendly alternative sources and refinement in technology has taken place to a stage so that economical and reliable power can be produced.

The signal reconstruction system based compressed sensing was showed in Figure 1.For a signal X of size N , the nth element of the signal vector is referred to as $X(n)$, where $n=1,\dots,N$. Let us assume that the basis $\Psi=[\psi_1,\dots,\psi_N]$ provides a K-sparse representation of x:

$$x=\Psi\theta \quad (1)$$

Where θ is a $N\times 1$ vector with K-nonzero elements. Many different basis expansions can achieve sparse approximations of the signal, including DCT, wavelets, and Gabor frames. In other words, a signal does not result in exactly K-sparse representation; instead its transform coefficients decay exponentially to zero.

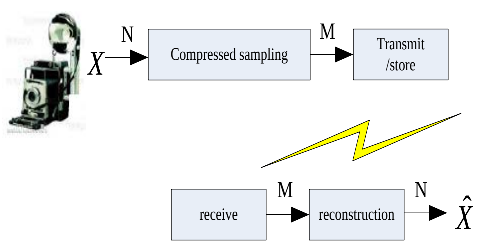

Figure 1: Signal Reconstruction Based On Compressed Sensing

In the CS framework, it is assumed that the K-largest $\theta(n)$ is not measured directly. Rather, $M<N$ linear projections of the signal vector x onto another set of vectors $\Phi=[\phi_1^{'}\dots,\phi_M^{'}]$ are measured:

$$y=\Phi x=\Phi\Psi\theta \quad (2)$$

Where the vector y ($M\times 1$) constitutes the compressive samples and matrix $\Phi(M\times N)$ is called the measurement matrix. Since $M \prec N$, recovery of the signal x from the compressive samples y is underdetermined; however, the additional sparsity assumption makes recovery possible.

The sparsity can be used to recover a signal that is a solution of the following minimization

$$\arg\min\|\theta\|_0 \text{ subject to } \Phi\Psi\theta=y \quad (3)$$

The minimization (3) is however combinatorial and thus intractable. It is relax by using ℓ^1 norm $\|\theta\|_1$ of the coefficients of x in Ψ. The recovered signal x^* is a solution of the following convex problem

$$x^*\in\arg\min\|\theta\|_1 \text{ subject to } \Phi\Psi\theta=y \quad (4)$$

This optimization problem, also known as Basis Pursuit, can be efficiently solved using polynominal time algorithms. The CS literature has mostly focused on problems involving single sensors, one dimensional (1-D) signals, or 2-D images. Until now the theory of compressed sensing has only been developed for classes of signals that have a very sparse representation in an orthogonal basis (ONB). This is a rather stringent restriction. Indeed, allowing the signal to be sparse with respect to a redundant dictionary adds a lot of flexibility and significantly extends the range of applicability.

3. ADAPTIVE WAVELET TRANSFORM

Compressed sensing includes three parts: sparse representation, measurement, reconstruction. If the signal length is bigger, the measurement matrix size is bigger, so the computational complex of the signal reconstruction is higher.

Journal of Theoretical and Applied Information Technology
10th February 2013. Vol. 48 No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

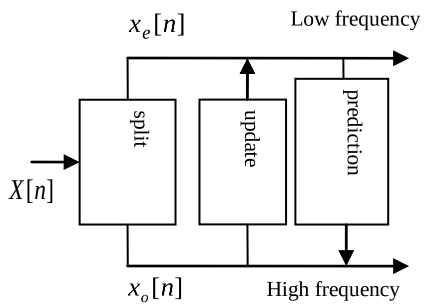

Figure 2: Update-Then-Prediction Scheme

For reducing the size of the measurement matrix in compressed sensing, we hope that the signal length can be shortened. It is well known that the wavelet transforms can concentrate the large wavelet coefficients in the low frequencies of the signal, so when the precision meets the demand of application, the high frequency coefficient can be discarded in signal reconstruction processes, and then the signal length in compressed sensing is smaller.

Wavelet transforms based on lifting schemes [10] have achieved large recognition in the last years. In general, lifting splits a signal into two sub samples, followed by at least two lifting steps, Prediction and Update. A general lifting scheme may comprise any sequence of basic lifting steps being alternatively of prediction and update type. For the adaptive wavelet transform based on lifting scheme, the wavelet transform framework (Figure 2) by first updating and then predicting has been presented in [11], so the update-then-predict lifting scheme has been adopted in this paper. In order to implement adaptive prediction algorithm, there are two crucial points in designing wavelet transform scheme: The first is to detect the jumps in the signal. The second is how to use one-sided data near jumps to avoid oscillations. Assuming that β_{2i+1} □s the jump (predicted point), α_{2i} is its left side data (updated data) and α_{2i+2} is its right side data (updated data). The adaptive prediction algorithm consists of the following steps [12]:

For each index i:
(1) Calculate the linear error e_{2i} sequence of the update data α_{2i} sequence from

$$e_{2i} = \alpha_{2i} - (\alpha_{2i-2} + \alpha_{2i+2})/2$$

(2) For the e_{2i} sequence, the multiplying value of the two adjacent numbers is calculated, that is the value of $e_{2i} \times e_{2i+2}$.

If this value is negative, β_{2i+1} are the jumps,

Then the next step will be performed.

Else

$\beta'_{2i+1} = \beta_{2i+1} - (\alpha_{2i} + \alpha_{2i+2})/2$, we get the high frequency coefficient sequence.

(3) If the α_{2i} is the jump of the updated data, the value of $|e_{2i-2}|$ is larger. Otherwise, the value of $|e_{2i+4}|$ is larger. Comparing with the value between $|e_{2i-2}|$ and $|e_{2i+4}|$, the prediction algorithm using the left side data or the right side data of the jump can be determined.

If $|e_{2i-2}| > |e_{2i+4}|$ then

$$\beta'_{2i+1} = \beta_{2i+1} - \alpha_{2i}$$

Else If $|e_{2i-2}| > |e_{2i+4}|$ then

$$\beta'_{2i+1} = \beta_{2i+1} - \alpha_{2i+2}$$

Else

$$\beta'_{2i+1} = \beta_{2i+1} - (\alpha_{2i} + \alpha_{2i+2})/2$$

Through the previous discussion, we know that the adaptive prediction algorithm can be reconstructed without using extra additional information. The inverse transform algorithm is the inverse process of the forward wavelet transform.

4. OVERCOMPLETE DICTIONARY

Most work on compressed sensing so far assumes sparsity with respect to the canonical basis, or at least with respect to an orthogonal basis. However, this can be a rather stringent restriction in practice. A recent direction of interest in compressed sensing concerns problems where signals are sparse in an over-complete dictionary D instead of a basis.

Let $D \in \Re^{N \times K}, K \geq N$, be a over-complete dictionary, i.e., its columns span $\Re^N$. A vector x is said to be K-sparse with respect to $x = D\theta$ for a K-sparse $\theta \in \Re^K$. Given a suitable measurement matrix $\Phi \in \Re^{n \times N}$ our task is then to reconstruct x from $y = \Phi x$. This is accomplished if the coefficients x are recovered. Writing $\Psi = \Phi D \in \Re^{n \times N}$ we have

$$y = \Phi x = \Phi D\theta = \Psi\theta \quad (5)$$

Journal of Theoretical and Applied Information Technology
10th February 2013. Vol. 48 No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

An important observation on dictionaries is that given an overcomplete $D \in \Re^{N \times K}$ with $K \succ N$ and a signal x of length N, representations of signal x in D in terms of linear combinations of some atoms are not unique. This is to say that if D contains more than K nonzero atoms and if D is of full row rank, then the underdetermined system of linear equations $D\theta = x$ admits infinitely many solutions. Among these representations of x, we are particularly interested in finding the most economical one θ^*, that is the sparsest[13]. However, in most cases, it is hard to find a general complete basis dictionary that supplies the sparsest representation for any signal, and it also not easy to find such an over complete dictionary. The base using orthogonal transforms for 1-D and 2-D signals main include Wavelets, DCT and FFT matrix. Because overcomplete dictionary can acquire the sparser signal presentation, and can be used for the compressed sensing systems. In this section, through combing transform bases and units matrix, we design the implicit overcomplete dictionaries. In order to acquire signal sparser presents, the different bases must be inconherent in the compressed sensing system.

The coherence, which is also known as mutual coherence in the literature, between Λ and Θ in dictionary is defined as

$$\mu(\Lambda, \Theta) = \sqrt{n} \max_{1 \le k, j \le n} \left| \lambda_k \vartheta_j \right|. \tag{6}$$

In this paper, we design the followed overcomplete dictionaries:
$\mathbf{D} = \left[\mathbf{I}_n, \mathbf{C}_n^T \right]$, where I_n is the identity matrix of size $n \times n$, C_n is the 1-D DCT matrix of size n*n, and $\mu(I_n, F_n^H) = 1$.

For D overcomplete dictionaries, the signal sparse representations and the application for the compressed sensing system will be discussed in next section.

5. EXPERIMENT RESULTS

Next， we consider a piecewise smooth function defined by

$$f(x) = \begin{cases} 0 & 0 <= x < 0.2 \\ -50x - 5 & 0.2 <= x < 0.4 \\ 10\sin(4\pi x + 0.8\pi) - 1 & 0.4 <= x < 1.1 \\ 5e^{2x} - 100 & 1.1 <= x < 1.6 \\ 0 & 1.6 <= x <= 2.0 \end{cases}$$

Figure 3 shows the function $f(x)$. The length of the signal $f(x)$ is 512. In order to study on the performance of the optimal adaptive wavelet transform, using the adaptive wavelet transform scheme, one level wavelet decomposition for the f(x) is shown in Figure 4. The left part corresponds to the low frequency coefficients and the right part the high frequency coefficients. We notice that there are not some large high frequency coefficients. This illustrates that the adaptive wavelet coefficients have better distribution, i.e., no large coefficients in the high frequencies and the energy is concentrated in the low frequency. Therefore, we can reconstruct the signal by the low frequency coefficients.

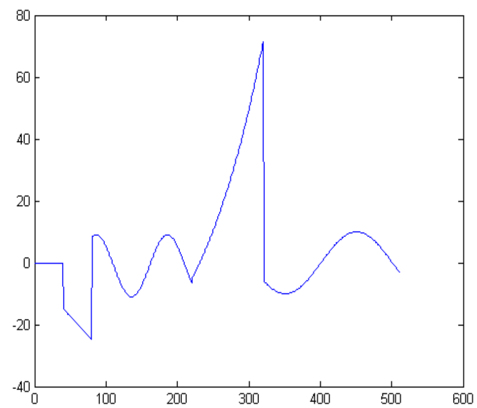

Figure 3: The Original Signal

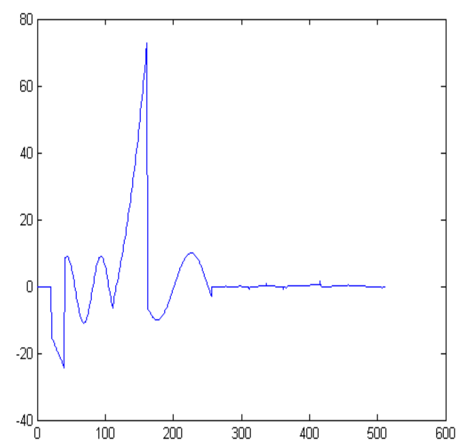

Figure 4: One Level Adaptive Wavelet Decomposition Of The Original Signal

10th February 2013. Vol. 48 No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

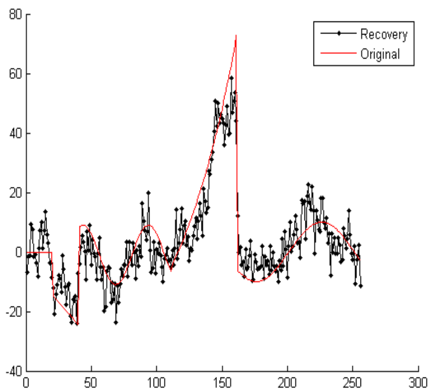

Figure 5: Reconstruction Signal(M=120, DCT)

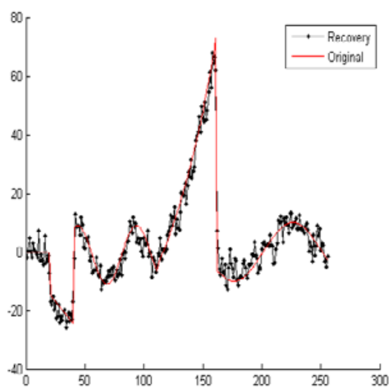

Figure 6: Reconstruction Signal(M=180, DCT)

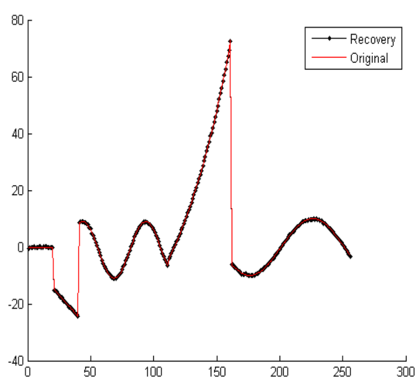

Figure 7: Reconstruction Signal (M=180, Overcomplete Dictionary)

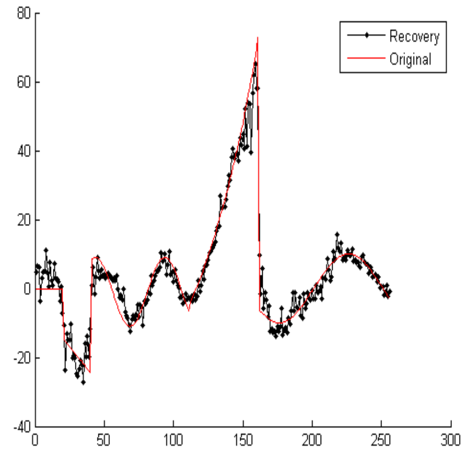

Figure 8: Reconstruction Signal (M=120, Overcomplete Dictionary)

For the low-frequency coefficients, the signal can be reconstructed using compressed sensing. Random matrix in compressed sensing was chosen as Measurement matrix. When M is 120 and 180 in the measurement matrix, and DCT was chosen as the sparse matrix, the low frequency coefficients were reconstructed. The reconstructed results were shown in Figure 5 and Figure 6.

For studying the performance of the overcomplete dictionaries, the overcomplete dictionary $D = \left[I_n, F_n^T \right]$ were chosen as the sparse transform matrix. When measurement value M=120 and M=180. The reconstruction signal was shown Figure 7 and Figure 8. The CVX software was used in the above signal reconstruction processes. In order to objectively evaluate the quality of reconstructed signals, the error can be calculated by the following equation.

$$Error = \sqrt{\sum_{i=1}^{N} (\tilde{x}_i - x_i)^2} \Big/ \sqrt{\sum_{i=1}^{N} \tilde{x}_i^2} \tag{7}$$

The reconstruction signal errors using the different sparse basis and the measurement value were listed in Table 1. From the table 1 and the reconstruction signal using the different sparse bases, we can know that the reconstruct signal using the overcomplete dictionary can acquired better signal approximation.

Journal of Theoretical and Applied Information Technology
10th February 2013. Vol. 48 No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

Finally, when the measurement value is 180, according to the low frequency coefficients reconstructed in compressed sensing, the signal can be reconstructed using the inverse adaptive wavelet transform. The reconstruction signal can be shown in Figure 9.

The simulation results demonstrated that the compressed sensing scheme proposed this paper can acquire better reconstruction effectiveness, and this compressed sensing scheme meet the demand of the signal reconstruction.

6. CONCLUSION

We know today that most of existing works in CS remain at the theoretical study. In particular, the implementation processes in compressed sensing have high computational complex. This paper presented new compressed sensing scheme which can reduce computational complex. This paper proposed the signal reconstruction scheme using adaptive wavelet transform and DCT. Taking advantage of the adaptive wavelet transform, the reconstruction signal length can be shortened. When the redundant dictionary was chosen as the sparse transform basis, the signal acquired better reconstruction precision.

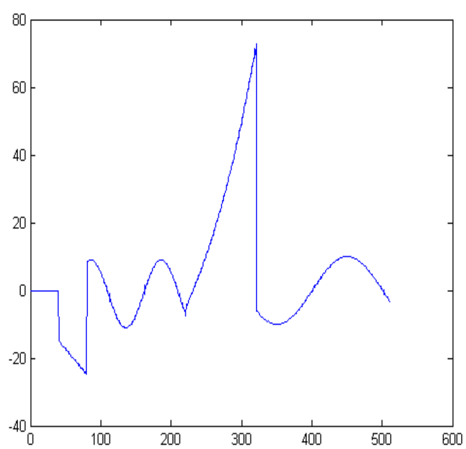

Figure 9: The Reconstruction Signal

Table 1: The Signal Reconstruction Errors

measurement value M	sparse basis	Reconstruction error
M=120	DCT	0.4369
M=180	DCT	0.2170
M=120	Overcomplete $D=[I_n, F_n^T]$	0.2343
M=180	Overcomplete $D=[I_n, F_n^T]$	0.0091

ACKNOWLEDGEMENTS

This work was supported by Zhejiang Provincial Natural Science Foundation of China (No.Y1110632) and (LY12F01017), supported by Scientific Research Fund of Zhejiang Provincial Education Department (Y201223096), supported by Teacher Scientific Research Fund (J-12006).

REFRENCES:

[1] Candes E., and TaoT., "Decoding by linear programming", *IEEE Trans. Inform. Theory*, Vol. 51, No.12, 2005, pp.4203-4215.

[2] Candes E., Romberg J., and Tao T., "Stable signal recovery from incomplete and inaccurate information", *Commun. Pure Appl. Math.*, Vol. 59, No. 9, 2005, pp.1207-1233.

[3] Candes E., Romberg J., and Tao T., "Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information", *IEEE Trans. Inform. Theory*, Vol.52, No.2, 2006, pp.489-590.

[4] Donoho D., "Compressed sensing", *IEEE Trans. Inform. Theory*, Vol.52, No.4, 2006, pp.1289-1306.

[5] Gabriel Peyr´e, "Best Basis Compressed Sensing", *IEEE Transaction on Signal Processing*, Vol.58, No.1, 2010, pp.1-11.

[6] candes E., Donoho D., "Curvelets-A surprising effective nonadaptive representation for objects with edges". *Curves and Surfaces*, Nashvielle: Vanderbilt University Press, 2000, pp.105-120.

[7] Rauhut H., Schnass K., and Vandergheynst P., "Compressed sensing and redundant dictionaries", *IEEE Trans. Info. Theory*, Vol.54, No.5, 2008, pp.2210–2219.

[8] M. F. Duarte, M. B. Wakin and R. G. Baraniuk, "Wavelet-domain compressive signal reconstruction using a Hidden Markov Tree model", *IEEE International Conference on Acoustics, Speech and Signal Processing*, (Las Vegas), March 30-April 4, 2008, pp.5137-5140.

[9] C. La and M. Do, "Signal reconstruction using sparse tree representations". *Proc. SPIE Wavelets XI*. August 30, 2005,pp.273-283.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

[10] W. Sweldens, "The lifting scheme: a construction of second generation wavelets", *SIAM J. Math. Anal.*, Vol.29, No.2, 1997, pp. 511-546.

[11] R. Claypoole, R. Baraniuk, and R. Nowak, "Adaptive wavelet transforms via lifting". *Proceedings of the IEEE International Conference on Acoustics, Speech,and Signal Processing*, (Seattle, Washington), May 12-15, 1998, pp.1513-1516.

[12] Gao,Guang-chun, Zhao. Sheng-ying, Zhu.Hong-li, Shang. Li-na, "Based on lifting scheme adaptive prediction wavelet transform algorithms", *Journal of Circuits and Systems*, Vol.15, No.5, 2010, pp.31-34.(in Chinese)

[13] Wu-Sheng Lu, "Compressed sensing and Sparse Signal processing", University of Victoria, Canada, November, 2010.