Journal of Theoretical and Applied Information Technology
10th February 2013. Vol. 48 No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

AN INVENTORY MODEL FOR NON-INSTANTANEOUS DETERIORATING ITEMS UNDER TWO LEVELS OF TRADE CREDIT AND TIME VALUE OF MONEY

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ABSTRACT

In this study, in order to investigate the influence of the time value of money strategy, an inventory system for non-instantaneous deteriorating items under two-level trade credit is considered by using the discounted cash-flows (DCF) approach. The purpose of this paper is to find the optimal replenishment policies for minimizing the total present value of all future cash-flow cost for the retailer. Useful theorems to characterize the optimal solutions have been derived. Numerical examples are provided to demonstrate the results.

Keywords: *Two Levels Of Trade Credit; Non-Instantaneous Deteriorating Items; Time Value Of Money; Inventory Model*

1. INTRODUCTION

In the real markets, to motivate retailer to increase their order qualities, the supplier often provides a permissible delay in payment to the retailers. Furthermore, the retailers may offer their customers a permissible delay period when they received a trade credit by the supplier, that is two-level trade credit. Goyal [1] was the first to explore an EOQ model under permissible delay in payment. Huang [2] extended [1] to develop an EOQ model under the two-level trade credit. Chung [3] discussed an inventory model that trade credit depended on the ordering quantity. Mahata [4] investigated an EPQ model with deteriorating items, which assumed that the retailer offered the partial trade credit to his/her customers when he/she received the full trade credit by the supplier. Liao [5] established an EOQ model for deteriorating items with two-storage facilities where trade credit was linked to order quantity.

The inventory models above ignored the effects of the time value of money. In practice, all cash outflows have different values at different points of time. Therefore, it is necessary to take the time value of money on the inventory policy into consideration. Chung [6] analyzed an EOQ model for deteriorating items and talked about the time value of money; Chung [7] discussed the effect of trade credit depending on the order quantity by using the DCF approach; Liao [8] extended the inventory model by considering the factors of two levels of trade credit, deterioration and time discounting.

In the above deteriorating items inventory literatures assumed that the deterioration of the items in inventory starts from the instant of their arrival in stock. In fact, most goods would have a span of maintaining quality before they deteriorated. Ouyang [9] established an inventory model for non-instantaneous deteriorating items with permissible delay in payments.

In this paper, an inventory system for non-instantaneous deteriorating item is investigated under two levels of trade credit and time value of money. The method of finding the optimal cycle time is proposed. Sensitivity analysis of major parameters is given to obtain some managerial insights.[1]

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Journal of Theoretical and Applied Information Technology
10th February 2013. Vol. 48 No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

2. NOTATIONS AND ASSUMPTIONS

2.1 Notations

The following notations are used throughout this paper.

A	the ordering cost one order;
c	unit purchasing cost per item;
p	unit selling price per item $p > c$;
h	holding cost per unit time excluding int erest charges;
r	the continuous rate of discount;
D	demand rate per year;
Q	the retailer' order quantity per cycle;
T	the cycle time;
$PV_{\infty}(T)$	the present value of all future cash-flow cost.

2.2 Assumptions

The mathematical model in this paper is developed under the following assumptions

(1) Time horizon is infinite, and the lead time is negligible;

(2) Replenishments are instantaneous, and shortage is not allowed;

(3) T_{θ} is the length of time during which the product has no deterioration. If $T \le T_{\theta}$, then the product is no deterioration; else, a constant θ ($0<\theta<1$) fraction of the on-hand inventory deteriorates per unit of time and there is no repair or replacement of the deteriorated inventory;

(4) When $T \ge M$, the account is settled at $t = M$ and the retailer would pay for the interest charges on items in stock with rate I_p (per \$ per year) during the interval $[M, T]$; when $T < M$, the account is also settled at $t = M$ and the retailer does not need to pay any interest charge of items in stock during the whole cycle;

(5) The retailer can accumulate revenue and earn interest after his/her customer paying for the amount of purchasing cost to the retailer till the end of the trade credit period offered by the supplier. That is, the retailer can accumulate revenue and earn interest during the period N to M with rate I_e (per \$ per year) under the condition of trade credit; meanwhile the fixed credit period offered by the supplier to the retailer is not less to his/her customers, i.e. $0 \le N \le M$.

(7) For $T > T_{\theta}$, $I_1(t)$ denotes the level of inventory at time $t \in (0, T_{\theta})$; $I_2(t)$ denotes the inventory level at time $t \in [T_{\theta}, T]$; $I(t)$ denotes the level of inventory at time $t \in (0, T]$.

(8) By convenience, we Let

$$rce^{-rM} + h - pI_e\left(e^{-rN} - e^{-rM}\right) \ge 0 .$$

3 . MATHEMATICAL MODEL

Based on the assumptions above, when $0 < T \le T_{\theta}$, the items are depleted in the interval $(0, T]$ due to the effect of demand. At time $t = T$, the inventory level reaches zero. Thus, the inventory level can be described by the following differential equation

$$I^{'}(t) = -D,\ 0 \le t \le T \text{ and } T < T_{\theta},\ I(T) = 0 .$$

The solution to the equation is

$$I(t) = D(T - t),\ 0 \le t \le T \text{ and } T < T_{\theta} .$$

In this case, the retailer's order size per bicycle

$$Q = I(0) = DT .$$

When $T > T_{\theta}$, if $t \in (0, T_{\theta}]$, then the level of inventory will decrease only owing to demand. Thus, the inventory level, $I_1(t)$, is given by the differential equation

$$I_1^{'}(t) = -D,\ 0 \le t \le T_{\theta},\ I_1(0) = Q .$$

The solution to the equation is

$$I_1(t) = Q - Dt \quad (1)$$

At the interval $t \in (T_{\theta}, T]$, the inventory level depletes to zero due to the effect of demand as well as deterioration. Hence, the inventory level, $I_2(t)$, is given by

$$I_2^{'}(t) = -D - \theta I_2(T),\ T_{\theta} \le t \le T,\ I_2(T) = 0 . \quad (2)$$

The solution to the equation is

$$I_2(t) = D\left[e^{\theta(T-t)} - 1\right]\Big/\theta,\ T_{\theta} \le t \le T . \quad (3)$$

Considering the continuity of $I(t)$ at time $t = T_{\theta}$, i.e.

$$I_1(T_{\theta}) = I_2(T_{\theta}) .$$

Which implies that

$$Q = DT_{\theta} + D\left[e^{\theta(T-T_{\theta})} - 1\right]\Big/\theta .$$

Thus, we obtain

$$I_1(T) = D(T_{\theta} - t) + D\left[e^{\theta(T-T_{\theta})} - 1\right]\Big/\theta .$$

$PV_{\infty}(T)$ consists of the following elements

(1) The present value of order cost $V_O = A\big/\left(1 - e^{-rT}\right)$;

(2) The present value of holding cost excluding interest charges.

When $0 < T \le T_{\theta}$,

$$V_H = hD\left(e^{-rT} + rT - 1\right)\Big/\left[r^2\left(1 - e^{-rT}\right)\right];$$

When $T > T_{\theta}$,

Journal of Theoretical and Applied Information Technology
10th February 2013. Vol. 48 No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

$$V_H = \frac{hD}{r\left(1-e^{-rT}\right)}\left[\frac{1}{r}\left(e^{-rT_\theta}+rT_\theta-1\right)-\frac{1}{\theta}+\frac{e^{\theta(T-T_\theta)}}{\theta}\left(1-\frac{\theta e^{-rT_\theta}}{\theta+r}\right)+\frac{e^{-rT}}{\theta+r}\right];$$

(3) The present value of purchasing cost.
When $0<T\le T_\theta$,

$$V_C = cDTe^{-rM}\Big/\left(1-e^{-rT}\right);$$

When $T>T_\theta$,

$$V_C = cDe^{-rM}\left[\theta T_\theta + e^{\theta(T-T_\theta)}-1\right]\Big/\left[\theta\left(1-e^{-rT}\right)\right].$$

(4) The present values of interest charged.
When $M\le T\le T_\theta$,

$$V_{IP} = cI_pD\left[e^{-rM}\left(rT-rM-1\right)+e^{-rT}\right]\Big/\left[r^2\left(1-e^{-rT}\right)\right];$$

When $M\le T_\theta\le T$,

$$V_{IP} = \frac{cI_pD}{r\left(1-e^{-rT}\right)}\left\{\frac{1}{r}\left[e^{-rM}\left(rT_\theta-rM-1\right)+e^{-rT_\theta}\right]-\frac{1}{\theta}e^{-rM}+e^{\theta(T-T_\theta)}\left(\frac{e^{-rM}}{\theta}-\frac{e^{-rT_\theta}}{\theta+r}\right)+\frac{e^{-rT}}{\theta+r}\right\};$$

When $T_\theta\le M\le T$,

$$V_{IP} = \frac{cI_pD}{r\theta(\theta+r)\left(1-e^{-rT}\right)}\left[re^{\theta T-(r+\theta)M}+\theta e^{-rT}-(\theta+r)e^{-rM}\right].$$

(5) The present values of interest earned.
When $0<T\le N$,

$$V_{IE} = pI_eDT\left(e^{-rN}-e^{-rM}\right)\Big/\left[r\left(1-e^{-rT}\right)\right];$$

When $N\le T\le M$,

$$V_{IE} = \frac{pI_eD}{r^2\left(1-e^{-rT}\right)}\left[(rN+1)e^{-rN}-rTe^{-rM}-e^{-rT}\right];$$

When $M\le T$,

$$V_{IE} = \frac{pI_eD}{r^2\left(1-e^{-rT}\right)}\left[(rN+1)e^{-rN}-(rM+1)e^{-rM}\right].$$

From the arguments above, $PV_\infty(T)$ can be expressed as

$$PV_\infty(T) = V_O+V_H+V_C+V_{IP}-V_{IE}.$$

Three situations can be set as

$$0<T_\theta\le N;\ N\le T_\theta\le M;\ M\le T_\theta.$$

When $0<T_\theta\le N$, $PV_\infty(T)$ is given by

$$PV_\infty(T)=\begin{cases}PV_{11}(T), & 0<T\le T_\theta,\\ PV_{12}(T), & T_\theta\le T\le N,\\ PV_{13}(T), & N\le T\le M,\\ PV_{14}(T), & M\le T,\end{cases}$$

where

$$PV_{11}(T)=\frac{1}{1-e^{-rT}}\left\{A+\frac{hD}{r^2}\left(e^{-rT}-1\right)+\frac{DT}{r}\left[rce^{-rM}+h-pI_e\left(e^{-rN}-e^{-rM}\right)\right]\right\};$$

$$PV_{12}(T)=\frac{1}{1-e^{-rT}}\left[E+\frac{DF}{\theta}e^{\theta(T-T_\theta)}+\frac{hDe^{-rT}}{r(\theta+r)}-\frac{D}{r}pI_eT\left(e^{-rN}-e^{-rM}\right)\right];$$

$$PV_{13}(T)=\frac{1}{1-e^{-rT}}\left\{E+\frac{DF}{\theta}e^{\theta(T-T_\theta)}+\frac{hDe^{-rT}}{r(\theta+r)}-\frac{pI_eD}{r^2}\left[(rN+1)e^{-rN}-rTe^{-rM}-e^{-rT}\right]\right\};$$

$$PV_{14}(T)=\frac{1}{1-e^{-rT}}\left\{E+\frac{D}{\theta}Fe^{\theta(T-T_\theta)}+\frac{hDe^{-rT}}{r(\theta+r)}+\frac{cI_pD}{\theta}\left[\frac{1}{\theta+r}e^{\theta T-(r+\theta)M}+\frac{\theta e^{-rT}}{r(\theta+r)}-\frac{e^{-rM}}{r}\right]-\frac{pI_eD}{r^2}\left[(rN+1)e^{-rN}-(rM+1)e^{-rM}\right]\right\}.$$

$$E=A+\frac{c}{\theta}De^{-rM}\left(\theta T_\theta-1\right)+\frac{h}{r^2\theta}D\left[\theta\left(e^{-rT_\theta}+rT_\theta-1\right)-r\right];$$

$$F=ce^{-rM}+h\left[(\theta+r)-\theta e^{-rT_\theta}\right]\Big/\left[r(\theta+r)\right].$$

Since $PV_{11}(T_\theta)=PV_{12}(T_\theta)$, $PV_{12}(N)=PV_{13}(N)$ and $PV_{13}(M)=PV_{14}(M)$, moreover $PV_\infty(T)$ is continuous and well-defined.

When $N\le T_\theta\le M$, $PV_\infty(T)$ is given by

$$PV_\infty(T)=\begin{cases}PV_{21}(T), & 0<T\le N,\\ PV_{22}(T), & N\le T\le T_\theta,\\ PV_{23}(T), & T_\theta\le T\le M,\\ PV_{24}(T), & M\le T,\end{cases}$$

where

$$PV_{21}(T)=PV_{11}(T);\ PV_{23}(T)=PV_{13}(T);$$

$$PV_{24}(T)=PV_{14}(T);$$

$$PV_{22}(T)=\frac{1}{1-e^{-rT}}\left[A-\frac{hD}{r^2}-\frac{pI_eD}{r^2}(rN+1)e^{-rN}+\frac{1}{r}DT\left(rce^{-rM}+h+pI_ee^{-rM}\right)+\frac{D}{r^2}e^{-rT}\left(h+pI_e\right)\right].$$

Since $PV_{21}(N)=PV_{22}(N)$, $PV_{22}(T_\theta)=PV_{23}(T_\theta)$ and $PV_{23}(M)=PV_{24}(M)$, and $PV_\infty(T)$ is continuous and well-defined.

When $M\le T_\theta$, $PV_\infty(T)$ is given by

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

$$PV_\infty(T)=\begin{cases}PV_{31}(T), & 0<T\le N,\\ PV_{32}(T), & N\le T\le M,\\ PV_{33}(T), & M\le T\le T_\theta,\\ PV_{34}(T), & T_\theta\le T,\end{cases}$$

where

$PV_{31}(T)=PV_{11}(T);\ PV_{32}(T)=PV_{22}(T);$

$$PV_{33}(T)=\frac{1}{1-e^{-rT}}\left\{A-\frac{D}{r^2}\left[h+cI_p(rM+1)\right]+\frac{1}{r}DT\left(rce^{-rM}+h+cI_pe^{-rM}\right)+\frac{1}{r^2}\left(h+cI_p\right)e^{-rT}-\frac{pI_eD}{r^2}\left[(rN+1)e^{-rN}-(rM+1)e^{-rM}\right]\right\};$$

$$PV_{34}(T)=\frac{1}{1-e^{-rT}}\left\{E+\frac{D}{\theta}Fe^{\theta(T-T_\theta)}+\frac{hDe^{-rT}}{r(\theta+r)}+\frac{cI_pD}{r^2\theta}\left[\theta e^{-rM}(rT_\theta-rM-1)+\theta e^{-rT_\theta}-re^{-rM}\right]+\frac{cI_p}{r}De^{\theta(T-T_\theta)}\left(\frac{1}{\theta}e^{-rM}-\frac{1}{\theta+r}e^{-rT_\theta}\right)+\frac{cI_pD}{r(\theta+r)}e^{-rT}-\frac{pI_eD}{r^2}\left[(rN+1)e^{-rN}-(rM+1)e^{-rM}\right]\right\}.$$

Since $PV_{31}(N)=PV_{32}(N)$, $PV_{32}(M)=PV_{33}(M)$ and $PV_{33}(T_\theta)=PV_{34}(T_\theta)$, and $PV_\infty(T)$ is continuous and well-defined.

4. THEORETICAL RESULTS

Lemma 1. Let x^* denote the minimizing value of $F(x)$.

If $f(x)$ is continuous and increasing on $[a,b]$, and $F'(x)=f(x)e^{-rx}/(1-e^{-rx})^2$.

(a) if $f(a)\ge 0$, then $x^*=a$; (b) if $f(a)<0<f(b)$, then $x^*=x_0$, where x_0 is the unique solution of $f(x)=0$ on $[a,b]$; (c) if $f(b)\le 0$, then $x^*=b$.

Theorem 1. When $0<T_\theta\le N$, the optimal cycle time T^* will be determined by the following steps.

(a) if $f_{11}(T_\theta)\ge 0$, $f_{12}(N)\ge 0$ and $f_{13}(M)\ge 0$, then $T^*=T_{11}^*$; (b) if $f_{11}(T_\theta)<0$, $f_{12}(N)\ge 0$ and $f_{13}(M)\ge 0$, then $T^*=T_{12}^*$; (c) if $f_{11}(T_\theta)<0$, $f_{12}(N)<0$ and $f_{13}(M)\ge 0$, then $T^*=T_{13}^*$; (d) if $f_{11}(T_\theta)<0$, $f_{12}(N)<0$ and $f_{13}(M)<0$, then $T^*=T_{14}^*$.

Theorem 2. When $N\le T_\theta\le M$, the optimal cycle time T^* will be determined by the following steps.

(a) if $f_{21}(N)\ge 0$, $f_{22}(T_\theta)\ge 0$ and $f_{23}(M)\ge 0$, then $T^*=T_{21}^*$; (b) if $f_{21}(N)<0$, $f_{22}(T_\theta)\ge 0$ and $f_{23}(M)\ge 0$, then $T^*=T_{22}^*$; (c) if $f_{21}(N)<0$, $f_{22}(T_\theta)<0$ and $f_{23}(M)\ge 0$, then $T^*=T_{23}^*$; (d) if $f_{21}(N)<0$, $f_{22}(T_\theta)<0$ and $f_{23}(M)<0$, then $T^*=T_{24}^*$.

Theorem 3. When $M<T_\theta$, the optimal cycle time T^* will be determined by the following steps.

(a) if $f_{31}(N)\ge 0$, $f_{32}(M)\ge 0$ and $f_{33}(T_\theta)\ge 0$, then $T^*=T_{31}^*$; (b) if $f_{31}(N)<0$, $f_{32}(M)\ge 0$ and $f_{33}(T_\theta)\ge 0$, then $T^*=T_{32}^*$; (c) if $f_{31}(N)<0$, $f_{32}(M)<0$ and $f_{33}(T_\theta)\ge 0$, then $T^*=T_{33}^*$; (d) if $f_{31}(N)<0$, $f_{32}(M)<0$ and $f_7(T_\theta)<0$, then $T^*=T_{34}^*$.

Proof. See Appendix.

5. NUMERICAL EXAMPLES

To illustrate the results obtained in this paper, we provide the following numerical examples.

Let $A=350$, $c=15$, $p=17$, $I_p=0.15$, $I_e=0.1$, $h=0.5$, $\theta=0.08$, $D=1000$, $M=0.5$, $N=0.3$, $r=0.08$, $T_\theta=0.2$.

Example: when $T_\theta=0.2$, (a) if $A=10$, then $f_{11}(T_\theta)>0$, $f_{12}(N)>0$ and $f_{13}(M)>0$, according to Theorem 1(a) $T^*=T_{11}^*=0.1107$ and $PV_\infty(T^*)$ $=PV_{11}(T_{11}^*)=178290$; (b) if $A=100$, then $f_{11}(T_\theta)<0$, $f_{12}(N)>0$, $f_{13}(M)>0$ and according to Theorem 1 (b) $T^*=T_{12}^*=0.2960$ and $PV_\infty(T^*)=PV_2(T_{12}^*)=183560$; (c) if $A=350$, then $f_{11}(T_\theta)<0$ and $f_{12}(N)<0$, according to Theorem 1(c) $T^*=T_{13}^*=0.4453$ and $PV_\infty(T^*)=PV_{13}(T_{13}^*)=192090$; (d) if $A=1000$, then $f_{11}(T_\theta)<0$, $f_{12}(N)<0$ and $f_{13}(M)<0$, according to Theorem1(d) $T^*=T_{14}^*=0.6661$ and $PV_\infty(T^*)$ $=PV_{14}(T_{14}^*)=206340$.

When $T_\theta=0.4$ and $T_\theta=0.7$, the theroem2 and 3 are illustrated.

Next, we study further on the effects of changes of parameters M and N on the optimal solutions.

Journal of Theoretical and Applied Information Technology
10th February 2013. Vol. 48 No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

Table 1. *the impacts of change of M and N on T^* and $PV_\infty(T^*)$.*

M	N	$PV_\infty(T^*)$	T^*
0.5	0.30	192090	0.4453
	0.35	192850	0.4582
	0.40	193700	0.4727
0.6	0.30	188550	0.4467
	0.35	189310	0.4598
	0.40	190150	0.4743
0.7	0.30	185040	0.4482
	0.35	185800	0.4613
	0.40	186640	0.4759

The following inferences can be made based on table1.

(1) When other parameters are fixed, it shows that $PV_\infty(T^*)$ increase when the value of N increase, and decrease when the value of M increase.

(2) When other parameters are fixed, it shows that T^* increase when the value of M and N increase.

Figures 1-3 demonstrate the change of T^*, $PV_\infty(T^*)$, K_1 and K_2 when r is changed from $(0,1]$. T_* and PV_* are the value of the optimal cycle time and the value of optimal present value when $r=1$, $K_1=(T^*-T_*)/T^*$ and $K_2=\left[PV_\infty(T^*)-PV_*\right]/PV_\infty(T^*)$.

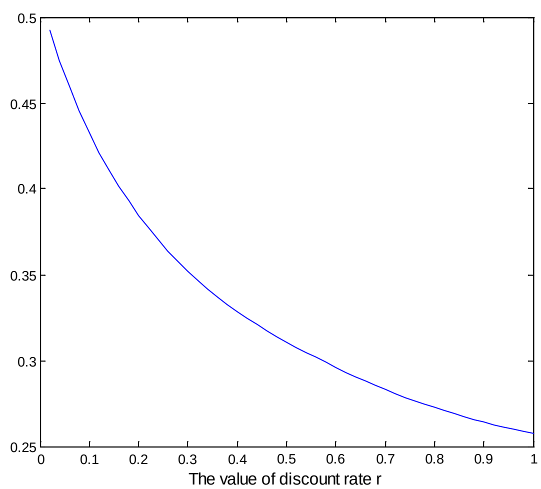

Fig. 1. The Impact Of Change Of r On T^*.

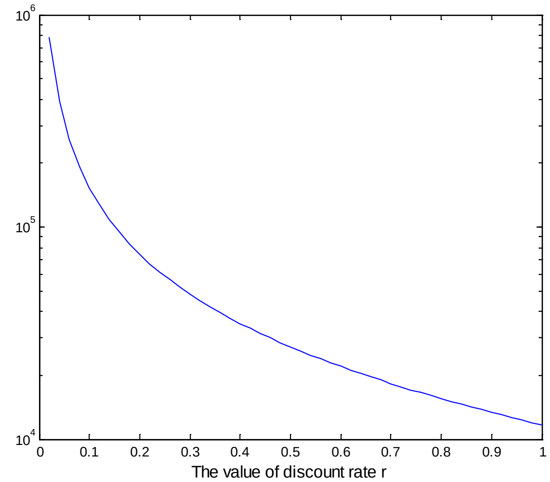

Fig. 2. The Impact Of Change Of r On $PV_\infty(T^*)$.

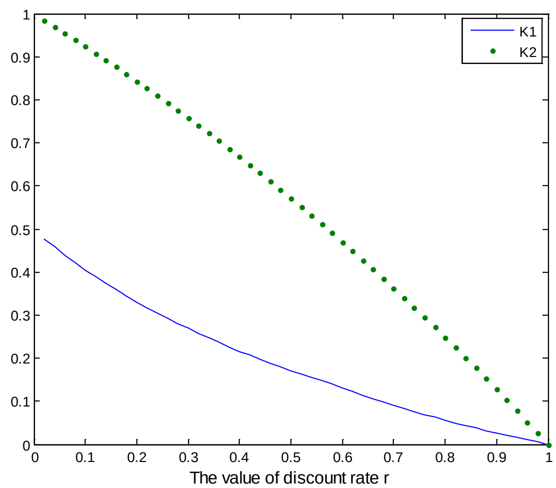

Fig. 3. The Impact Of Change Of r On K_1 And K_2.

Figs. 1-3 give the following results:
When r increases, T^*, $PV_\infty(T^*)$, K_1 and K_2 decreasing.

6. CONCLUSIONS

In this paper, an inventory model for non-instantaneous deteriorating items with two-level trade credit is established by DCF approach. By analyzing the present value of all future cash-flow cost, we developed theoretical results to obtain optimal solutions. Finally, some numerical examples are given to illustrate the theoretical results, and sensitive analyses of key parameters are given some managerial insights for the retailers. The presented model can be further extended to some practical

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

situations under two-level trade credit and time value of money, such as the demand depending on the selling price, limited storage space, etc.

ACKNOWLEDGMENT

This work was supported by the national specific Subject of the Ministry of Education and the Ministry of Finance of the PRC (TS11496), the National Natural Science Foundation of China (No. 11226200) , and the national statistics scientific research project (2011LY094), and the key Project of natural science of universities in Anhui Province (No. 2005Kj023ZD).

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Journal of Theoretical and Applied Information Technology
10th February 2013. Vol. 48 No.1

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

Appendix. Proof: Taking derivative of $PV_{ij}(T)$ $(i=1,2,3;\ j=1,2,3,4)$ with respect to T, we obtain

$$PV_{ij}^{'}(T)=\frac{f_{ij}(T)e^{-rT}}{\left(1-e^{-rT}\right)^{2}},$$

where

$$f_{11}(T)=-r(1-e^{-rT})PV_{11}(T)+\left(e^{rT}-1\right)D\Big\{ce^{\theta T-rM}+h\frac{e^{\theta T}-e^{-rT}}{\theta+r}-\frac{pI_e}{r}\left[\alpha e^{-rT}+(1-\alpha)e^{-rN}-e^{-rM}\right]\Big\},$$

$$f_{12}(T)=-r(1-e^{-rT})PV_{12}(T)+\left(e^{rT}-1\right)D\Big[Fe^{\theta(T-T_\theta)}-\frac{he^{-rT}}{\theta+r}-\frac{pI_e}{r}\left(e^{-rN}-e^{-rM}\right)\Big],$$

$$f_{13}(T)=-r(1-e^{-rT})PV_{13}(T)+\left(e^{rT}-1\right)D\Big[Fe^{\theta(T-T_\theta)}-\frac{he^{-rT}}{\theta+r}-\frac{pI_e}{r}D\left(e^{-rT}-e^{-rM}\right)\Big],$$

$$f_{14}(T)=-r(1-e^{-rT})PV_{14}(T)+\left(e^{rT}-1\right)D\Big\{Fe^{\theta(T-T_\theta)}+\frac{1}{\theta+r}\left[cI_pe^{\theta T-(r+\theta)M}-\left(h+cI_p\right)e^{-rT}\right]\Big\},$$

$$f_{22}(T)=-r(1-e^{-rT})PV_{22}(T)+\left(e^{rT}-1\right)D\Big[-\frac{1}{r}e^{-rT}\left(h+pI_e\right)+\frac{1}{r}\left(rce^{-rM}+h+pI_ee^{-rM}\right)\Big],$$

$$f_{33}(T)=\left(e^{rT}-1\right)D\Big[\frac{1}{r}\left(rce^{-rM}+h+cI_pe^{-rM}\right)-\frac{1}{r}e^{-rT}\left(h+cI_p\right)\Big]-r(1-e^{-rT})PV_{33}(T),$$

$$f_{34}(T)=\left(e^{rT}-1\right)D\Big\{e^{\theta(T-T_\theta)}F-\frac{e^{-rT}}{\theta+r}\left(h+cI_p\right)+e^{\theta(T-T_\theta)}\frac{cI_p}{r(\theta+r)}\left[(\theta+r)e^{-rM}-\theta e^{-rT_\theta}\right]\Big\}-r(1-e^{-rT})PV_{34}(T),$$

$$f_{23}(T)=f_{13}(T),\ f_{14}(T)=f_{24}(T),\ f_{22}(T)=f_{32}(T),$$

$f_{11}(0)=-rA$ and $\lim\limits_{T\to+\infty}f_{14}(T)=+\infty.$ $\lim\limits_{T\to+\infty}f_{34}(T)=+\infty.$

Taking derivative of $f_{1i}(T)$ $(i=1,2,3,4)$ with respect to T, we find that

$$f_{11}^{'}(T)=f_{21}^{'}(T)=f_{31}^{'}(T)=De^{\theta T}\left[rce^{-rM}+h-pI_e\left(e^{-rN}-e^{-rM}\right)\right]\left(e^{rT}-1\right),$$

$$f_{12}^{'}(T)=\left(e^{rT}-1\right)D\left[(\theta+r)Fe^{\theta(T-T_\theta)}-pI_e\left(e^{-rN}-e^{-rM}\right)\right],$$

$$f_{13}^{'}(T)=f_{23}^{'}(T)=\left(e^{rT}-1\right)D\left[(\theta+r)Fe^{\theta(T-T_\theta)}+pI_eDe^{-rM}\right],$$

$$f_{14}^{'}(T)=f_{24}^{'}(T)=\left(e^{rT}-1\right)De^{\theta T}\left[(\theta+r)Fe^{-\theta T_\theta}+cI_pe^{-(r+\theta)M}\right],$$

$$f_{22}^{'}(T)=f_{32}^{'}(T)=\left(e^{rT}-1\right)D\left(rce^{-rM}+h+pI_ee^{-rM}\right),$$

$$f_{33}^{'}(T)=\left(e^{rT}-1\right)D\left(rce^{-rM}+h+cI_p\right),$$

$$f_{34}^{'}(T)=\left(e^{rT}-1\right)De^{\theta(T-T_\theta)}(\theta+r)\Big\{F+\frac{cI_p}{r(\theta+r)}\left[(\theta+r)e^{-rM}-\theta e^{-rT_\theta}\right]\Big\}.$$

From assumption (6), we know that $f_{11}^{'}(T)\geq 0,\ f_{12}^{'}(T)\geq 0,\ f_{13}^{'}(T)\geq 0,\ f_{14}^{'}(T)\geq 0$ and from lemma 1, the theorem 1 is proofed. Similarly, theorem 2 and 3 are proofed.