PROBABILITY DETECTION FOR STATISTICAL MIMO RADAR: ALGORITHMS AND THEIR ANALYSIS

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ABSTRACT

Multiple-input–multiple-output (MIMO) radar is an emerging technology that has significant potential for advancing the state-of-the-art of modern radar. This is a multiple antenna radar system capable of transmitting arbitrary waveform from each antenna element. Like MIMO communications, MIMO radar offers a new paradigm for signal processing research. It's significant potentials for fading mitigation, resolution enhancement, and interference and jamming suppression, which can result in significantly improved target detection, parameter estimation, target tracking and recognition performance. To identify a target in statistical MIMO radar system multiple antennas, not closely placed, for transmission and reception is used. This paper introduces the signal processing issued in statistical MIMO radar and coherent MIMO radar for improved target detection and recognition performance. The fundamental difference between statistical MIMO and other radar array systems is that the latter seek to maximize the coherent processing gain, while the former capitalizes on the diversity of target scattering to improve radar performance. Compared to coherent MIMO radar, statistical MIMO radar can offer better parameter identification , higher resolution which allows for much flexibility for transmit beam-pattern design and waveform optimization. This paper focuses on the application of the target spatial diversity to improve detection performance. The optimal detector in the Neyman-Pearson sense is developed and analyzed for the statistical MIMO radar in case of stationary target. The performance improvement achieved by the use of angular diversity in statistical MIMO radar is investigated. The results show that at high and low SNR values statistical MIMO radar provides great improvements of target detection performance over coherent MIMO radar. Finally, a number of numerical examples demonstrated for the effectiveness of the proposed approaches. Therefore, statistical MIMO radar can be applied to enhance radar resolution by allowing the measurement of one scatter at a time.

Keywords: *MIMO, Statistical, Coherent, Radar, Probability Detection and SNR.*

1. INTRODUCTION:

Standard MIMO radar [1]-[2] takes the opposite direction of the phased-array radar. The approach is to employ multiple uncorrelated waveforms that are radiated via omnidirectional transmission, in compare to phased-array radar [3], where a single probing waveform is sent via directional transmission. Inspired by several publications have advocated the concept of MIMO radar from the system implementation point of view, processing techniques for target detection and parameter estimation. Target parameters of interest in radar systems include target strength, location, and Doppler characteristics. MIMO radar systems employ multiple antennas to transmit multiple waveforms and engage in joint processing of the received echoes from the target. MIMO radar may be configured with its antennas co-located or widely [9] distributed over an area and able to provide independent diversity paths. Hence, we can find two types of MIMO radar: The co-located MIMO [10], [11] radar where the transmitter and the receiver are close enough so that they share the same angle variable, i.e., coherent MIMO radar, and The uncolocated MIMO [3], [4]-[6], radar with widely separated antennas, i.e., statistical MIMO radar.

In particular, the term "statistical MIMO radar," refers to the signal model where the signals measured at different antennas are uncorrelated. If the antennas are separated far enough, the target radar cross sections (RCS) for different transmitting

paths become independent random variables. Thus, each orthogonal waveform carries independent information about the target; spatial diversity about the target is thus created. Exploiting the independence between signals at the array elements, MIMO radar achieves improved detection performance [7], [8] and increased radar sensitivity. Full exploitation of these potentials can result in significant improvement in target detection, parameter estimation, target tracking and recognition performance. This is the motivation for using multiple inputs multiple output (MIMO) radar. Thus, each orthogonal waveform carries independent information about the target; spatial diversity about the target is thus created. Exploiting the independence between signals at the array elements, MIMO radar achieves improved detection performance [8]   and increased radar sensitivity.

2. PERFORMANCE ANALYSIS

2.1 Coherent MIMO Radar

Coherent MIMO radar uses antenna arrays for transmitting and receiving signals. These arrays may be co-located and even transmit and receive functions can be performed by the same array or the arrays may be separated. The separation between the elements may be uniform or non-uniform. The arrays can be filled or sparse depending on the application type. But the separation is always small compared to the range extent of the target. Whatever the separation between the array elements is, the important point in coherent MIMO radar is that the array elements are close enough so that every element sees the same aspect of the target i.e. the same RCS. As a result, point target assumption is generally used in coherent MIMO radar applications.

2.1.1. Signal model
Consider a coherent MIMO radar system that has transmit and a receive array consisting of M and N elements respectively.
Then the received signal can be written as

$$y(t) = \sqrt{\frac{E_t}{M}} Hx(t - \tau) + w(t) \dots \dots (1)$$

Where, $\sqrt{E_t/M}\, Hx(t - \tau)$ denote the discrete time baseband signal transmitted by the transmit antenna elements where $x(t - \tau)$ is the input message signal with delay time, E_t is the total average transmitted energy and $w(t)$ is the noise vector.
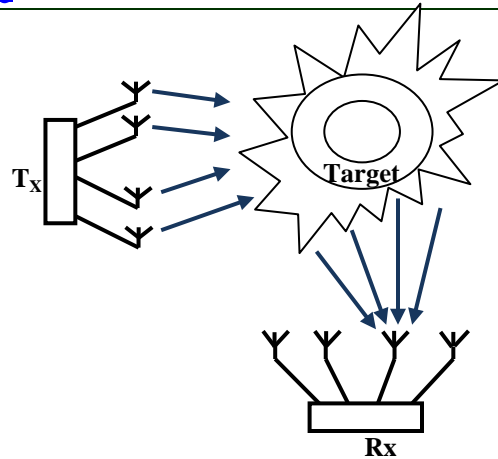

Fig.1. Coherent MIMO Radar Configuration.

2.1.2. Probability detection
The detection problem here can be formulated as binary hypothesis testing problem as follows:

$$\left. \begin{array}{l} H_0: \bar{y} = \bar{w} \\[1em] H_1: \bar{y} = \sqrt{\frac{E_t}{M}}\, \bar{\alpha} + \bar{w} \end{array} \right\} \dots \dots \dots (2)$$

Where H_0 indicates absence of signal and H_1 indicates presence of signal.
It is well known that the optimum solution to this hypothesis testing problem under Neyman-Pearson criterion is the Likelihood Ratio Test (LRT).
The likelihood ratio test becomes,

$$\|\bar{y}\|^2 \underset{H_0}{\overset{H_1}{\gtrless}} T' \dots \dots \dots (3)$$

To see the performance limit of coherent MIMO radar, the vector α become identical and coherent integration of the received samples becomes possible before detection process. The modified binary hypothesis testing problem turns in the form,

$$\left. \begin{array}{l} H_0: \quad y = w \\[1em] H_1: \quad y = \sqrt{\frac{E_t}{M_t}}\, MN\alpha + w \end{array} \right\} \dots \dots \dots (4)$$

Where **w** is now a complex number.
The probability of false alarm rate, P_{fa} can be calculated as,

$$P_{fa} = Prob \left\{ exp\left(\frac{1}{MNo_w^2}\right) > T' \right\}$$
$$= exp\left(\frac{T'}{MNo_w^2}\right) \ \dots\dots (5)$$

Then P_d can be written in terms of SNR and probability of false alarm rate as,

$$P_d = exp\left(\frac{ln(P_{fa})}{(SNR)N + 1}\right) \dots\dots (6)$$

So, the probability of detection does not depend on the number of transmit antennas but depends only on number of receive antennas and SNR.

2.1.3. Results:

To compare with the detection performance of Coherent MIMO Radar, the detector in (6) is implemented. P_{fa} value is set to 10^{-6}.

If the number of receive elements is held constant at the value of 5, and the number of transmit elements is increased, the P_d vs SNR curve in Fig. 2 is obtained.
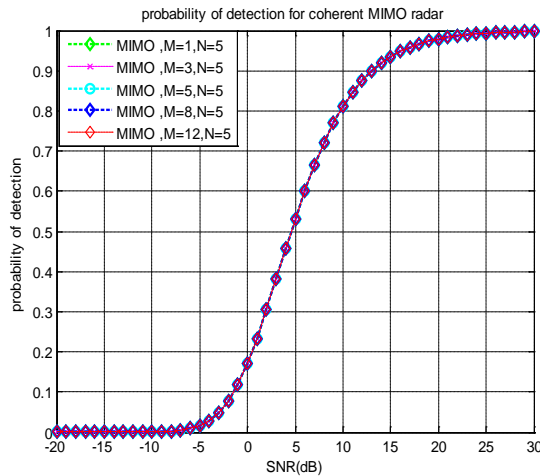

Fig. 2. Probability Of Detection For Coherent MIMO Radar, Changing M

The graphics in Fig. 2 show that the detection performance does not change with increasing M. This is because the transmitted power is normalized and it does not change with the number of transmit elements, and also because the noise power and the signal power in the received signal after coherent summation increase at the same rate.

If the number of transmit elements is held constant at the value of 5 and the number of receive elements is increased, the P_d vs SNR curve in Fig. 3 is obtained. We can see from the graph that as number of receiving antennas is increased the probability of detection increases, because the total received energy increases.
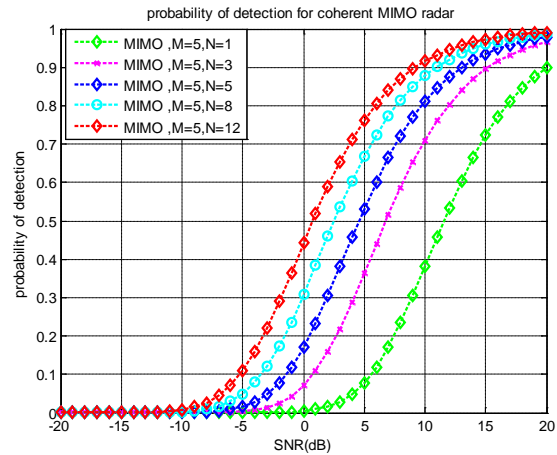

Fig 3: Probability Of Detection For Coherent MIMO Radar, Changing N.

2.2. Statistical MIMO radar

Statistical MIMO radar employs antenna arrays which are widely separated. The inter element spacing in an array is also so large that each transmit-receive pair sees a different aspect of the target and thus sees different RCS due to target's complex shape. If the spacing between the antennas elements is wide enough, received signals from each transmit receive pair become independent. This is called spatial or angular diversity. Statistical MIMO radar focuses on this property.

2.2.1. System Model

The model of the statistical MIMO radar system is shown in Fig. 4 where transmitters and receivers be widely separated.
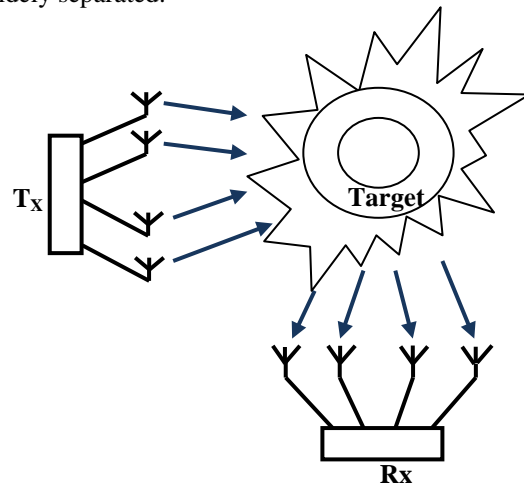

Fig. 4. Statistical MIMO Radar Configuration.

If the received signal is fed to a bank of matched filters each of which is matched to $x_m(t)$, and the corresponding output is sampled at the time instant,

then the output of the matched filter bank can be written in the vector form as

$$\bar{y} = \sqrt{\frac{E_t}{M}} \bar{\alpha} + \bar{w} \qquad \dots\dots\dots(7)$$

Where $\bar{y}$ is a $MN \times 1$ complex vector whose entries correspond to the output of the each matched filter at every receiver, $\bar{\alpha}$ is a $MN \times 1$ complex vector that contains all the elements of channel matrix (H) and $\bar{w}$ is a $MN \times 1$ complex noise vector.

2.2.2. Probability detection

Statistical MIMO Radar can be formulated as binary hypothesis testing problem for H_0 indicates absence of signal and H_1 indicates presence of signal.

$$\left.\begin{array}{l} H_0: \quad \bar{y} = \bar{w} \\[2mm] H_1: \quad \bar{y} = \sqrt{\frac{E_t}{M}} \bar{\alpha} + \bar{w} \end{array}\right\} \dots\dots\dots(8)$$

It is well known that the optimum solution to this hypothesis testing problem under Neyman-Pearson criterion is the Likelihood Ratio Test (LRT) as,

$$\frac{p(\bar{y}|H_1,C,A)}{p(\bar{y}|H_0,C)} \mathop{\gtrless}\limits_{H_0}^{H_1} T \dots\dots\dots\dots(9)$$

The P_{fa} can be calculated as in terms of threshold T',

$$\begin{aligned} P_{fa} &= Prob\left\{\frac{\sigma_w^2}{2}\chi_{2MN}^2 > T'\right\} \\[2mm] &= Prob\left\{\chi_{2MN}^2 > \frac{2}{\sigma_w^2}T'\right\} \\[2mm] &= 1 - Q_{\chi_{2MN}^2} exp\left(\frac{2}{\sigma_w^2}T'\right)\dots\dots(10) \end{aligned}$$

Where, $Q_{\chi_{2MN}^2}$ represents the cumulative distribution function of a Chi-squared random.

The P_d can be calculated in terms of P_{fa} and SNR as,

$$P_d = 1 - Q_{\chi_{2MN}^2}\left(\frac{1}{\left(\frac{SNR}{M}+1\right)}Q_{\chi_{2MN}^2}^{-1}\left(1 - P_{fa}\right)\right)\dots\dots(11)$$

2.2.3. Results and Observation

To compare with the detection performance of statistical MIMO radar, the probability detection in equation (11) is implemented for which P_{fa} value is set to 10^{-2}. If the number of receiving elements is held constant at the value of 5, and the number of

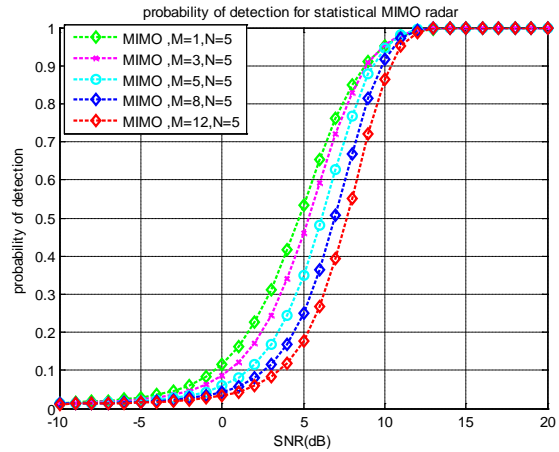transmitting elements is increased, the P_d vs. SNR curve in Fig. 5 is obtained.

Fig. 5. Probability Of Detection For Statistical MIMO Radar, Changing M.

It is interesting to see that the detection performance decreases as the number of transmitting antennas increases. Because of this decrease in the detection performance; using more widely separated receiving antennas instead of increasing the number of spatially diverse transmitting antennas seems more reasonable.

If the number of transmitting elements is held constant at the value of 5, and the number of receiving elements is increased, the p_d vs. SNR curve in Fig. 6 is obtained.
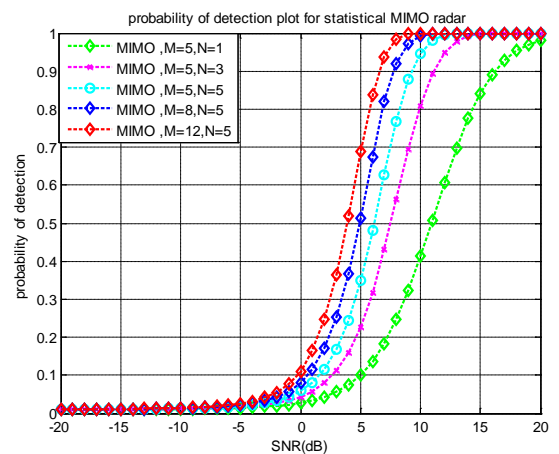

Fig. 6. Probability Of Detection For Statistical MIMO Radar, Changing N.

The ROC of coherent MIMO radar versus statistical MIMO radar and also comparison of probability detection of both type of MIMO radar are is given in Fig. 7 and Fig. 8 respectively. These figures are obtained using the analytical expressions given in

Equations (6), (11) for $= N = 5$. In the figure the blue lines belong to statistical MIMO radar and the red lines belong to coherent MIMO radar.

The results in Fig.7 and 8 show that at high SNR values and at high detection probabilities, the detection performance of statistical MIMO radar is better than coherent MIMO radar.
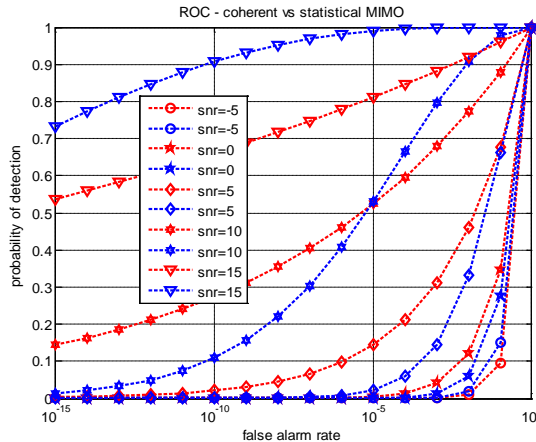

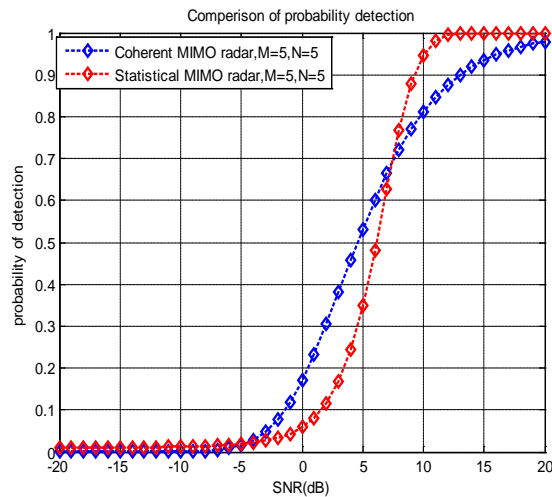Fig. 7. ROC-Coherent MIMO Radar Vs. Statistical MIMO Radar.

Fig. 8**.** Comparison Of Probability Detection Of Statistical MIMO Radar Vs. Phased Array Radar.

7. CONCLUSION

In this paper, we have presented a wide variety of signal processing algorithms for statistical MIMO radar and coherent MIMO radar. A novel algorithm on the space-time adaptive processing is proposed for improving the performance of statistical MIMO radar system. Derivations of the respective optimal detectors are shown when the target and noise level are either known or unknown. The statistical MIMO radar outperforms the coherent MIMO radar

at high detection rates which enables RCS fluctuation smoothing is demonstrated. The waveform design problem with information about the target and the clutter responses are being dealt. We have provided several numerical examples which show that the statistical MIMO radar has much better performance. We have proposed an iterative algorithm for jointly designing the transmitting waveforms and the receiving filters to maximize the SNR in statistical and coherent MIMO radar and also the algorithm converge faster. We also proposed a new metric to analyze the performance of these systems. Development of an adaptive optimal energy allocation mechanism is done to get significant improvement in performance. Finally we simulated a realistic scenario to analyze the performance of the proposed system.

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