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THE ALGORITHM OF REGION POLE PLACEMENT ON THE SAFETY INTERVAL

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ABSTRACT

This paper is concerned with the design problem for linear system. For a class of linear systems relation to the sector region pole placement, we put forward the concept of gain fault safety interval for each actuator channel. By using mathematical model of gain fault with single actuator channel, we present the algorithm of the actuator gain fault safety interval in the representative sector pole placement, we also analyze the impact on the stability and the dynamic performance of the system with each actuator channel, and provide an important theory evidence for the designer to design and place on the number of hardware. A practical example is given to illustrate the algorithm procedures and the approach's effectiveness.

Keywords: *Safety Interval, Pole Placement, Effectiveness, Actuator Fault*

1. INTRODUCTION

In the control theory, the most basic is to design the controller, and linear system pole placement of controller design is one of the important means. This paper is concerned with the design problem for linear system. In Ref. [1], all poles of a class of linear system are in complex plane on a proper disk, which will ensure its stability and dynamic performance. In recent years, some approaches have been put out for all poles of linear systems placement in a given disk [1, 2]. Recently, for uncertain systems with disk area closed-loop poles robust control research also made some progress [3 ~ 6]. In Ref. [7], the system poles keep in left half plane, disk area and sector area, the system matrix to satisfy the necessary and sufficient condition is comprehensively provided. Different regions reflect different system dynamic performance.

System component (sensor and actuator) and control process itself which exist in the small fault may cause the performance deterioration of the whole system even collapse, resulting in casualties and property loss of material. Therefore, the reliable control of complex system has become a hot research field in the current domestic and foreign engineering [8 ~ 13]. A control system designed to tolerate faults of sensors or actuators, while maintaining an acceptable level of the closed-loop system stability and performance. It is called a reliable control system. Reliable control can resist system components faults of sensors or actuators, but, this conservative design approach increases the design cost and control consumption to a certain extent, however, the fault tolerant hardware redundancy design not only solves the problem of economic and energy consumption, but also this approach of design is simple and can protect the environment.

Fault tolerant control includes active fault tolerant control and passive fault tolerance control. Active fault tolerant is on the basis of detection and diagnosis, according to the system's fault diagnosis for system to implement compensation. Reliable control is also called passive fault tolerant control. In this paper, by using active fault tolerant design conception, a definition for safety interval in the different signal channel of actuator fault is presented Under the condition of the system itself performance is not affected, the system also keep asymptotic stability and good performance in this security interval. Furthermore, the safety interval size reveals the strength of deviation fault tolerance for actuator in a signal channel. Therefore, this conclusion that put forwarded, has important guiding significance to the actual control system.

2. FORMATTING INSTRUCTIONS

Consider a linear system

$$\dot{x} = Ax + Bu \tag{1}$$

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where $x \in R^n$ is the state, $u \in R^m$ is the input of actuator, A 、 B are constant matrices with appropriate dimension. Where

$$u = Kx \tag{2}$$

Lemma 1: For system (1), when all eigenvalues of matrix A are in the sector area with an angle of θ, the necessary and sufficient conditions is existing positive symmetric matrix X, we have

$$\begin{pmatrix} \sin\theta(AX + XA^T) & \cos\theta(AX - XA^T) \\ \cos\theta(XA^T - AX) & \sin\theta(AX + XA^T) \end{pmatrix} < 0 \tag{3}$$

Lemma 2: For system (1), if existing controller K makes the system poles placement in sector area θ, the necessary and sufficient conditions is existing positive symmetric matrix X and matrix P ,we have

$$\begin{pmatrix} \sin\theta(Z + Z^T) & \cos\theta(Z - Z^T) \\ \cos\theta(-Z + Z^T) & \sin\theta(Z + Z^T) \end{pmatrix} < 0 \tag{4}$$

where, $Z = AX + BP$

The feasible solution is (X, P), the controller gain matrix is

$$K = PX^{-1}$$

Take consider of system actuator fault, the system is described as

$$\dot{x} = Ax + Bu^f \tag{5}$$

The actual gain fault mode with a signal channel i is described as

$$u^f = M_i u \tag{6},$$

$$M_i = diag\left(\underbrace{1,\ldots,1}_{i-1}, f_i, \underbrace{1,\ldots,1}_{m-i}\right),$$

$$f_i \in \left(\underline{f_i}, \overline{f_i}\right), i = 1, 2, \ldots, m$$

Introducing the following mathematical signals:

1) The eigenvalue of matrix A is described as $eig(A)$;

2) When actuator i channel fault occurs, the j pole of closed-loop system is described as λ_{ij};

3) The real value of λ is described as $\mathrm{Re}(\lambda)$;

4) The imaginary value of λ is described as $\mathrm{Im}(\lambda)$;

5) Maximum in a, b is described as $Max(a,b)$;

6) Minimum in a, b is described as $Min(a,b)$.

3. MAIN RESULTS

Definition: Consider the gain fault model (6) , in the event of

$$\underline{\delta_i} = \min(f_i), \overline{\delta_i} = \max(f_i),$$

and $f_i \in (\underline{\delta_i}, \overline{\delta_i})$ is given,

$$\dot{x} = A_c(f_i)x \tag{7}$$

The closed-loop system poles can be placed in sector area with the angle of θ, where

$$A_c(f_i) = A + BM_iK$$

So we definite the area $(\underline{\delta_i}, \overline{\delta_i})$ for safety interval in the different signal channel of actuator fault. According to this definition, we know that

$$(\underline{\delta_i}, \overline{\delta_i}) \subseteq \left(\underline{f_i}, \overline{f_i}\right).$$

Generally speaking, we can get $\underline{\delta_i} \ge \underline{f_i}$, $1 \le \overline{\delta_i} \le \overline{f_i}$. The smaller safety interval is, the weaker tolerate fault system for the channel signal is. That is to say, the more hardware redundancy is needed, the more important the reliability of this channel to the stability of the system is and vice versa. If the actuator safety interval is $(\underline{f_i}, \overline{f_i})$, we know $\underline{\delta_i} = \underline{f_i}$, $\overline{\delta_i} = \overline{f_i}$, so there is no effect on the system stability and performance when the actuator fault occurs in this channel.

Theorem: For system (7), focus on the problem of the sector region pole placement with a angle of θ, in the case of the actuator channel safety interval is $(\underline{\delta_i}, \overline{\delta_i})$, and if and only if

$$\tan\alpha_{i_{\max}} = \tan\theta \tag{8}$$

the solution is f_{i1}, f_{i2}, at the same time,

$$\underline{\delta_i} = Min(f_{i1}, f_{i2}),\ \overline{\delta_i} = Max(f_{i1}, f_{i2})$$

$$\lambda_{ij}(f_i) = eig(A_c(f_i)),\ x_{ij} = \mathrm{Re}\left(\lambda_{ij}(f_i)\right),$$

$$y_{ij} = \mathrm{Im}\left(\lambda_{ij}(f_i)\right),\ \tan\alpha_i = \frac{|y_{ij}|}{|x_{ij}|}$$

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$\tan\alpha_{i_{\max}} = \max(\tan\alpha_i)$, $j = 1,2,\ldots,n$.

Proof: For system (6), in view of the system actuator fault of the channel, we have the system matrix

$$A_c = (A + BM_i K)$$

the eigenvalue is

$$\lambda_{ij}(f_i) = eig(A_c(f_i)).$$

where $\tan\alpha_{\max i} < \tan\theta$, it explains when actuator channel i fault occur, the closed-loop poles are still in the provision sector region; where $\tan\alpha_{i_{\max}} > \tan\theta$, some and even all closed-loop poles are out of the provision sector region; where $\tan\alpha_{i_{\max}} = \tan\theta$, some and even all closed-loop poles are just at the provision sector region, and this is critical state.

4. NUMERICAL EXAMPLE

Consider a CH - 47 twin rotor helicopter linear model, the speed of level flight is standard 40 section (section 1.85 kilometers per hour), where

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

$$A = \begin{pmatrix} -0.02 & 0.005 & 2.4 & -32 \\ -0.14 & 0.44 & -1.3 & -30 \\ 0 & 0.018 & -1.6 & 1.2 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} 0.14 & -0.12 \\ 0.36 & -8.6 \\ 0.35 & 0.009 \\ 0 & 0 \end{pmatrix}, C = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 57.3 \end{pmatrix}$$

Control output:

y_1 : vertical speed (section /hour);

y_2 : angle of tilt (radians);

Control input:

u_1 : collector propeller thrust;

u_2 : differential collector propeller thrust;

Assume the range of fault is $0.1270 \le f_1, f_2 \le 1.8820$, we can receive the poles of the matrix are

$$v_1 = -2.2279$$
$$v_2 = 0.0652$$
$$v_3 = 0.4913 \text{ -}0.4151i$$
$$v_4 = 0.4913 \text{ +}0.4151i$$

It is easy to see that the system is unstable.

By lemma 2, we design the controller K ,where

$$K = \begin{pmatrix} 0.3221 & -0.2311 & -8.7655 & -23.2331 \\ -0.0490 & 0.0229 & -0.3521 & -4.4353 \end{pmatrix}$$

the system pole can be placement in the sector region with the angle of θ , where $\theta = 30^\circ$. (Figure1)

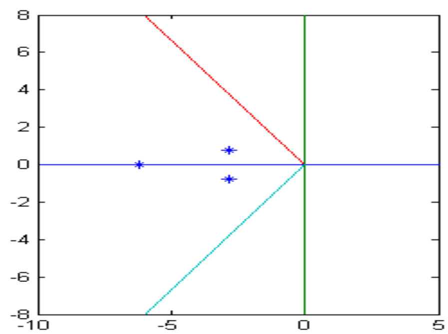

Figure1 Distribution Of The Constant Close Loop System Pole

By theorem, we can get the influence of sector region pole placement of actuator every channel gain range.(Figure2)

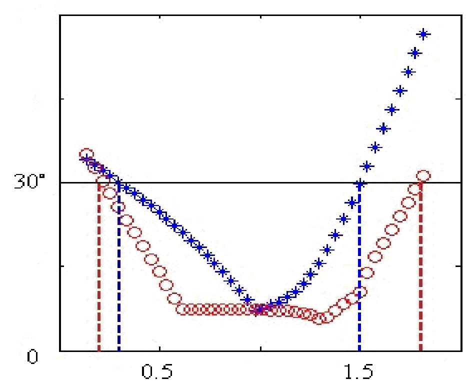

Figure2 The Influence Of Gain Error Actuator To The Angle Of The Sector Region Pole

It can be seen in Figure 2 that the solid line expresses the size of the sector region, where $\theta = 30^\circ$, $*$ expresses that when the actuator

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channel 1 fault parameters f_1 are change within a certain area, the closed-loop system pole's largest angle changes . ○ expresses that when the actuator channel 2 fault parameters f_1 are change within a certain area, the closed-loop system pole's largest angle changes .

It can be easily seen from Figure2 that actuator channel 1 fault gain fluctuate safety interval is (0.3563,1.5105) , that is to say $\underline{\delta}_1 = 0.3563, \overline{\delta}_1 = 1.5105$. Actuator channel 2 fault gain fluctuate safety interval is (0.2810,1.8270) , that is to say $\underline{\delta}_2 = 0.2810, \overline{\delta}_2 = 1.8270$.

From that we can see the fault gain fluctuation influences of channel 1 are larger than channel 2. It explains that channel 1 is more important for the reliability of the system. So, engineering designers can accord to this data to strengthen hardware redundancy of actuator channel 1, to reduce the failure of this channel, to improve the system reliability, in order to ensure the normal operation of the flight system

5. PAPER SUBMISSION

Generally, in a practical engineering control system design, engineering designers increase actuator quantity for a signal channel, to improve the channel reliability, in order to ensure this channel signal can be accurate and effective transferred, so that the normal operation of the control system. However, increase blindly actuator quantity, not only makes a system design complicated, but also causes unnecessary waste. Therefore, according to the difference of different signal channel permits signal fluctuation safety interval, we can know that each channel gain fault of actuator influence of the system is not the different. This paper gives a concept of the actuator of the signal channel gain deviation safety interval for the sector region pole placement, to reflect the importance of the signal channel in the system reliability, The smaller the safety interval is, the smaller system on the channel deviation signal fluctuation range is, and the signal channel needs more hardware quantity and vice versa. In all, this paper provides guidance advice to the engineering designers on the system control design, and a specific example proves the correctness and necessity of the algorithm.

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