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STATICS ANALYSIS OF CYLINDRICAL GEAR DRIVE AT ANY MESHING POSITION BASED ON ANSYS

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ABSTRACT

The transform relation between the meshing position and rotation angle of a cylindrical gear pair is firstly derived according to the theory of gear meshing. Then the technique of building the assembly model of a cylindrical gear pair at specified meshing position in ANSYS is researched. On this basis, a new method to perform static contact analysis of cylindrical gear at any meshing location is proposed. In comparison with the conventional method, the maximum working stress of the gear in a meshing cycle can be accurately calculated by this method. Simulation results show that the proposed method is reasonable, and it provides the foundation for further study of the fatigue life analysis and structural optimization of the cylindrical gear transmission.

Keywords: *Statics Analysis, Finite Element Method, Gear Strength, Meshing Position*

1. INTRODUCTION

As an important kind of transmission, gear drive is widely used in the industries of mining machinery and agricultural machinery *etc*. With the advantages of tight structure, high transmission efficiency, high reliability and stable transmission ratio, forces and motions can be transmitted through the successive meshing teeth between two shafts, which can satisfy the demands of high speed and power in modern industry. As the key component of the gearing system, the gears' performances can directly decide the reliability of the whole system. So it's significant to calculate the gear strength accurately.

The calculation of gear strength is usually performed at the position of pitch circle by using the traditional methods. To ensure the accuracy of the calculation, the single-tooth contact factors Z_B and Z_D, have been introduced to convert the contact stress at the pitch point to LPSTC of the pinion and wheel correspondingly. And stress correction factor Y_s and Y_{sa} are also used to apply the load at HPSTC and the addendum top to calculate the tooth bending strength. The finite element method is used by many scholars to take deep researches for the accurate calculation of gear strength. A dynamics analysis of helical cylindrical gear has been conducted by Li *et al.* [1] based on ANSYS. And time-histories of stress, strain and the most worst meshing positions have been obtained during the process of engagement. However, the result of the dynamics analysis is not accurate for the ignorance of the inertia force and the damping force. Gu *et al.* [2] has presented that the bending fatigue strength of helical gear is closely related to the location of the most worst meshing line, and the location of the most worst meshing line is directly determined by the position of the meshing point when they took an investigation of the bending fatigue of helical gear. To take an investigation of tooth contact stress, Hassan [3] selected 10 meshing points every 3 degrees of rotating angle on the action line. And a series of simulations for the tooth contact stress have been carried out based on ANSYS. The finite element method is also used by JongBoon *et al.* [4] to perform the modal and stress analysis at different meshing points. The positions of start of the active profile (SAP), LPSTC, HPSTC, end of the active profile (EAP) have also been selected to investigate the dynamic behavior of spur gears with FEM by some scholars [5]. The above scholars have attempted to perform the accurate calculation of gear strength via special meshing positions. However, the process of gear engagement is dynamic and continuous, and stress analysis at special meshing points cannot be depended on to obtain the accurate results of gear strength. The calculations by using the traditional methods or at special meshing points are approximate which will cause great error. Therefore, it's necessary to determine the position of any different meshing status on the action line and perform statics analysis to calculate the gear strength more accurately.

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In this paper, according to the theories of gear geometry and engagement [6], combined with the conditions of continuous transmission, the accurate equations of the relationship between the meshing radius, which means the distance from the meshing point to the center of pinion, and the rotating angle are derived. Based on the equations, the position of meshing point can be accurately determined during the meshing process. Then, the models of a pair of gears are established in ANSYS. Statics analysis is performed at different meshing positions after adjusting the models to the actual meshing position based on the specified meshing radius. And stresses at different meshing points have been compared. The result has shown that there are great gaps among the strength at different meshing positions and it's more reasonable to calculate the gear strength at the positions obtained by the dynamics analysis.

2. METHODOLOGY

2.1 Theoretical Analysis

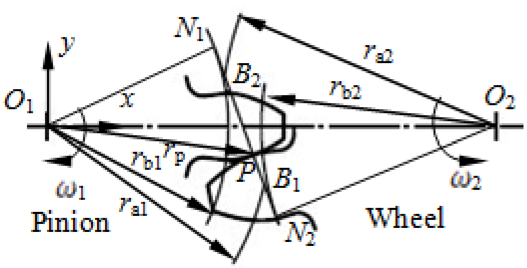

Figure 1: The Reference Meshing Position

Before deriving the relationship between the meshing radius and the rotating angle, the reference meshing position, which means the initial meshing position of two gears, should firstly be defined. As Fig.1 shown, in this paper, a pair of mating cylindrical gears is used to take the investigation. The reference meshing position is defined as the location where the central line of the pinion's tooth thickness is coincided with the x-axis in the global coordinate system. And the rotate direction of the pinion is clockwise. In Fig.1, N_1 and N_2 are two end points on the theoretical action line, while B_1 and B_2 are two end points on the actual action line. The pitch point P is the meshing point of two gears at the current position. And r_{b1} , r_{b2} accordingly stand for the base radius of the pinion and wheel, while r_{a1} and r_{a2} denote the addendum radius of two gears. Taking the whole meshing process, which means from the meshing point acting on the line to demeshing on the line, into consideration, two situations when $r_{p1} \geq r_p$ and $r_{p1} < r_p$ are correspondingly discussed at the actual meshing point P_1. Before the discussion, the reference meshing radius r_p should firstly be calculated.

According to the equation of the constant chord, the constant chord s_c of pinion can be calculated as:

$$s_c = s' cos^2 \alpha'_t \quad (1)$$

Where, s' stands for the arc tooth thickness at the radius of the pitch circle, and α' denotes the pressure angle at the pitch cylinder.

Based on this equation and combined with the law of cosines, the reference meshing radius r_p when two gears are meshing at the point P can be calculated as the equation:

$$r_p = \sqrt{r_1'^2 + \left(\frac{s'}{2} cos^2 \alpha'_t\right)^2 + r_1' s' cos\alpha'_t sin\alpha'_t} \quad (2)$$

Where, r_1' is the pitch radius of pinion.

2.1.1 Situation when $r_{p1} \geq r_p$

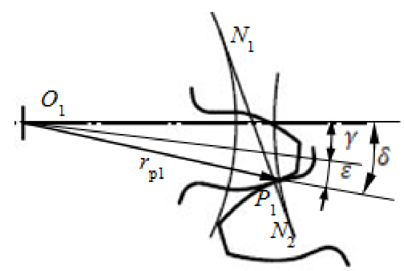

Figure 2: The Meshing Position When $r_{p1} \geq r_p$

Fig. 2 has shown the position when the actual meshing radius r_{p1} is equal or greater than the reference meshing radius r_p . At the current position, two gears are mating at the point P_1 . From the geometrical relationship in Fig.2, equation (3) can be derived as:

$$\gamma = \delta - \varepsilon \quad (3)$$

Where, angle γ stands for the rotating angle from the initial reference meshing position to the current meshing position. ε is half the central angle of tooth thickness at the radius of meshing point P_1 , while δ is the included angle between the line of

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centers and the line from the meshing point to pinion center O_1.

According to the equation of tooth thickness at any radius, the tooth thickness s_{p1} at the radius of meshing point P_1 can be obtained as:

$$s_{p1} = r_{p1} \cdot \left[\frac{s}{r} - 2\left(\theta_{p1} - \theta_1\right) \right] \tag{4}$$

Where, r_{p1} is the meshing radius, and s , r correspondingly denote the teeth thickness at pitch circle and the pitch radius. While θ_{p1} and θ_1 stand for the abduction angles at meshing point P_1 and pitch point P .

Based on the involute function, the angle θ_{p1} and θ_1 can be calculated according to the following equations:

$$\theta_{p1} = tan\left(arccos\left(\frac{r_b}{r_{p1}} \right) \right) - arccos\left(\frac{r_b}{r_{p1}} \right) \tag{5}$$

$$\theta_1 = tan\alpha - \alpha \tag{6}$$

Where, r_b is the base radius and α is the pressure angle at the pitch circle.

Combined with the equations (4) ~ (6), the angle ε can be calculated by the following equation:

$$\varepsilon = \frac{s}{2r} - \left(\theta_{p1} - \theta_1\right) \tag{7}$$

While angle δ can be determined by law of sines as the equation:

$$\delta = \frac{\pi}{2} - \alpha_t' - arcsin\left(\frac{r_1'}{r_{p1}} \cdot cos\alpha_t' \right) \tag{8}$$

From the equations (3), (7) and (8), we can derive the rotating angle γ given the specified meshing radius r_{p1}, as equation (9) shown:

$$\gamma = \frac{\pi}{2} - \alpha_t' - arcsin\left(\frac{r_1'}{r_{p1}} \cdot cos\alpha_t' \right) - \frac{s}{2r} + tan\left(arccos\left(\frac{r_b}{r_{p1}} \right) \right) - arccos\left(\frac{r_b}{r_{p1}} \right) - tan\alpha + \alpha \tag{9}$$

2.1.2 situation when $r_{p1} < r_p$

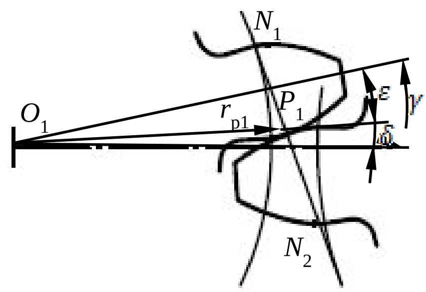

Figure 3: The Meshing Position When $r_{p1} < r_p$

Fig.3 has shown the position when the actual meshing radius r_{p1} is less than the reference meshing radius r_p . At the current moment, two gears are meshing at the point P_1. According to the geometrical relationship, equation (10) can be obtained:

$$\gamma = \delta + \varepsilon \tag{10}$$

Where, angle γ stands for the rotating angle from the initial reference meshing position to the current meshing position. ε is half the central angle of tooth thickness at the radius of meshing point P_1, while δ is the included angle between the line of centers and the line from the meshing point to pinion center.

Situation when $r_{p1} \geq r_p$ can be referred to in order to calculate the angle ε :

$$\varepsilon = \frac{s}{2r} - tan\left(arccos\left(\frac{r_b}{r_{p1}} \right) \right) + arccos\left(\frac{r_b}{r_{p1}} \right) + tan\alpha - \alpha \tag{11}$$

Then, angle δ can be derived as the following equation:

$$\delta = -\frac{\pi}{2} + \alpha_t' + arcsin\left(\frac{r_1'}{r_{p1}} \cdot cos\alpha_t' \right) \tag{12}$$

According to equation (10) ~ (12), we can obtain the relationship between the rotating angle γ and the meshing radius r_{p1} as equation (13) shown:

$$\gamma = -\frac{\pi}{2} + \alpha_t' + arcsin\left(\frac{r_1'}{r_{p1}} \cdot cos\alpha_t' \right) + \frac{s}{2r} - tan\left(arccos\left(\frac{r_b}{r_{p1}} \right) \right) + arccos\left(\frac{r_b}{r_{p1}} \right) + tan\alpha - \alpha \tag{13}$$

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2.2 Statics Analysis

2.2.1 Eestablishment of the simplified model

Before the simulation based on ANSYS, a pair of accurate gear model is essential to perform a statics analysis. Theoretically, a pair of whole teeth gears will help solve the problem more accurately, and precise result can be obtained through the performance of meshing and analysis. But this will produce a model with a huge number of elements which has high demands for the computer and will cause a low solving efficiency. For the process of gear meshing is continuous and periodic, and taking the calculation of the contact ratio into consideration, we can draw out that the number of teeth taking part into the meshing process is between one and three. Based on this fact, a simplified gear model with 3 pairs of teeth is made to take the investigation.

A model produced by third party CAD softwares can be used to carry out the simulation during the establishment of gear modeling. But in this way, many problems will occur like the loss of the geometry features due to the interface between ANSYS and other CAD software, which will cause negative effects on the meshing process. While a basic function of free-form surface can be used in ANSYS, the model cannot satisfy an accurate analysis due to the high demands for the establishment of cylindrical gear surface especially helical cylindrical gear. To meet the needs of analysis precision, a method of cubic uniform B-spline through extracting special nodes in ANSYS is used to produce the complicated surfaces of teeth [7]. Through commands of boolean operations in ANSYS, a solid tooth model can be obtained based on the surfaces. Then, through the operations of rotation and duplication, the pinion model can be produced and used to mesh the grids. In the same way, the wheel model can also be established in ANSYS. By adjusting the gear models to the initial reference meshing position, a pair of simplified model can be used to take the investigation as Fig. 4 shown.

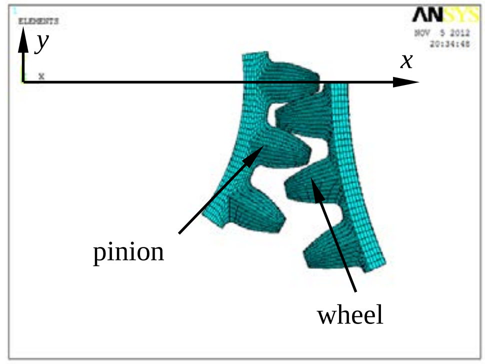

Figure 4: Gear Model At The Reference Position

2.2.2 Adjustment of the meshing position

According to equations (9) and (13), a corresponding rotating angle γ_1 of pinion can be calculated given any meshing radius. Then, based on the transmission ratio, the rotating angle γ_2 can also be obtained using the following equation:

$$\gamma_2 = \pm \frac{z_2}{z_1} \cdot \gamma_1 \tag{14}$$

Where, z_1 and z_2 accordingly stand for teeth number of pinion and wheel. The operator " + " is used in the transmission of internal gear drive while operator " – " is used in the transmission of external drive.

After the determinations of angle γ_1 and γ_2 when the specified meshing radius is given, the two angles can be used to adjust the pinion and wheel model to the actual meshing position. Fig. 5 shows the position after the adjustment. And this model can be used to take further statics analysis in ANSYS.

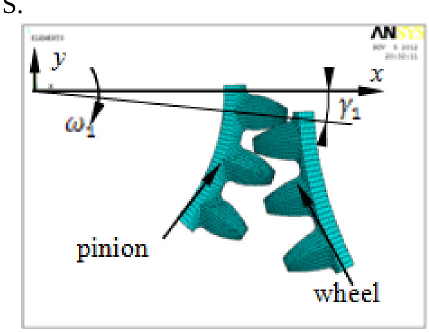

Figure 5: Gear Model After The Adjustment

2.2.3 Applying load and solving

During the meshing process, the lowest stress has little influence on the contact stress and the root bending stress. Therefore, the method of rigid coupling can be applied to the internal surface to improve the calculation efficiency [8]. To achieve the process, two Pilot nodes should firstly be defined at two centers and then coupled with the

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nodes on the internal surfaces to produce the rigid coupling areas. In this way, the nodes on the internal surfaces have the same degree of freedom (DOF) with the Pilot nodes.

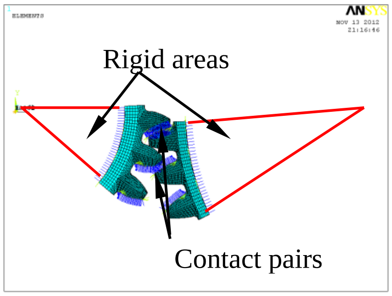

Figure 6: Contact Pairs And Rigid Areas

Fig. 6 has shown the rigid areas and contact pairs. Compared with the stress in the internal rigid areas, stress and strain in tooth root and tooth surface change greatly during the meshing process. To simulate the time-history of stress and strain, contact pair is used in ANSYS. Contact surface is defined on the pinion tooth surface while target surface is defined on the wheel tooth surface. By selecting the corresponding contact and target elements, changes of stress and strain can be accurately simulated.

After the definitions of contact pairs and rigid areas, a statics analysis can be carried out by applying the boundary conditions to the Pilot nodes. Since the analysis type is statics, all the DOFs of the wheel Pilot node should be restrained. The torque is applied to the pinion Pilot node, and in this way, forces and moments can be transmitted to the whole gear model. After the statics analysis, the result can be obtained for further investigation. And Fig.7 and Fig. 8 have shown the contact stress and equivalent bending stress after the statics analysis.

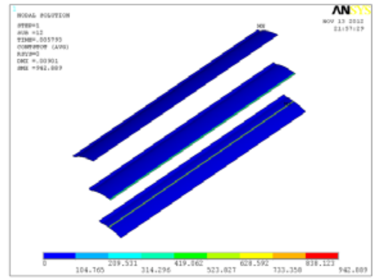
Figure 7: Contact Stress Of Pinion

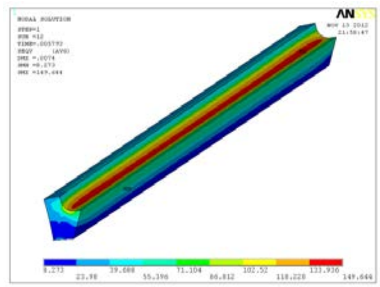
Figure 8: Equivalent Bending Stress Of Pinion

3. CASE STUDY

A pair of standard spur gears is used to take the investigation of statics analysis. And the basic design and working parameters, including teeth number z_1 and z_2, gear width b_1 and b_2, module m, pressure angle α, addendum coefficient h_a^*, tip clearance coefficient c^*, input power P and input rotating speed n, are shown as Table 1.

TABLE 1 : BASIC DESIGN AND WORKING PARAMETERS

Item	z_1	z_2	b_1 (mm)	b_2 (mm)	m (mm)	α (°)	h_a^*	c^*	P (KW)	n (rpm)
Value	25	45	40	40	2	20	1	0.25	30	1450

To compare with the results that calculated at the general meshing positions, a series of points, which include LPSTC B and HPSTC D of pinion, SAP B_2 and EAP B_1, pitch point P, maximum contact stress meshing point P_1, maximum equivalent root stress meshing point P_2 of pinion and maximum equivalent root stress meshing point P_3 of wheel, are taken to perform the statics simulation. The positions of the 8 points are shown as Fig. 9.

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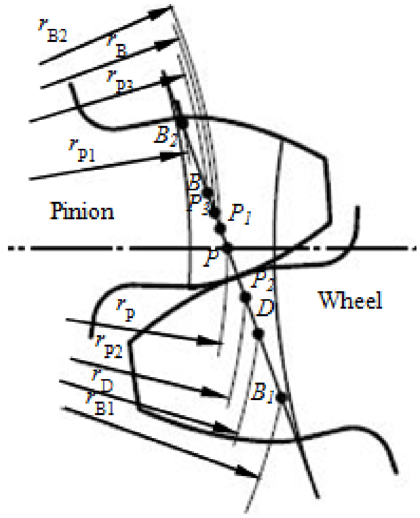

Figure 9: Positions Of The Meshing Points

As Fig.9 shown, the radii at pitch point P, LPSTC B of pinion, HPSTC D of pinion, SAP B_2 and EAP B_1 on the action line can be calculated by theoretical equations [9]. While the positions of maximum contact stress point P_1, maximum equivalent root stress point P_2 of pinion and maximum equivalent root stress point P_3 of wheel can be obtained by a full dynamics analysis [10].

In ANSYS, by checking the values of contact stress and equivalent root stress in every step, the above three positions can be determined indirectly. And based on the radii of all the 8 meshing positions, the corresponding rotating angles can be calculated through equations from (9) to (13), which can be used to adjust the models to different meshing positions. According to the results of the statics analysis, the radii r at all the points, the corresponding rotating angles γ, contact stress σ_H, pinion equivalent root stress σ_{F1}, and wheel equivalent root stress σ_{F2} can be collected as Table 2 shown.

Table 2 : Positions And Results

Position	B_2	B	P_3	P_1	P	P_2	D	B_1
r (mm)	23.74082	24.63146	24.74332	24.98232	25	25.15232	25.27733	27
γ (°)	-16.09811	-6.39611	-5.50831	-3.726442	-3.6	-2.53760	-1.69811	8.00384
σ_H (MPa)	667.903	920.849	1335	1424	1283	1002	708.68	942.889
σ_{F1} (MPa)	73.502	160.167	231.947	242.969	243.941	252.213	188.53	149.644
σ_{F2} (MPa)	103.802	148.065	205.006	191.158	190.622	184.178	130.273	62.896

From the results of statics analysis, maximum contact stress occurs at point P_1, rather than the general calculating point P or B. And maximum equivalent stresses of pinion and wheel correspondingly occur at point P_2 and P_3, not HPSTC or EAP. Therefore, this method is more reasonable to calculate the gear strength at the most worst meshing positions of P_1, P_2 and P_3 than the pitch point P or any other meshing points.

4. CONCLUSION

In this paper a new method for the accurate calculation of gear strength is presented. The corresponding relationship between the meshing position and rotating angle is derived, and a series of statics analysis are performed using a pair of cylindrical gear based on ANSYS. The result has shown that the gear strength at different meshing positions change greatly. And stresses at the most worst meshing positions obtained by the dynamics analysis are greater than those at the traditional positions, for example, the pitch point, LPSTC or HPSTC of pinion. Therefore, this method is more accurate than the traditional calculation methods, which provides a new approach to calculate the gear strength accurately.

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