

## DAMAGE FOLDING MUTATION MODEL OF FRACTURING ROCK MASS ON THE OIL-GAS RESERVOIR

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### ABSTRACT

According to the core problem of the oil-gas reservoir fracturing crack formation, this paper proposes that the essence of the oil-gas reservoir fracturing crack formation is the deformation energy accumulation increase and suddenly release of the rock mass which is resulted by fracturing pump work. Based on the principle of conservation of energy, the reservoir fracturing rock mass damage folding mutation model is established. The standard form of balance equation and energy release about the reservoir fracturing rock mass mutating is given. Combined with the actual fracturing of a block of Daqing Oilfield, the energy release quantity is calculated and the calculated result is reasonable which provides a new method for the study of the hydraulic fracturing crack parameters.

**Keywords:** *Hydraulic Fracturing, Folding Mutation, Damage Theory*

### 1. INTRODUCTION

Hydraulic fracturing as the main technical means of the reservoir stimulation, the formation mechanism of whose crack has been the key issue and the bottleneck problem of the study. According to the wellhead pump pressure operation curve and the bottom hole fracturing pressure analysis, in the process of the fracturing pump injecting fracturing fluid into the underground hole, rock cannot produce the continuity damage as the injecting, but performance for the unceasing increase of stress - strain when stress is not up to rock burst, namely for the accumulation of the rock mass deformation energy. It can be shown in Figure.1 and Figure.2.

A lot of research has been done by the scholars both at home and abroad, which is about the description problems of reservoir fracturing cracking up crack and extension and formation [1-8]. In recent years, based on the analysis of the technology of hydraulic fracturing the writer thinks that using catastrophe theory can give rock mass fracturing exhibited burst very good explanation, and the catastrophe theory is a powerful tool to the study of jump instability phenomenon. In the rock mechanics theory, the representative application has existed. For example, Yue Pan[9], etc, put forward the method of using work and principle of conservation of energy researching rock mass dynamic instability, and analyze the application conditions of different mutation model under different state; Yue Pan[10],etc,established the

folding mutation model of circular coyote blasting, and deduce and give the calculation expression of coyote blasting releasing the seismic energy. The above research results give a good description to mutation model, and inspire the author to deeply understand the connotation of the mutation model. Combined with the actual engineering of reservoirs hydraulic fracturing putting forward the folding mutation model of hydraulic fracturing fracture formation, the energy release quantity are calculated under the condition of the different mechanics parameters of tuff and tuffaceous conglomerate, and the settlement result is reasonable. This paper studies provide the new research methods for the formation and parameters calculation of subsequent hydraulic fracture. The research results have important significance to solve the problem of reservoir fracturing fracture formation.

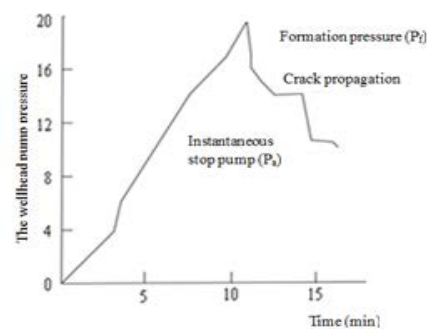


Figure 1. The Diagrammatic Sketch Of Hydraulic Fracturing Wellhead Operation

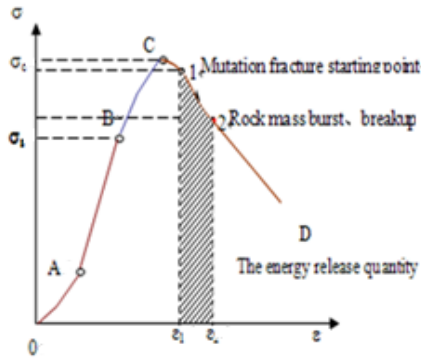


Figure 2. The Stress - Strain Curve Of The Hydraulic Fracturing Rock Mass

## 2. THE ENERGY EQUATION OF THE HYDRAULIC FRACTURING ROCK MASS

Hydraulic fracturing effect is that in the process of the rock mass energy gathering and without considering the influence of temperature, fracturing fluid potential and fracturing pump power convert to rock mass deformation energy and rock mass fractures when energy gather to a certain degree, then hydraulic cracks are formed. According to the principle of conservation of energy, the total potential energy of the hydraulic fracturing rock before cracking is:

$$W_b - \Pi_e - \Pi_p = 0 \quad (1)$$

Where,  $W$  is the total work of pump and fracturing fluid doing for the rock in process of fracturing;  $\Pi_e$  is the strain energy when fracturing rock mass occurs the elastic deformation;  $\Pi_p$  is the strain energy when the fracturing rock is into the plastic deformation.

### 2.1 Outside Load Work Model

In the process of fracturing, the total work of fracturing pump is:

$$W_b = \sum_{i=1}^n \eta P_i t_i \quad (2)$$

Where,  $W_b$  is the total work of pump doing in the process of fracturing;  $P_i$  is the fracturing pump power in the  $i$  stage;  $t_i$  is the work time of fracturing pump in the  $i$  stage;  $\eta$  is the pump efficiency,  $\eta=0.6$ .

### 2.2 Rock Mass Elastic Deformation Energy

According to the principle of conservation of energy, the deformation energy in the stage of rock mass elastic deformation is (unit volume rock):

$$\Pi_D = \int_0^{\epsilon_d^e} \sigma(\epsilon) d\epsilon \quad (3)$$

### 2.3 Rock Mass Plastic Deformation Energy

The deformation energy in the stage of rock mass plastic deformation is:

$$\Pi_p = \int_{\epsilon_d^e}^{\epsilon_d^p} \sigma(\epsilon) d\epsilon \quad (4)$$

According to the full stress-strain curve characteristics of the rock mass fracturing, the rock mass deformation satisfy the continuity damage characteristics, its stress-strain constitutive relationship meet this form:

$$\begin{cases} \sigma(\epsilon) = E\epsilon(1-D) & \epsilon < \epsilon_d^e, D = 0 \\ \sigma(\epsilon) = E\epsilon(1-D) & \epsilon_d^e \leq \epsilon \leq \epsilon_d^p, D = e^{-\left(\frac{\epsilon}{k}\right)^m} \end{cases} \quad (5)$$

$$\text{Where: } k = \frac{\epsilon_{pk}}{\sqrt[m]{m-1}}, \quad m = \frac{1}{\ln E\epsilon_c - \ln \sigma_c}$$

$E$  is the rock mass elastic modulus;  $K$  and  $m$  are the rock material parameters;  $\epsilon_c$  is the peak strain of the rock mass fracturing;  $\sigma_c$  is the peak stress of the rock mass fracturing.

## 3. THE STUDY OF MUTATION MODEL

Bring the Eq. 3 and Eq. 4 into the Eq. 1, then:

$$W_b - \int_0^{\epsilon_d^e} \sigma(\epsilon) d\epsilon + \int_{\epsilon_d^e}^{\epsilon_d^p} \sigma(\epsilon) d\epsilon = 0 \quad (6)$$

Make the Eq. 6 differential, and express it as the differential form of stress, then:

$$\frac{dW_b}{d\epsilon} - \frac{\sigma(\epsilon)}{E} \frac{d\sigma(\epsilon)}{d\epsilon} - \sigma(\epsilon) = 0 \quad (7)$$

The Eq. 7 is the differential form of the energy-balance equation of hydraulic fracturing fluid on rock mass work and before rock mass not formed the hydraulic fracture. According to the analysis of the stress - strain curve of the fracturing rock mass, the inflection point exists in the curve and hypothesize the inflection point strain as

$$\sigma(\epsilon) = \left[ \sigma(\tilde{\epsilon}) + \sigma'(\tilde{\epsilon})(\epsilon - \tilde{\epsilon}) + \frac{\sigma''(\tilde{\epsilon})}{2!}(\epsilon - \tilde{\epsilon})^2 \right] + O(\epsilon - \tilde{\epsilon})^3$$

. Determine the position of  $\tilde{\epsilon}$  by  $\sigma'(\tilde{\epsilon})$  and do the Eq.7 Taylor expansion, then eliminates the higher

order term according to the deterministic law. Get the Taylor expansion form shown as the Eq.8 and Eq.9:

$$\sigma(\varepsilon) \frac{d\sigma(\varepsilon)}{d\varepsilon} = \sigma(\tilde{\varepsilon})\sigma'(\tilde{\varepsilon}) + [\sigma(\tilde{\varepsilon})\sigma''(\tilde{\varepsilon}) + [\sigma'(\tilde{\varepsilon})]^2](\varepsilon - \tilde{\varepsilon}) + \frac{[2\sigma'(\tilde{\varepsilon})\sigma''(\tilde{\varepsilon}) + \sigma'(\tilde{\varepsilon})\sigma'''(\tilde{\varepsilon}) + \sigma(\tilde{\varepsilon})\sigma'''(\tilde{\varepsilon})]}{2!}(\varepsilon - \tilde{\varepsilon})^2 + O(\varepsilon - \tilde{\varepsilon})^3 \quad (8)$$

$$\sigma(\varepsilon) = \left[ \sigma(\tilde{\varepsilon}) + \sigma'(\tilde{\varepsilon})(\varepsilon - \tilde{\varepsilon}) + \frac{\sigma''(\tilde{\varepsilon})}{2!}(\varepsilon - \tilde{\varepsilon})^2 \right] + O(\varepsilon - \tilde{\varepsilon})^3 \quad (9)$$

Considering the assumption that  $\tilde{\varepsilon}$  is the inflection point of the process of fracturing rock mass damage softening, which meets the requirements of  $f''(\tilde{\varepsilon}) = 0$ , eliminate the item of  $f''(\tilde{\varepsilon}) = 0$ , then get:

$$-\frac{1}{2} \left[ (\varepsilon - \tilde{\varepsilon}) + \frac{[\sigma'(\tilde{\varepsilon})]^2}{\sigma(\tilde{\varepsilon})\sigma'''(\tilde{\varepsilon})} \right]^2 + \frac{[\sigma'(\tilde{\varepsilon})]^4}{[\sigma(\tilde{\varepsilon})]^2 [\sigma'''(\tilde{\varepsilon})]^2} + \frac{E \frac{dW_b}{d\tilde{\varepsilon}}}{\sigma(\tilde{\varepsilon})\sigma'''(\tilde{\varepsilon})} - \frac{\sigma(\tilde{\varepsilon})[\sigma'(\tilde{\varepsilon}) + E]}{\sigma'''(\tilde{\varepsilon})} = 0 \quad (10)$$

Where:  $\sigma(\tilde{\varepsilon}) = E\tilde{\varepsilon}e^{-[\frac{\tilde{\varepsilon}}{k}]^m}$

$$\sigma'(\tilde{\varepsilon}) = Ee^{-[\frac{\tilde{\varepsilon}}{k}]^m} - E \frac{m}{a^m} \tilde{\varepsilon}^m e^{-[\frac{\tilde{\varepsilon}}{k}]^m}$$

$$\sigma''(\tilde{\varepsilon}) = E \frac{m}{k^m} \tilde{\varepsilon}^{m-1} \exp[-(\frac{\tilde{\varepsilon}}{k})^m] \left( \frac{m}{k^m} \tilde{\varepsilon}^m - m - 1 \right) = 0$$

$$\sigma'''(\tilde{\varepsilon}) = Ee^{-[\frac{\tilde{\varepsilon}}{k}]^m} \left[ \frac{3m^3}{k^{2m}} \tilde{\varepsilon}^{2m-2} - \frac{m^3}{k^{3m}} \tilde{\varepsilon}^{3m-2} - \frac{m(m^2-1)}{k^m} \tilde{\varepsilon}^{m-2} \right]$$

Do the Eq.10 variable substitution and make:

$$x = (\varepsilon - \tilde{\varepsilon}) + \frac{[\sigma'(\tilde{\varepsilon})]^2}{\sigma(\tilde{\varepsilon})\sigma'''(\tilde{\varepsilon})} \quad (11)$$

$$a = -2 \left[ \frac{[\sigma'(\tilde{\varepsilon})]^4}{[\sigma(\tilde{\varepsilon})]^2 [\sigma'''(\tilde{\varepsilon})]^2} + \frac{EJ}{\sigma(\tilde{\varepsilon})\sigma'''(\tilde{\varepsilon})} \right] \left[ \frac{\sigma(\tilde{\varepsilon})[\sigma'(\tilde{\varepsilon}) + E]}{\sigma'''(\tilde{\varepsilon})} \right] \quad (12)$$

Write the Eq.12 the standard form of the folding mutation model:

$$x^2 + a = 0 \quad (13)$$

The Eq.13 is a. According the Eq.13, it is known that  $a \leq 0$ . When  $a = 0$  is the parabolic branch point, the two points correspond by the arbitrary symmetric solutions in the parabola indicate the two equilibrium states before and after the rock mass mutating. When  $x < 0$ , that is

$$(\varepsilon - \tilde{\varepsilon}) + \frac{[\sigma'(\tilde{\varepsilon})]^2}{\sigma(\tilde{\varepsilon})\sigma'''(\tilde{\varepsilon})} < 0, \text{ which corresponds the}$$

parabolic underside, the rock mass is in the corresponding equilibrium state of the crack formation before fracturing; When  $x > 0$ , that is

$$(\varepsilon - \tilde{\varepsilon}) + \frac{[\sigma'(\tilde{\varepsilon})]^2}{\sigma(\tilde{\varepsilon})\sigma'''(\tilde{\varepsilon})} > 0, \text{ which corresponds the}$$

parabolic upside, the rock mass fractures and hydraulic fracture forms, as shown in Fig.3.

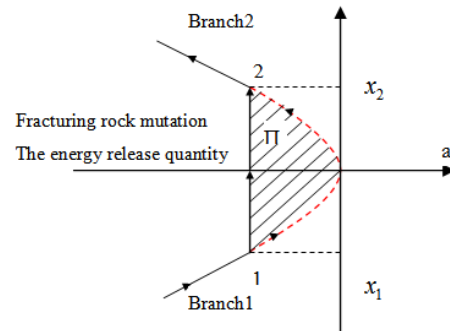


Figure 3. The Damage Mutation Of Hydraulic Fracturing Rock Mass

According to the standard form of the Eq.13, two roots that is  $x_1 (< 0)$  and  $x_2 = -x_1 (> 0)$  of the balance equation can be pointed out, as:

$$x_1 = - \sqrt{-2 \left[ \frac{[\sigma'(\tilde{\varepsilon})]^4}{[\sigma(\tilde{\varepsilon})]^2 [\sigma'''(\tilde{\varepsilon})]^2} + \frac{EJ}{\sigma(\tilde{\varepsilon})\sigma'''(\tilde{\varepsilon})} \right] \left[ \frac{\sigma(\tilde{\varepsilon})[\sigma'(\tilde{\varepsilon}) + E]}{\sigma'''(\tilde{\varepsilon})} \right]} \quad (14)$$

$$x_2 = \sqrt{-2 \left[ \frac{[\sigma'(\tilde{\varepsilon})]^4}{[\sigma(\tilde{\varepsilon})]^2 [\sigma'''(\tilde{\varepsilon})]^2} + \frac{EJ}{\sigma(\tilde{\varepsilon})\sigma'''(\tilde{\varepsilon})} \right]} \quad (15)$$

Do the Eq.13 integral, the total potential energy function of the folding mutation model is:

$$\Pi = \frac{1}{3}x^3 + ax \quad (18)$$

$$\Pi_1 = \frac{1}{3}x_1^3 + ax_2 \quad (19)$$

$$\Pi_2 = \frac{1}{3}x_2^3 + ax_2 \quad (20)$$

$$\Delta\Pi = \frac{1}{3}(x_2^3 - x_1^3) + a(x_2 - x_1) \quad (21)$$

Where:  $x_2 = -x_1$ ,  $x_2^2 = x_1^2 = -a$ . Through finishing, the energy release quantity of the fracturing rock mass mutation is:

$$\Delta\Pi = -\frac{4}{3}x_2^3 = -\frac{4}{3} \left[ 2 \left[ \frac{\sigma(\tilde{\varepsilon})[\sigma'(\tilde{\varepsilon})+E]}{\sigma'''(\tilde{\varepsilon})} - \frac{[\sigma'(\tilde{\varepsilon})]^4}{[\sigma(\tilde{\varepsilon})]^2 [\sigma'''(\tilde{\varepsilon})]^2} \right] \right]^{\frac{3}{2}} \quad (22)$$

According to the curve characteristics of fig.3, it is known that if  $x$  curve is from branch 1 to branch 2 through the original point, the rock mass fracturing will be asymptotic type damage; and if  $x$  curve along the branch 1 transits to the branch 2 through some point suddenly, rock mass will mutate.

In the process of hydraulic fracturing, the energy accumulation of rock mass increases when fracturing fluid is on rock mass effect. If the rock mass occurs mutations, the external force work of fracturing fluid from limiting numerical value suddenly is reduced and  $J=0$  when mutating. At this time, the numerical value of the branch point is:

$$(\varepsilon - \tilde{\varepsilon}) + \frac{[\sigma'(\tilde{\varepsilon})]^2}{\sigma(\tilde{\varepsilon})\sigma'''(\tilde{\varepsilon})} = x_{1(J=0)}$$

$$= -\sqrt{2 \left[ \frac{\sigma(\tilde{\varepsilon})[\sigma'(\tilde{\varepsilon})+E]}{\sigma'''(\tilde{\varepsilon})} - \frac{[\sigma'(\tilde{\varepsilon})]^4}{[\sigma(\tilde{\varepsilon})]^2 [\sigma'''(\tilde{\varepsilon})]^2} \right]} \quad (16)$$

$$(\varepsilon - \tilde{\varepsilon}) + \frac{[\sigma'(\tilde{\varepsilon})]^2}{\sigma(\tilde{\varepsilon})\sigma'''(\tilde{\varepsilon})} = x_{2(J=0)}$$

$$= \sqrt{2 \left[ \frac{\sigma(\tilde{\varepsilon})[\sigma'(\tilde{\varepsilon})+E]}{\sigma'''(\tilde{\varepsilon})} - \frac{[\sigma'(\tilde{\varepsilon})]^4}{[\sigma(\tilde{\varepsilon})]^2 [\sigma'''(\tilde{\varepsilon})]^2} \right]} \quad (17)$$

#### 4. CASE STUDY

Taking the tuff and tuffaceous conglomerate core of a block of Daqing oilfield, the basic mechanical parameters and stress strain curve of rock is measured through the test. According to the different test conditions and test results, the energy release quantity of rock mass fracturing mutations is calculated, as shown in table 1.

Table1 The Experimental Results Of Different Rock Mechanics Parameters

Specimen lithology	Specimen Group number	Surrounding rock stress (MPa)	Peak stress (MPa)	Elastic modulus (MPa)	Rock mass fracture strain	Rock mass release energy (10 <sup>6</sup> J)
Tuff	H1	10	67.57	8100	0.074285	-7.20516
	H2	20	105.35	8600	0.092465	-15.1893
	H3	40	119.4	10500	0.092847	-24.5478
Tuffaceous conglomerate	L1	10	122.0	16200	0.075785	-27.7113
	L2	20	144.90	17000	0.079547	-32.4829
	L3	40	185.45	17800	0.094949	-73.9291



## 5. CONCLUSION

(1) It is discussed that the essential characteristics of reservoir fracturing fracture formation is the increase and suddenly release of rock mass deformation caused by the fracturing pump work. Using folding mutation model to study the rationality and validity of the problem is proposed.

(2) In this passage, the folding mutation model of the rock mass fracturing fracture formation is established based on damage theory. Through the principle of conservation of energy, this model demonstrates the energy gathered and release of rock mass in the process of fracturing, and gives the balance equation of reservoir fracturing rock burst.

(3) Combined with the tuff and tuffaceous conglomerate of a block of Daqing oilfield, the rock mass energy release quantity is calculated respectively under the condition of different fracturing and the calculated result is reasonable and reliable. The new method is provided for the study of subsequent hydraulic fracturing fracture parameters.

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