Journal of Theoretical and Applied Information Technology
20th January 2013. Vol. 47 No.2

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

EXPONENTIAL STABILITY OF STRONG SOLUTION FOR A STOCHASTIC DISTURBANCE PREDATOR-PREY SYSTEM OF THREE SPECIES WITH AGE-STRUCTURE

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ABSTRACT

This thesis first introduces stochastic disturbance factor into a model and has a discussion of it on the basis of a predator-prey system of three species with age-structure proposed by Zhixue Luo. Sufficient conditions of a stochastic disturbance predator-prey system of three species with age-structure have been given and exponential stability of strong solution has been proved according to Gronwall's lemma and $It\hat{o}$ formula. The conclusion is a further development of pertinent literature.

Keywords: *Exponential Stability, Stochastic Disturbance, $It\hat{o}$ Formula, Gronwall's Lemma, Three Species Subject Classification:(AMS) 35B10, 45D05, 60H15, 34K40*

1. INTRODUCTION

The study of age-structured single species was initiated by Rorres and Fair[1]. Since then, the control problem has received many attentions from several authors[2-9]. In [2], Chan and Guo studied optimal birth control policies for the model of Mckendrick type. Maximum principles for problems with free ends, the time optimalcontrol problem, problems with target sects and infinite horizon problems had beenderived respectively. Motivated by the idea of Chan and Gou[2], the aim of this paperis to establish necessary optimality conditions for the above mentioned optimalcontrol problems by using a powerful functional approach first suggested for generalexternal problems.In [10], Webb studied the stability of nontrivial equilibrium solution of the model.In [11], Zhixue Luo, Ze-Rong He and Wan-Tong Li considered optimal birth control for predator-prey system of three species with age-structure. Considering the effect of age factor for control problems of the interacting species. They introduce the following food chain system composed of three age-dependent species.

$$\begin{cases} \frac{\partial p_1}{\partial t}+\frac{\partial p_1}{\partial a}=-\mu_1(a,t)p_1-\lambda_1(a,t)P_2(t)p_1, \\ \frac{\partial p_2}{\partial t}+\frac{\partial p_2}{\partial a}=-\mu_2(a,t)p_2+\lambda_2(a,t)P_1(t)p_2-\lambda_3(a,t)P_3(t)p_2, \\ \frac{\partial p_3}{\partial t}+\frac{\partial p_3}{\partial a}=-\mu_3(a,t)p_3+\lambda_4(a,t)P_2(t)p_3, \\ p_i(0,t)=\beta_i(t)\int_{a_1}^{a_2} m_i(a,t)p_i(a,t)da, \\ p_i(a,0)=p_{i0}(a) \\ P_i(t)=\int_0^{a_+} p_i(a,t)da, i=1,2,3 \qquad (a,t)\in Q. \end{cases} \tag{1}$$

where $Q=(0,a_+)\times(0,+\infty),[a_1,a_2]$ is the fertility interval , and the other parameters mean as follow: (for the sake of convenience, throughout this paper we suppose that i=1,2,3).

$p_i(a,t)$: the density of ith population of age a at time t, $p_i := p_i(a,t)$;;

$\mu_i(a,t)$: the average mortality of ith population;

$\beta_i(t)$: the average fertility of ith population;

$\lambda_k(a,t)$: the interaction coefficients(k = 1, 2, 3, 4);

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

$m_i(a,t)$: the ratio of females in ith population;

$p_{i0}(a)$: the initial of age distribution of ith population;

a_+ : the life expectancy, $0 < a_+ < +\infty$. Here without loss of generality.

we assume that the three population have the same life expectancy.

There has been much recent interest in a three-species Predator-Prey System. For example, Tan DJ and Zhanp predator-prey system with impulsive perturbations on predators, Liu ZJ[13] researched existence of periodic solutions for a delayed ratiod-ependent three-species predator-prey diffusion system on time scales, In [14], Xu R and Chen LS discussed persistence and global stability for a three-species ratio-dependent predator-prey system with time delays in two-patch environments, In [15], Zhang YP and Sun JT studied persistence in a three species Lotka-Volterra non-periodic predator-prey system, Dong LZ, Yuan CD and Chen LS[16] discussed persistence of a three-species predator-prey-chain autonomous system with diffusion.

This thesis first introduces stochastic disturbance to a predator-prey system of three species with age-structure.

Suppose that in Eq(1), add $f_1(p_1,t)dw$ to $-\mu_1(a,t)p_1 - \lambda_1(a,t)P_2(t)p_1$, add $f_2(p_2,t)dw$ to $-\mu_2(a,t)p_2 + \lambda_2(a,t)P_1(t)p_2 - \lambda_3(a,t)P_3(t)p_2$,ad d $f_3(p_3,t)dw$ to $-\mu_3(a,t)p_3 + \lambda_4(a,t)P_2(t)p_3$, where $\omega(t)$ is white noise. Then this environmentally perturbed system may be described by this system of three species with age-structure:

$$\begin{cases} \frac{\partial p_1}{\partial t} + \frac{\partial p_1}{\partial a} = -\mu_1(a,t)p_1 - \lambda_1(a,t)P_2(t)p_1 + f_1(p_1,t)dw, \\ \frac{\partial p_2}{\partial t} + \frac{\partial p_2}{\partial a} = -\mu_2(a,t)p_2 + \lambda_2(a,t)P_1(t)p_2 \\ -\lambda_3(a,t)P_3(t)p_2 + f_2(p_2,t)dw, \\ \frac{\partial p_3}{\partial t} + \frac{\partial p_3}{\partial a} = -\mu_3(a,t)p_3 + \lambda_4(a,t)P_2(t)p_3 + f_3(p_3,t)dw, \\ p_i(0,t) = \beta_i(t)\int_{a_1}^{a_2} m_i(a,t)p_i(a,t)da, \\ p_i(a,0) = p_{i0}(a) \\ P_i(t) = \int_0^{a_+} p_i(a,t)da, \\ i = 1,2,3 \end{cases} \tag{2}$$

where $f_1(p_1,t)dw$, $f_2(p_2,t)dw$, $f_3(p_3,t)dw$ denote stochastically perturbed, $\omega(t)$ is white noise.

A new stochastic differential equation model (2) for a predator-prey system of three species with age-structure is derived. It is an extension of Eq. (1).

In this paper, we shall discussion the exponential stability of strong solution for stochastic predator-prey system of three species with age structure.

In Section 2, we begin with some preliminary results which are essential for our analysis. In Section 3, we shall establish some criteria for the exponential stability of strong solution for stochastic predator-prey system of three species with age structure.

2. PRELIMINARIES

Let $V = H^1([0,a_+]) = \{\varphi \mid \varphi \in L^2([0,a_+]),$ $\frac{\partial \varphi}{\partial a} \in L^2([0,a_+]),$ Where $\frac{\partial \varphi}{\partial a}$ are generalized partial derivatives $\}$, V is the Sobolev space. $H = L^2([0,a_+])$ such that $V \to H \equiv H' \to V'$. V' is the dual space of V .We denote by $\|\cdot\|, |\cdot|$ and $\|\cdot\|_*$ the norms in V , H,and V' respectively; by $\langle \cdot,\cdot \rangle$ the duality product between V , V' and $\langle \cdot,\cdot \rangle$ the scalar product in H , and m a constant such that $|x| \le m\|x\|$, $\forall x \in V$. For an operator $B \in \psi(M,H)$ be the space of all bounded linear

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

operators from M into H, $\|B\|_2$ denotes the Hilbert-Schmidt norm, i.e. $\|B\|_2^2 = tr(BWB^T)$. Let ω_t be a Wiener process defined on complete probability space (Ω, F, P) and taking its values in the separable Hilbert space K,with increment convariance operator .Throughout this paper, we always assume that:

(H1) $\mu_i \in L(Q),\ 0 \le \mu_{i0} \le \mu_i(a,t) \le \mu_i^0$,

μ_{i0}, μ_i^0 are constants,

$\int_0^{a_+} \mu_i(a, t+a)da = +\infty, (a,t) \in Q$;

(H2) $0 \le B_k \le \lambda_k(a,t) \le A_k, (a,t) \in Q, A_k, B_k$ are constants, (k=1,2,3,4);

(H3) $0 \le m_i(a,t) \le M_i, (a,t) \in Q,\ M_i$ are constants, and $m_i(a,t) \equiv 0$, when $a < a_1$ or $a > a_2$;

(H4) $\beta_i \in U_i := \{h_i \in L^\infty(0,\infty) : 0 \le \beta_{i0} \le h_i(t) \le \beta_i^0$, $\forall t > 0\}, U = U_1 \times U_2 \times U_3$.

(H5) $p_{i0} \in L^\infty(0, a_+), p_{i0}(a) \ge 0, \forall a \in (0, a_+)$;

(H6) $r_i \le P_i(t) \le R_i, P_i := P_i(t)$. For any given $T > 0$ and $p = (p_1, p_2, p_3) \in C(0,T; L^2(0,a_+; R^3)) \bigcap L^\infty(Q_T; R^3)$, where $Q_T = (0,a_+) \times (0,T)$;

(H7)Lipschitz:
$\forall p_i \in C(0,T; L^2(0,a_+; R^3)) \bigcap L^\infty(Q_T; R^3)$,
$\exists K_i > 0$, such that $\| f_i \|_2^2 \le K_i | p_i |^2, i = 1,2,3$,

For any given $T > 0$ and
$p = (p_1, p_2, p_3) \in C(0,T; L^2(0,a_+; R^3))$
$\bigcap L^\infty(Q_T; R^3)$ $Q_T = (0,a_+) \times (0,T)$,

where $Q_T = (0,a_+) \times (0,T)$.

3. EXPONENTIAL STABILITY OF SOLUTIONS

In this section, we shall establish some criteria for the exponential stability of stochastic competitive population system Eq(2). we assume there exists aprocess $p = (p_1, p_2, p_3)$ $\in C(0,T; L^2(0,a_+; R^3)) \bigcap L^\infty(Q_T; R^3)$,

which is the strong solution of Eq(2).

Theorem3.1 Assume the preceding hypotheses and $2Br + 2\mu_0 - (\beta^0)^2 M^2 a_+^2 - 2AR - K > 0$ hold, if (p_1, p_2, p_3) is a global strong solution to Eq(2).

There exist contants $\tau = 2Br + 2\mu_0 - (\beta^0)^2 M^2 a_+^2$
$-2AR - K > 0, C > 0$

such that

$E(|p_1|^2 + |p_2|^2 + |p_3|^2) \le Ce^{-\tau t}, \forall t \ge 0$.

Proof. We can choose $\delta > 0$ small enough such that $\xi - \delta > 0$. Then, by $It\hat{o}$ formula , we have

$$\begin{aligned}
&e^{(\xi-\delta)t}(|p_1|^2 + |p_2|^2 + |p_3|^2) \\
&-(|p_{10}|^2 + |p_{20}|^2 + |p_{30}|^2) \\
&= (\xi-\delta)\int_0^t e^{(\xi-\delta)s}(|p_1|^2 + |p_2|^2 + |p_3|^2)ds \\
&+2\int_0^t e^{(\xi-\delta)s}\langle dp_1, p_1\rangle ds \\
&+2\int_0^t e^{(\xi-\delta)s}\langle dp_2, p_2\rangle ds \\
&+2\int_0^t e^{(\xi-\delta)s}\langle dp_3, p_3\rangle ds \\
&+\int_0^t e^{(\xi-\delta)s}(\|f_1\|_2^2 + \|f_2\|_2^2 + \|f_3\|_2^2)ds.
\end{aligned}$$
(3)

then

$$\begin{aligned}
&e^{(\xi-\delta)t}E(|p_1|^2 + |p_2|^2 + |p_3|^2) \\
&= E(|p_{10}|^2 + |p_{20}|^2 + |p_{30}|^2) \\
&+(\xi-\delta)\int_0^t e^{(\xi-\delta)s}E(|p_1|^2 + |p_2|^2 + |p_3|^2)ds \\
&+2E\int_0^t e^{(\xi-\delta)s}\langle dp_1, p_1\rangle ds \\
&+2E\int_0^t e^{(\xi-\delta)s}\langle dp_2, p_2\rangle ds \\
&+2E\int_0^t e^{(\xi-\delta)s}\langle dp_3, p_3\rangle ds \\
&+E\int_0^t e^{(\xi-\delta)s}(\|f_1\|_2^2 + \|f_2\|_2^2 + \|f_3\|_2^2)ds.
\end{aligned}$$
(4)

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

$$2E\int_0^t e^{(\xi-\delta)s}\langle dp_1, p_1\rangle ds$$
$$= -2E\int_0^t e^{(\xi-\delta)s}\langle -\frac{\partial p_1}{\partial a}, p_1\rangle ds$$
$$-2E\int_0^t e^{(\xi-\delta)s}\langle \mu_1(a,s)p_1, p_1\rangle ds \quad (5)$$
$$-2E\int_0^t e^{(\xi-\delta)s}\langle \lambda_1(a,s)P_2(t)p_1, p_1\rangle ds.$$

By (H_1) − (H_6)

$$2E\int_0^t e^{(\xi-\delta)s}\langle dp_1, p_1\rangle ds$$
$$\le [(\beta_1^0)^2 M_1^2 a_+^2 - 2\mu_{10} + 2A_1 r_1 - 2B_1 r_1] \quad (6)$$
$$\int_0^t e^{(\xi-\delta)s} E(|p_1|^2 + |p_2|^2 + |p_3|^2)ds.$$

$$2E\int_0^t e^{(\xi-\delta)s}\langle dp_2, p_2\rangle ds$$
$$\le [(\beta_2^0)^2 M_2^2 a_+^2 - 2\mu_{20} + 2A_2 r_2 - 2B_3 r_3] \quad (7)$$
$$\int_0^t e^{(\xi-\delta)s} E(|p_1|^2 + |p_2|^2 + |p_3|^2)ds.$$

$$2E\int_0^t e^{(\xi-\delta)s}\langle dp_3, p_3\rangle ds$$
$$\le [(\beta_3^0)^2 M_3^2 a_+^2 - 2\mu_{30} + 2A_3 r_2] \quad (8)$$
$$\int_0^t e^{(\xi-\delta)s} E(|p_1|^2 + |p_2|^2 + |p_3|^2)ds.$$

By Lipschitz(H_7),

$$\| f_i \|_2^2 \le K_i |p_i|^2, i = 1,2,3.$$

Then

$$e^{(\xi-\delta)t} E(|p_1|^2 + |p_2|^2 + |p_3|^2)$$
$$\le E(|p_{10}|^2 + |p_{20}|^2 + |p_{30}|^2)$$
$$+(\xi-\delta)\int_0^t e^{(\xi-\delta)s} E(|p_1|^2 + |p_2|^2 + |p_3|^2)ds$$
$$+[(\beta_1^0)^2 M_1^2 a_+^2 - 2\mu_{10} + 2A_1 r_1 - 2B_1 r_1 + K_1]$$
$$\int_0^t e^{(\xi-\delta)s} E(|p_1|^2 + |p_2|^2 + |p_3|^2)ds$$
$$+[(\beta_2^0)^2 M_2^2 a_+^2 - 2\mu_{20} + 2A_2 r_2 - 2B_3 r_3 + K_2]$$
$$\int_0^t e^{(\xi-\delta)s} E(|p_1|^2 + |p_2|^2 + |p_3|^2)ds$$
$$+[(\beta_3^0)^2 M_3^2 a_+^2 - 2\mu_{30} + 2A_3 r_2 + K_3]$$
$$\int_0^t e^{(\xi-\delta)s} E(|p_1|^2 + |p_2|^2 + |p_3|^2)ds \quad (9)$$
$$\le E(|p_{10}|^2 + |p_{20}|^2 + |p_{30}|^2) + (\xi-\delta-\tau)$$
$$\int_0^t e^{(\xi-\delta)s} E(|p_1|^2 + |p_2|^2 + |p_3|^2)ds.$$

where,

$$\tau = 2Br + 2\mu_0 - (\beta^0)^2 M^2 a_+^2 - 2AR - K,$$
$$K = maxK_i, \quad A = maxA_i, \quad R = maxr_i,$$
$$\beta_0 = max\beta_{i0}, \quad \mu_0 = min\mu_{i0}, \quad B = minB_i,$$
$$r = minr_i, \ i = 1,2,3.$$

On applying the by Gronwall's lemma， we then have the following inequality:

$$E(|p_1|^2 + |p_2|^2 + |p_3|^2) \le Ce^{-\tau t}, \forall t \ge 0$$

The proof is proved.

We shall assume the following generalized coercivity condition (H):

There exist constants $\alpha > 0, \xi > 0, \lambda \in R,$ and a nonnegative continuous function $\gamma(t), t \in R^+,$

such that

$$\| f(t,p_i)\|_2^2 \le -\alpha \| p_i \|^2 + \lambda |p_i|^2 + \gamma(t)e^{-\xi t},$$
$$\forall p_i \in C(0,T; L^2(0,a_+;R^3)) \bigcap L^\infty(Q_T;R^3), \ a.e.t,$$

where,for arbitrar $\delta > 0, \ \gamma(t)$ satisfies

$$\gamma(t) = o(e^{\delta t}), \text{ as } t \to \infty, i.e., \lim_{t\to\infty} \gamma(t)/e^{\delta t} = 0,$$

As follows we give another theorem of this paper.

Theorem 3.2 Assume the preceding hypotheses and (H) hold, if (p_1, p_2, p_3) is a global strong solution to Eq(2). there exist contants $\nu > 0, C > 0$ such that

$$E(|p_1|^2 + |p_2|^2 + |p_3|^2) \le Ce^{-\nu t}, \forall t \ge 0 \quad (10)$$

Proof. We can choose $\delta > 0$ small enough such that $\xi - \delta > 0$ Then, by $It\hat{o}$ formula , we have

$$e^{(\xi-\delta)t} E(|p_1|^2 + |p_2|^2 + |p_3|^2)$$

Journal of Theoretical and Applied Information Technology
20th January 2013. Vol. 47 No.2

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

$$= E(|p_{10}|^2 + |p_{20}|^2 + |p_{30}|^2)$$
$$+(\xi-\delta)\int_0^t e^{(\xi-\delta)s} E(|p_1|^2 + |p_2|^2 + |p_3|^2)ds$$
$$+2E\int_0^t e^{(\xi-\delta)s}\langle dp_1, p_1\rangle ds + 2E\int_0^t e^{(\xi-\delta)s}\langle dp_2, p_2\rangle ds$$
$$+2E\int_0^t e^{(\xi-\delta)s}\langle dp_3, p_3\rangle ds$$
$$+E\int_0^t e^{(\xi-\delta)s}(\|f_1\|_2^2 + \|f_2\|_2^2 + \|f_3\|_2^2)ds.$$
(11)

By $(H_1)-(H_6)$

$$e^{(\xi-\delta)t}E(|p_1|^2 + |p_2|^2 + |p_3|^2)$$
$$\le E(|p_{10}|^2 + |p_{20}|^2 + |p_{30}|^2)$$
$$+(\xi-\delta)\int_0^t e^{(\xi-\delta)s} E(|p_1|^2 + |p_2|^2 + |p_3|^2)ds$$
$$+[(\beta_1^0)^2 M_1^2 a_+^2 - 2\mu_{10} + 2A_1r_1 - 2B_1r_1]$$
$$\int_0^t e^{(\xi-\delta)s} E(|p_1|^2 + |p_2|^2 + |p_3|^2)ds$$
$$+[(\beta_2^0)^2 M_2^2 a_+^2 - 2\mu_{20} + 2A_2r_2 - 2B_3r_3]$$
$$\int_0^t e^{(\xi-\delta)s} E(|p_1|^2 + |p_2|^2 + |p_3|^2)ds$$
$$+[(\beta_3^0)^2 M_3^2 a_+^2 - 2\mu_{30} + 2A_3r_2]$$
$$\int_0^t e^{(\xi-\delta)s} E(|p_1|^2 + |p_2|^2 + |p_3|^2)ds$$
$$+E\int_0^t e^{(\xi-\delta)s}(\|f_1\|_2^2 + \|f_2\|_2^2 + \|f_3\|_2^2)ds$$

$$\le E(|p_{10}|^2 + |p_{20}|^2 + |p_{30}|^2)$$
$$+(\xi-\delta+(\beta^0)^2M^2a_+^2 + 2AR - 2Br - 2\mu_0)$$
$$\int_0^t e^{(\xi-\delta)s} E(|p_1|^2 + |p_2|^2 + |p_3|^2)ds$$
$$+E\int_0^t e^{(\xi-\delta)s}(\|f_1\|_2^2 + \|f_2\|_2^2 + \|f_3\|_2^2)ds. \quad (12)$$

$where, A = maxA_i, R = maxr_i, \beta_0 = max\beta_{i0}$, $M = maxM_i, \mu_0 = min\mu_{i0}$, $B = minB_i$, $r = minr_i$, i=1,2,3.

By (H)

$$e^{(\xi-\delta)t}E(|p_1|^2 + |p_2|^2 + |p_3|^2)$$
$$\le E(|p_{10}|^2 + |p_{20}|^2 + |p_{30}|^2)$$
$$+(\xi-\delta+(\beta^0)^2M^2a_+^2 + 2AR - 2Br - 2\mu_0)$$
$$\int_0^t e^{(\xi-\delta)s} E(|p_1|^2 + |p_2|^2 + |p_3|^2)ds$$
$$-\alpha\int_0^t e^{(\xi-\delta)s} E(\|p_1\|_2^2 + \|p_2\|_2^2 + \|p_3\|_2^2)ds$$
$$+\lambda\int_0^t e^{(\xi-\delta)s} E(|p_1|^2 + |p_2|^2 + |p_3|^2)d$$
$$+3\int_0^t \gamma(s)e^{-\delta s}ds$$

$$\le E(|p_{10}|^2 + |p_{20}|^2 + |p_{30}|^2)$$
$$+(\xi-\delta-\nu)\int_0^t e^{(\xi-\delta)s} E(|p_1|^2 + |p_2|^2 + |p_3|^2)ds$$
$$+3\int_0^t \gamma(s)e^{-\delta s}ds. \quad (13)$$

Where $\nu = \alpha/m^2 - \lambda + 2Br + 2\mu_0 - (\beta^0)^2 M^2 a_+^2 - 2AR,$

If $\xi - \nu \le 0$, it follows immediately

$$e^{(\xi-\delta)t}E(|p_1|^2 + |p_2|^2 + |p_3|^2)$$
$$\le E(|p_{10}|^2 + |p_{20}|^2 + |p_{30}|^2) + 3\int_0^t \gamma(s)e^{-\delta s}ds. \quad (14)$$

which means that there exists a positive constant $k = k(\delta)$ such that

$$E(|p_1|^2 + |p_2|^2 + |p_3|^2)$$
$$\le (E(|p_{10}|^2 + |p_{20}|^2 + |p_{30}|^2) + k(\delta)e^{-(\xi-\delta)t}. \quad (15)$$

On the other hand , if $\xi - \nu > 0$, we can choose $\delta > 0$ small enough such that $\xi - \delta - \nu > 0$. Then , from (13) and by Gronwall's lemma can obtain

$$e^{(\xi-\delta)t}E(|p_1|^2 + |p_2|^2 + |p_3|^2)$$
$$\le (E(|p_{10}|^2 + |p_{20}|^2 + |p_{30}|^2)$$
$$+3\int_0^t \gamma(s)e^{-\delta s}ds)e^{(\xi-\delta-\nu)t}.$$

and, once again, there exists a positive constant $k(\delta) > 0$ such that

$$E(|p_1|^2 + |p_2|^2 + |p_3|^2)$$
$$\le (E(|p_{10}|^2 + |p_{20}|^2 + |p_{30}|^2) + k(\delta)e^{-\nu t}.$$

The proof of this Theorem3.2 is complete.

Then we obtain the exponential stability of strong solution for stochastic predator-prey system of three

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

species with age structure by Theorem3.1and Theorem3.2.

4. CONCLUSION

In this paper, we discussed the exponential stability of strong solution for Eq(2). As the exponential stability of strong solution for a stochastic disturbance predator-prey system of three species with age-structure is reached out under the condition of requirements from (H1) to (H7) and (H) being satisfied, we plan to weaken the conditions in the future to substitute local Lipschitz to global Lipschitz and make a further proof of exponential stability of Eq(2).

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