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# ANALYSIS ON THE CONVERGENCE OF A NOVEL BILINEAR ADAPTIVE FILTER

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**ABSTRACT**

Although the volterra filter is known to be an efficient adaptive filter in a large variety application for the non-linear system, but its computational complexity is also very high, especially as the orders of the filter increase. To overcome the computational complexity of the Volterra filter, a novel adaptive filter using layered bilinear architecture is proposed in this paper. Compared with the conventional second-order Volterra filter and Direct Bilinear adaptive filter, the layered bilinear adaptive filter exhibits a slightly better convergence performance in terms of convergence speed and steady-state error.

**Keywords:** *Adaptive, Nonlinear System, Bilinear, Volterra Filter*

## 1. INTRODUCTION

The classical adaptive filter has been widely applied in the linear signal processing fields for its well developed theory and easy realization. But to the nonlinear system such as weather forecasting and target tracking, its performance will decrease dramatically. However, a series of nonlinear adaptive filter algorithm have been set up for the application of nonlinear system including communication channel equalization and echo cancellation, among which the volterra filter with its truncated version and the bilinear filter with polynomial series structure become more and more popular recently.

The algebraic model of the volterra filter can be described as the following equation.

$$y(n) = h_0 + \sum_{\tau_1=0}^{\infty} h_1(\tau_1)x(n-\tau_1) + \sum_{\tau_1=0}^{\infty}\sum_{\tau_2=\tau_1}^{\infty} h_2(\tau_1,\tau_2)x(n-\tau_1) x(n-\tau_2) + \ldots\ldots + \sum_{\tau_1=0}^{\infty}\sum_{\tau_2=0}^{\infty}\ldots\ldots\sum_{\tau_m=0}^{\infty} h_m(\tau_1,\tau_2,\ldots\ldots\tau_m) x(n-\tau_1)x(n-\tau_2)\ldots\ldots x(n-\tau_m) \quad (1)$$

The relationship between the output signal y(n) of the nonlinear system and its input x(n) is a little bit clear in such model. That is to say the volterra filter depends linearly on the coefficients of the filter itself. In other words, the volterra filter may be interpreted as extensions of linear filters to the nonlinear case. In this paper, we only concentrate on truncated second-order volterra filters of the following form:

$$y(n) = h_0 + \sum_{\tau_1=0}^{\infty} h_1(\tau_1)x(n-\tau_1) + \sum_{\tau_1=0}^{\infty}\sum_{\tau_2=\tau_1}^{\infty} h_2(\tau_1,\tau_2)x(n-\tau_1)x(n-\tau_2) \quad (2)$$

The bilinear filter is the other case of the adaptive polynomial filter. The algebraic model of the bilinear filter can be described as the following equation.

$$y(n) = \sum_{i=0}^{\infty} a_n(i)x(n-i) + \sum_{i=0}^{\infty}\sum_{j=1}^{\infty} b_n(i,j)x(n-i) y(n-j) + \sum_{j=1}^{\infty} c_n(j)y(n-j) \quad (3)$$

where the coefficients an(i) , bn(i,j)and cn(j) can have different lengths. However, we use the same length for simplicity in deriving the corresponding bilinear algorithm.

It is easy to find that the bilinear filter is the recursive form of the volterra filter. Based on the analysis of the algebraic model of the bilinear filter, a novel architecture of the filter is proposed here as shown in the figure 1.1. In this paper, we also concentrate on truncated second-order bilinear filter of the following form:

$$y(n) = \sum_{i=0}^{\infty} a_1(i)x(n-i) + \sum_{i=0}^{\infty} c_1(i)y_1(n-i) + \sum_{i=0}^{\infty}\sum_{j=1}^{\infty} b_2(i,j) x(n-i)y_1(n-j) + \sum_{j=1}^{\infty} d_2(j)y_2(n-j) \quad (4)$$

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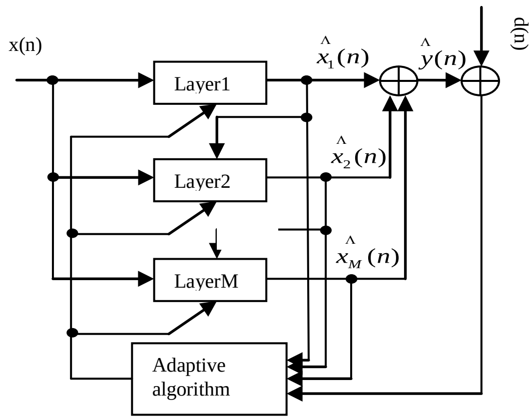


*Fig. 1 Layered Bilinear Adaptive Filter*

## 2. ADAPTIVE ALGORITHM OF THE PROPOSED NOVEL FILTER

Based on the analysis of the novel architecture of the bilinear polynomial filter, it is easy to find that it is actually a layered architecture which is designed by modularization technology, different layer means different order and the coefficients of different layer are irrelated. Therefore, according to the Least Mean Square(LMS) algorithm, we can get the updating equation of each layer's coefficient as following:

$$p(n+1) = p(n) + \tau e(n)\frac{\partial y(n)}{\partial p(n)} = p(n) + \tau e(n)\frac{\partial y_k(n)}{\partial p(n)} \quad (5)$$

where p denotes the coefficient to be adapted of the certain layer and $\tau$ is the step-size parameter.

From the equation, we can find that the gradient of the overall filtering system output y(n) with respect to the coefficient p of a certain layer is equal to the gradient of the layer output $y_k(n)$ with respect to the coefficient p since the coefficient p of the certain layer is not related to the outputs of the other layers. Thus we can get the adaptive algorithms of the linear subsection and nonlinear subsection respectively as the quadratic filters are concerned.

### 2.1 Linear Subsection

As it is shown in equation 4 and figure 1, the algorithm of the linear subsection is to minimize the instantaneous squared error between the output y(n)and the desired response d(n). Thus the updating equation of corresponding weight vector a(n) and c(n) can be derived as the following form:

$$a_1(n+1) = a_1(n) + \tau_1 e(n)x(n)$$
$$c_1(n+1) = c_1(n) + \tau_1 e(n)y_1(n) \quad (6)$$

where $a_1(n)$ and $c_1(n)$ are the weight vectors to be adapted of the layered bilinear filter ,furthermore, we can get the updating equation of the vectors in accordance with the normalized least mean square (NLMS) algorithm:

$$a_1(n+1) = a_1(n) + \tau_1 e(n)\frac{x(n)}{\|x(n)\|_2^2}$$
$$c_1(n+1) = c_1(n) + \tau_1 e(n)\frac{y_1(n)}{\|y_1(n)\|_2^2} \quad (7)$$

where symbol $\|\bullet\|_2$ denotes the norm operator, and $\tau_1$ is the step-size parameter of the linear subsection.

### 2.2 Nonlinear Subsection

To the truncated second-order bilinear adaptive filter, the nonlinear subsection is the second-order subsection of the bilinear adaptive filter, shown as layer 2 of the layered bilinear filter in the Fig. 1 and according to the equation 5, the weight factors updating equation can be derived as the following form:

$$b_2(n+1) = b_2(n) + \tau_2 e(n)x^T(n)y_1(n)$$
$$d_2(n+1) = d_2(n) + \tau_2 e(n)y_2(n) \quad (8)$$

where $b_2(n)$ and $d_2(n)$ are the weight vectors to be adapted, and $\tau_2$ is the step-size parameter of the nonlinear subsection.

## 3. ANALYSIS OF THE CONVERGENCE

Since the layered bilinear adaptive filter is the recursive version of the volterra filter, the novel algorithm is subject to the possibility of unstable. To determine the condition for the stability of the layered bilinear adaptive filter algorithm, we use the algebraic model of the truncated second-order bilinear filter and analyze the linear subsection and nonlinear subsection respectively.

### 3.1 Linear Subsection

For the analysis of the linear subsection, we Choose the weight factor a1 as example and the weight factor c1 has the same situation.

We begin the analysis by rewriting the updating equation of weight factor a1 as the following form:

$$a_1(n+1) = a_1(n) + \tau_1 d(n)x(n) - \tau_1 x(n)x^T(n)a_1(n)$$
$$= [I_M - \tau_1 x(n)x^T(n)]a_1(n) + \tau_1 d(n)x(n) \quad (9)$$

Where IM is the $M \times M$ identity matrix.

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By defining the weight-error vector of $a_1$ at time n as equation 10, we can do some transformation to the equation 9 and get the updating equation of the weight-error vector $\hat{a}_1(n)$ shown as equation 11.

$$\hat{a}_1(n) = a_1(n) - a_{1opt} \quad (10)$$

where $a_{1opt}$ is the optimum value of the weight vector and $a_1(n)$ is the value of the weight vector at time n.

$$\hat{a}_1(n+1) = [I_M - \tau_1 x(n)x(n)^T]\hat{a}_1(n) + \tau_1 d(n)x(n) - \tau_1 x(n)x(n)^T a_{1opt}(n) \quad (11)$$

Based on the equation 1.11, we can get the the updating equation of the vector $\hat{a}_1(n)$ in accordance with the normalized least mean square (NLMS) algorithm as following:

$$\hat{a}_1(n+1) = [I_M - \tau_1 \frac{x(n)x(n)^T}{\|x(n)\|_2^2}]\hat{a}_1(n) + \tau_1 \frac{d(n)x(n)}{\|x(n)\|_2^2} - \tau_1 \frac{x(n)x^T(n)}{\|x(n)\|_2^2} a_{1opt}(n) \quad (12)$$

Accordingly, we can get the expectation of the vector $\hat{a}_1(n)$, and with the application of the principle of orthogonality, the third expectation minus the fourth expectation is equal to zero. Thus we may rewrite the equation 1.12 as the following form:

$$E\left[\hat{a}_1(n+1)\right] = [I_M - \tau_1 \frac{R_{xx}}{\|x(n)\|_2^2}]E\left[\hat{a}_1(n)\right] \quad (13)$$

where $R_{xx}$ is the correlation matrix of the tap inputs x(n), x(n-1), ……, x(n-M+1) in the layered bilinear filter of Fig. 1, that is

$$R_{xx} = E\left[x(n)x^T(n)\right] \quad (14)$$

Since the eigenvalues of the matrix $I_M - \tau_1 \frac{R_{xx}}{\|x(n)\|_2^2}$ are 1 and $1-\tau_1$ whose multiplicity are M-i and i respectively. Finally we can get the necessary and sufficient condition for the convergence or stability of the linear subsection of the layered bilinear adaptive algorithm is that the step-size parameter $\tau_1$ satisfy the double inequality:

$$0 < \tau_1 < 2 \quad (15)$$

### 3.2 Nonlinear Subsection

For the analysis of the nonlinear subsection of the truncated second-order bilinear filter, we choose the weight factor $b_2$ as example and the weight factor $d_2$ has the same situation.

We can begin the analysis by rewriting the updating equation of weight factor $b_2$ as the following form:

$$\begin{aligned} b_2(n+1) &= b_2(n) + \tau_2 e(n)x^T(n)y_1(n) \\ &= b_2(n) + \tau_2\{d(n) - y_1^T(n)x(n)\}x^T(n)y_1(n) \\ &= \left\{I_M - \tau_2 y_1^T(n)x(n)x^T(n)y_1(n)\right\}b_2(n) + \\ &\quad \tau_2 d(n)x^T(n)y_1(n) \end{aligned} \quad (16)$$

Furthermore, the equation 16 in accordance with the normalized least mean square (NLMS) algorithm can be rewritten as following:

$$b_2(n+1) = \left\{I_M - \tau_2 \frac{y_1^T(n)x(n)x^T(n)y_1(n)}{\|x^T(n)y_1(n)\|_2^2}\right\}b_2(n) + \tau_2 \frac{d(n)x^T(n)y_1(n)}{\|x^T(n)y_1(n)\|_2^2} \quad (17)$$

As the way we do in the analysis of convergence of the linear subsection, we define the weight-error vector of $b_2$ at time n as

$$\hat{b}_2(n) = b_2(n) - b_{2opt} \quad (18)$$

Thus we can get the updating equation of the weight-error vector $\hat{b}_2(n)$ as following:

$$b_2(\hat{n}+1) = \left\{I_M - \tau_2 \frac{y_1^T(n)x(n)x^T(n)y_1(n)}{\|x^T(n)y_1(n)\|_2^2}\right\}b_2(\hat{n}) + \tau_2 \frac{d(n)x^T(n)y_1(n)}{\|x^T(n)y_1(n)\|_2^2} - \tau_2 \frac{y_1^T(n)x(n)x^T(n)y_1(n)}{\|x^T(n)y_1(n)\|_2^2} b_{2opt} \quad (19)$$

Based on the equation 19, we can get the expectation of the vector $\hat{b}_2(n)$, and according to the principle of orthogonality, the third expectation minus the fourth expectation is equal to zero. Thus we may rewrite the equation 1.19 as the following form:

$$E\left[\hat{b}_2(n+1)\right] = [I_M - \tau_2 \frac{R_{vv}}{\|x^T(n)y_1(n)\|_2^2}]E\left[\hat{b}_2(n)\right] \quad (20)$$

Finally we can get the necessary and sufficient condition for the convergence or stability of the nonlinear subsection of the layered bilinear adaptive algorithm is that the step-size parameter $\tau_2$ satisfy the double inequality:

$$0 < \tau_2 < 2 \quad (21)$$

## 4. SIMULATIONS AND CONCLUSION

In this section, extensive simulation studies are carried out to evaluate the performance of the novel bilinear adaptive filter presented in this paper. The

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performance of the presented layered bilinear adaptive filter is compared to that of the direct bilinear adaptive filter and the volterra filter in terms of convergence speed, stable-state error.

In the simulation, we choose the unknown system models as following:

$$d(n) = x(n) + x(n)^2 + x(n)^3 \tag{22}$$

where d(n) is the output signal of the unknown system, and x(n) is the input signal. The measurement noise v(n) is zero-mean and white Gaussian sequence which is uncorrelated with x(n). Moreover, the step-size parameters $\tau_1$ and $\tau_2$ are set to be 0.1 and 0.2 respectively. In the simulation, we selected 2000 iterations for the case. By running 2000 iterations with 200 independent experiments, and the average experimental results are plotted in the following figure 2.

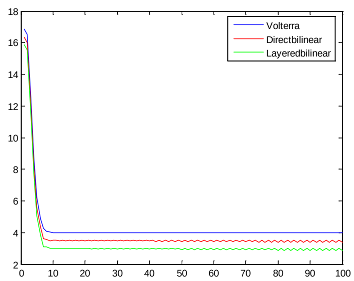


*Fig.2 Performance Of Layered Bilinear Adaptive Filter*

The novel bilinear adaptive filter with layered architecture presented in this paper is shown to be adaptive filter by the analysis and simulation. Therefore the layered bilinear adaptive filter is very attractive to be implemented in the practice.

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