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A THRESHOLD DENOISING METHOD BASED ON EMD

JIANZHAO HUANG, JIAN XIE, FENG LI, LIANG LI
Xi'an High-tech Research Institute, Xi'an 710025, Shanxi, China

ABSTRACT

The denoising method based on empirical mode decomposition (EMD) can be broadly divided into: IMF extraction method and IMF threshold approach. Aiming to the problems of how to select IMFs in extraction method and the processing of the selected IMFs, a threshold denoising method based on EMD is put forward. In this method, the standard of IMF selection in energy viewpoint is offered, and the IMFs upping to the standard are selected firstly, then, through comparing the average energy of all unselected IMFs with the energy of each selected IMF, the singular selected IMFs are confirmed, and denoised by threshold. Finally, the denoised signal is obtained by summing up all selected IMFs. The current method combines the soft threshold denoising method with the IMF selection together, compared with other denoising methods, the effectiveness and superiority of the method is validated. The result provides support for improving the denoising effect in engineering.

Keywords: *EMD, Threshold Denoising, IMF Selection, Singular IMF*

1. INTRODUCTION

The Empirical Mode Decomposition (EMD) has been proposed as an adaptive time-frequency data analysis method [1]. The major advantage of the EMD is that the basis functions are derived from the signal itself. It has been proved quite versatile in a broad range of applications for extracting signals from data generated in noisy nonlinear and non-stationary processes. By studying the filtering properties of the EMD, it is found that the EMD has the similar binary filter characteristics as the wavelet transform [2]. As in wavelet analysis, the energy will often be concentrated on the high frequency temporal modes and decreases towards the coarser ones [3]. According to this idea, there will be a mode after which the energy distribution of the signal overcomes that of the noise. This particular mode, allows us to separate signal from noise. Modes coarser than this particular mode are dominated by the signal, while finer modes are noise dominated.

Based on the principle mentioned above, the denoisng method based on EMD can be broadly divided into two directions: (1) IMF extraction method. IMFs are selected without any processing, and summed up to get the denoised signal. In article [4], the mini-neighboring root mean square error is used as the standard to select IMFs, the denosing result is compared with the average, median and wavelet filtering methods. Based on the EMD decomposition characteristics of white noise [5], the product of the energy density and the average period is calculated, the trip point of the product is considered as the standard to select IMFs [6], but the quantitative indicator of the trip point is not given in the paper. (2) The threshold approach. IMFs are dealt with the threshold function. For noise reduction of the speech signal [7], all decomposed IMFs are dealt with hard threshold function, and the method does better than the wavelet denosing method. In article [8], EMD and soft threshold denoising method are combined together, and all IMFs are dealt with the soft threshold function. A mode cell is defined as the signal between the two adjacent zero-crossings among an IMF [9], the denoising process is to make the cell's choice, and a single data is replaced by an oscillating unit in the article. The problem in this method is when the unit is rejected, the useful signal in the cell will be loss at the same time. Based on the work mentioned above, a threshold denoising method based on EMD is presented in this paper. Firstly, the algorithm based on the energy to determine the trip point is designed for IMF selection, then, by comparing the energy of the selected IMFs with excluded IMFs, singular selected IMFs are dealt with soft threshold function, and finally the denoised signal is obtained by summing up the selected IMFs. Compared with other denoising methods under different noise intensity, it is proved that the best IMFs can be summed up and properly denoised by the proposed method.

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2. THEORY OVERVIEW

2.1 EMD Basic

The EMD decomposes a given signal $x(t)$ into a series of IMFs through an iterative process called sifting; each one with a distinct time scale [1]. By definition, an IMF satisfies two conditions: (1) the number of extrema and the number of zeros crossings may differ by no more than one; (2) the average value of the envelope defined by the local maxima, and the envelope defined by the local minima, is zero.

Given a signal, the effective algorithm of EMD can be summarized as follows [1].

1) Identify all extrema of $x(t)$.

2) Interpolate between minima (resp. maxima), ending up with some "envelope" $e_{\min}(t)$ (resp. $e_{\max}(t)$).

3) Compute the average values $m(t)$, $m(t) = (e_{\min}(t) + e_{\max}(t))/2$.

4) Extract the detail $d(t) = x(t) - m(t)$.

5) Iterate on the residual $m(t)$.

At last, EMD ends up with a representation of the form:

$$x(t) = m_k(t) + \sum_{k=1}^{K} d_k(t) \tag{1}$$

Where $m_k(t)$ stands for a residual "trend" and the "modes" $\{d_k(t), k = 1, \ldots K\}$ are constrained to be zero-mean amplitude modulation frequency modulation waveforms.

2.2 IMF SELECTION

The statistic characteristics of white noise decomposed by EMD are summed up as follows [5]: the IMFs are all normally distributed, and the Fourier spectra of the IMFs are all identical and cover the same area on a semi-logarithmic period scale, and the product of the energy density of IMF and its corresponding averaged period is a constant, and that the energy-density function is chi-squared distributed. The energy density of IMF and its corresponding averaged period are defined as follows:

$$E_n = \frac{1}{N}\sum_{i=1}^{N}(c_n(i))^2 \tag{2}$$

$$\overline{T_n} = \frac{N}{N_{max}} \tag{3}$$

$$E_n\overline{T_n} = const \tag{4}$$

Where c_n is the n-th IMF, E_n is the energy density, and N is the length of the data; $\overline{T_n}$ is the average period, N_{max} is the maximum numbers of the c_n. The product of the energy density and the corresponding average period is a constant.

The E_n, $\overline{T_n}$ and $E_n\overline{T_n}$ of each IMF are calculated in accordance with equations (2)-(4). Because the $E_n\overline{T_n}$ of white noise is a constant and the high-frequency IMF is usually the noise. So there will be a trip point in the curve of $E_n\overline{T_n}$. Excluding all IMFs before the trip point, the summation of left IMFs is the denoised signal [6].

3. THRESHOLD DENOISING METHOD

3.1 The Problem

To study the characteristics of the energy-based IMF extraction method, a simulation signal $x(t)$ is used to do analysis. The simulation signal is superimposed by three sinusoidal signals corresponding to the period of 1200s, 600s and 50s, the sampling interval is 1s, and the sampling points are taken as 4,000 points. Adding different intensity of Gaussian white noise with the simulation signal $x(t)$. The noise variances σ are from 0.5 to 4, arithmetic increased by 0.5. The $E_n\overline{T_n}$ of each IMF is calculated under different noise intensity according to the equations (2)-(4). The simulation results show that: the total numbers of IMFs are different under different noise intensity, the variance greater the more. For the random of white noise generated by Matlab software, the results will be differences for each running, but this randomness does not affect the statistical properties of white noise. The average $E_n\overline{T_n}$ of top eight IMFs for 100 experiments is shown in Table 1.

As can be seen from the Table 1, the averages $E_n\overline{T_n}$ of IMFs under different noise intensity are difference. View from the portrait, the average $E_n\overline{T_n}$ of IMFs increases totally with the increased noise intensity, and there are individual circumstances, such as IMF7 column; View from the landscape, the average $E_n\overline{T_n}$ values don't obey the law of gradual increase with the decomposition numbers, there are singular IMFs, such as IMF6 column for each row. The judgment standard of trip point is not given, and there is no discussion of the singular IMFs in article [6]. So there are two problems need to be solved in practical applications: (1) the quantitative indicators of the trip point under different noise intensity; (2) the processing method of the singular IMFs.

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Table 1: The Average $E_n\overline{T_n}$ Of Top Eight Imfs For 100 Experiments

Noise Intensity \ IMFs	1	2	3	4	5	6	7	8
0.5	0.8726	0.6207	0.6474	11.78	14.27	2.653	117.2	2.27e+03
1.0	1.749	1.237	1.276	7.276	18.53	3.217	67.02	7.72e+04
1.5	2.631	1.861	1.918	6.091	20.95	4.143	43.22	1.37e+03
2.0	3.497	2.484	2.517	6.217	22.65	5.173	32.28	3.27e+05
2.5	4.379	3.086	3.131	6.588	22.84	6.191	20.41	292.4
3.0	5.277	3.700	3.738	6.935	23.54	6.578	21.50	5.59e+07
3.5	6.164	4.375	4.534	7.822	24.06	7.333	54.31	3.96e+07
4.0	6.997	4.948	5.028	7.608	25.21	8.729	19.74	361.23

3.2 Optimizd Threshold Denoising Method

Aiming to the two problems mentioned above, an optimized threshold denoising method based on EMD is put forward. The specific steps of the method are as follows:

1) Obtain the IMFs by EMD.

2) Calculate the energy density, the average period and the product for each IMF by using the equations (2)-(4).

3) Determine the trip point. Definite

$$Q_n = E_{n+1}\overline{T_{n+1}} / E_n\overline{T_n} \quad (n = 1, 2, 3\cdots, N-1) \tag{5}$$

Where N is the total number of IMFs. After large simulation experiments, the first IMF satisfying the condition $Q_n > 2$ is considered as the trip point.

4) Calculate the average value of all IMFs before the trip point:

$$ET_{ave} = mean(\sum_{i=1}^{n} E_n\overline{T_n}) \tag{6}$$

5) Definite the IMF component meeting the condition $E_m\overline{T_m} < 2 * ET_{ave} \quad (m = n+1, n+2, \cdots, N-1)$ as the singular IMF. If there is a singular IMF existing, do the soft threshold function [10]. The threshold is estimated by the following formula:

$$\mathrm{thr}_j = \frac{(median(abs(IMF_j))/0.6745) * \sqrt{2\ln M}}{\ln(j+1)} \tag{7}$$

Where M is the length of the signal, j means the $j-th$ IMF.

6) After steps 1) ~ 5), summing up all IMFs after the $n-th$ IMF and the “trend” component.

4. DENOISING EXPERIMENTS

To validate the feasibility and effectiveness of the proposed method, compared with the literature [4], [6] and [7], simulation experiments are used to do test. The trip point selecting method is not provided in the literature [6], so the proposed method in the article is employed for literature [6] in the programming. The signal to noise ratio (SNR), root mean square error (RMSE) and correlation coefficient (R) are served as the denoising evaluation index.

4.1 Low Frequency Experiment

The simulation signal $x(t)$ is superimposed by two sinusoidal signals with the period of 1200s and 600s, and each sinusoidal signal’s amplitude value is 1. The sampling interval is 1s, and the sampling points are taken as 4,000 points. Adding different intensity of Gaussian white noise with the simulation signal $x(t)$. The noise variances σ are from 0.5 to 4, arithmetic increased by 0.5. Due to limited space, only the denoised signals under two different noise variances conditions are shown in Fig.1 and Fig.2. The SNR, RMSE and R of the four different methods corresponding to different noise intensity are shown in Table 2.

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Table 2: Evaluation Indexes Of Four Methods Under Different Noise Intensity Experiments

evaluation index	method	noise variances							
		0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
SNR	literature [4]	17.35	18.54	13.60	15.61	12.76	14.27	12.78	10.14
	literature [6]	19.99	19.32	15.45	17.42	14.85	16.14	14.97	11.33
	literature [7]	17.74	15.30	12.45	11.30	10.81	9.972	7.713	9.093
	this article	19.99	19.32	15.45	17.42	14.85	16.14	14.97	11.33
RMSE	literature [4]	0.1389	0.1210	0.2136	0.1696	0.2354	0.1979	0.235	0.3184
	literature [6]	0.1024	0.1107	0.1728	0.1377	0.1851	0.1595	0.1825	0.2777
	literature [7]	0.1327	0.1757	0.2441	0.2787	0.2948	0.3245	0.4209	0.3591
	this article	0.1024	0.1107	0.1728	0.1377	0.1851	0.1595	0.1825	0.2777
R	literature [4]	0.9908	0.9929	0.9788	0.9862	0.974	0.9811	0.9755	0.9551
	literature [6]	0.9952	0.9943	0.986	0.9909	0.9837	0.9877	0.9863	0.9653
	literature [7]	0.9916	0.9856	0.9725	0.9619	0.9576	0.9481	0.9139	0.9412
	this article	0.9952	0.9943	0.986	0.9909	0.9837	0.9877	0.9863	0.9653

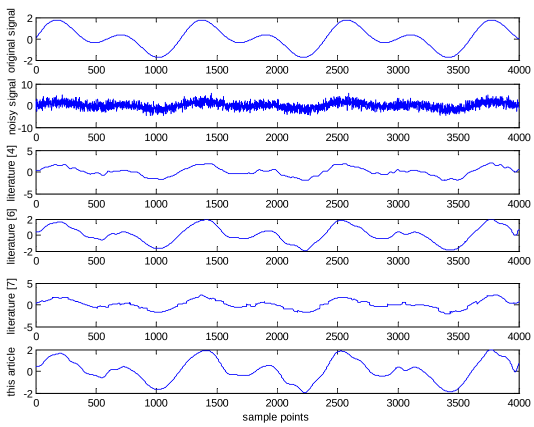

Figure 1: Denoising Under Noise Density 1.5

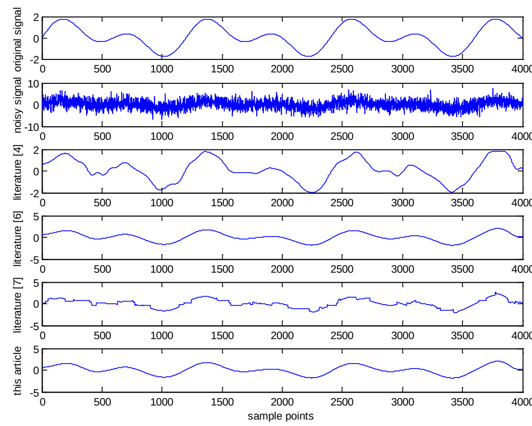

Figure 2: Denoising Under Noise Density 3.0

It can be seen from Table 2, under the same noise intensity, literature [6] and the article method have the same indicators, and they do better than literature [4] and [7]. The reason why literature [6] and the article method have the same indicators is that: through calculating the $E_n\overline{T_n}$ of all IMFs under different noise intensity, it is found that there is none singular IMF existing. That means the step 5) mentioned in optimized threshold de-noising method doesn't work, and the selecting methods of the trip point are the same in literature [6] and the article method. So the denoising indicators are no difference.

4.2 High Frequency Experiment

The simulation signal $x(t)$ is superimposed by three sinusoidal signals with the period of 1200s, 600s and 50s, and each sinusoidal signal's amplitude value is 1. The sampling interval is 1s, and the sampling points are taken as 4,000 points. Compared with the signal in experiment one, the high frequency component is added. Adding different intensity of Gaussian white noise with the simulation signal $x(t)$. The noise variances σ are from 0.5 to 4, arithmetic increased by 0.5. The denoised signals under the noise variances 1.5 and 3.0 are shown in Fig.3 and Fig.4. The SNR, RMSE and R of the four different methods corresponding to different noise intensity are shown in Table 3.

It can be seen from Table 3, the article method does better than the other three methods in all index. Compared with the low frequency experiment, the article method does better than literature [6], this demonstrates that the standard for singular IMF selection is proper and the soft threshold plays the role in the process. And this article method does better than other methods in SNR, RMSE and R.

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Table 3 : Evaluation Indexes Of Four Methods Under Different Noise Intensity Experiments

Evaluation index	method	noise variances 0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
SNR	literature [4]	4.569	4.607	4.345	4.549	4.237	4.237	4.153	3.538
	literature [6]	13.47	10.90	9.152	7.873	6.729	6.729	5.567	4.503
	literature [7]	7.481	4.851	4.466	4.475	3.787	3.787	2.893	3.644
	this article	13.47	11.04	9.118	8.980	8.530	8.530	7.859	6.734
RMSE	literature [4]	0.7349	0.7317	0.7541	0.7366	0.7635	0.7635	0.771	0.8275
	literature [6]	0.2637	0.3547	0.4336	0.5024	0.5731	0.5731	0.6551	0.7405
	literature [7]	0.5256	0.7114	0.7436	0.7429	0.8041	0.8041	0.8914	0.8175
	this article	0.2637	0.349	0.4353	0.4423	0.4658	0.4658	0.5032	0.5728
R	literature [4]	0.8055	0.8071	0.7960	0.8046	0.7905	0.7905	0.7869	0.7568
	literature [6]	0.9776	0.9611	0.9441	0.924	0.9071	0.9071	0.8844	0.8590
	literature [7]	0.9057	0.8194	0.8002	0.8004	0.7608	0.7608	0.6964	0.7586
	this article	0.9776	0.9622	0.9434	0.9345	0.9301	0.9301	0.9207	0.8997

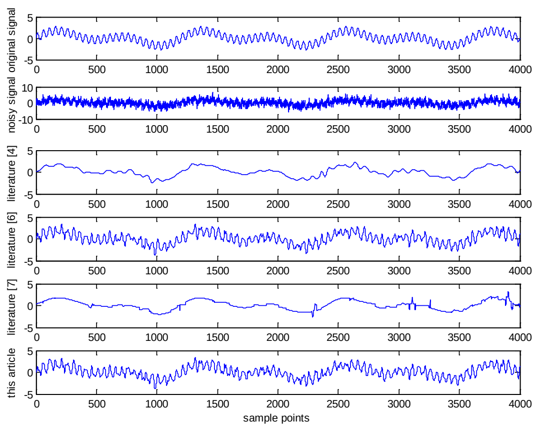

Figure 3: Denoising Under Noise Density 1.5

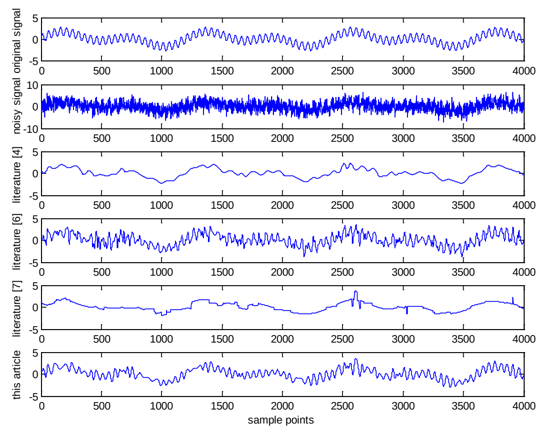

Figure 4: De-noising Under Noise Density 3.0

5. CONCLUSION

Based on the analysis of the denoising method based on EMD, aiming to the problems of the standard for the trip point and the processing of singular IMFs, a threshold denoising method based on EMD is presented. By taking signal to noise ratio, root mean square error and correlation coefficient as the evaluation index, this article method is applied to do denosing experiment for simulation signals with different noise intensity, and compared with other denoising methods. It is proved that this article method can optimize determine the location of the trip point, and do the threshold on singular IMFs. The method of this articlce does better than other three methods in denoising.

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