CHAOS OF HENON MAP BASED ON THE COUPLED NETWORKS

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ABSTRACT

In this paper, the relationship between the parameters of Henon map is studied, and the chaos of Henon map is also discussed based on different coupled networks. From the simulate results, we found with the increasing of parameter *a* the chaos of Henon map will be emerged. At the same time, the transition from non-chaotic to chaotic in coupled networks is exhibited, although the parameters in non-chaotic regions before they are coupled together.

Keywords: *Henon Map, Chaos, Lyapunov Exponent, Coupled Networks*

1. INTRODUCTION

The study of complex networks has most interests to the understanding of behaviors of the network dynamics. In a network, a individual can be represent by the node, the edge is defined relationship of two individuals[1]. With the continuous rise of the chaotic dynamics, chaos in coupled dynamical networks is an increasingly prominent issue, as well as an important work. Henon map is applied to energy forecasting, chemical production, and as well as have a good handle turbulence phenomena[2-3].According to these, we take two-dimensional Henon map as example to apply to some coupled networks, so as to verify the condition of chaotic phenomena in the superposition situation. Meanwhile, we add the nodes' number to N , which can apply to some big coupled networks[4]. Moreover, on this basis ,we analyze the situation of initial-chaotic Henon map in the coupled network, and research in different parameters, the different influences that different coupled networks offer, which can explain more clearly the influence of coupled networks in the Henon map by comparison.

The rest of this paper is organized as follows:
In section 2, the chaos of Henon map is discussed in detail. Based on simulation result, the chaos of three different networks are analyzed in section 3. Finally, conclusions are presents in section 4.

2. LYAPUNOV EXPONENT AND THE HENON MAP

2.1 Lyapunov Exponent

Lyapunov exponent is a kind of factor which can describe the way to get the chaotic state in the iterative process. We can definite as following when symbol λ stands for it[5-7],

$$\lambda = \lim_{n \to \infty} \frac{1}{n} \sum_{i=0}^{n-1} \ln | \frac{dF}{dx} |_{x=x_i} \qquad (1)$$

λ is stand for Lyapunov exponent ,which also means the separated index caused by each iteration index in the amounts of iterations[6]。

If λ>0, two adjacent points move closer to merger into one point, which stands for steady and periodic movement. On the other hand, if λ>0, it means the two adjacent points will eventually move apart, and can be local unstable. If the track also exists a stabilizing factor in the overall, it occur to a chaotic attractor by repeatedly folded [5]。

If the system is a two-dimensional map in the iterative process, it meets

$$\lambda = \lim_{n \to \infty} \frac{1}{n} \sum_{i=0}^{n-1} \ln \left| \frac{dF}{dX} \right|_{x=x_i}$$

$$= \lim_{n \to \infty} \frac{1}{n} \sum_{i=0}^{1} \ln \left| \frac{\|\Delta x_n\|}{\Delta x_0} \right|$$

$$= \lim_{n \to \infty} \frac{1}{n} \sum_{i=0}^{1} \ln \left| \frac{\|\Delta x_n\|}{\|\Delta x_{n-1}\|} \cdots \frac{\|\Delta x_1\|}{\|\Delta x_0\|} \right|$$

$$= \frac{1}{n} \ln(\lambda_0, \lambda_1)$$

(2)

According the definition of Lyapunov exponent, we can know

$$\lambda = \lim_{n \to \infty} \frac{1}{n} \sum_{i=0}^{n-1} \ln \left| \frac{dF}{dX} \right|_{x=x_i}$$

$$= \lim_{n \to \infty} \frac{1}{n} \sum_{i=0}^{1} \ln \left| \frac{\|\Delta x_n\|}{\Delta x_0} \right| \qquad (3)$$

$$= \lim_{n \to \infty} \frac{1}{n} \sum_{i=0}^{1} \ln \left| \frac{\|\Delta x_n\|}{\|\Delta x_{n-1}\|} \cdots \frac{\|\Delta x_1\|}{\|\Delta x_0\|} \right|$$

$$= \frac{1}{n} \ln(\lambda_0, \lambda_1)$$

So Lyapunov exponent

$$LE_1 = \frac{1}{n} \ln |\lambda_1|, LE_2 = \frac{1}{n} \ln |\lambda_2|$$

while λ is the eigenvalue of corresponding two-dimensional mapping of Jacobin matrix

$$J = \begin{bmatrix} \dfrac{\partial f_1}{\partial x_n} & \dfrac{\partial f_1}{\partial y_n} \\ \dfrac{\partial f_2}{\partial x_n} & \dfrac{\partial f_2}{\partial y_n} \end{bmatrix}$$

2.2 Henon map

Henon map is applied to energy forecasting, chemical production, and as well as have a good handle turbulence phenomena. For two-dimensional Henon map, the iterative equation is as follows:

$$\frac{n*(n-1)}{2} \qquad (4)$$

a and b are parameters. It will happen to repeatedly trace and chaotic with different parameters。
We select the parameters a=1.2, b=0.3[4]to make the relation chart of the top 50 map points iterations and Lyapunov exponent
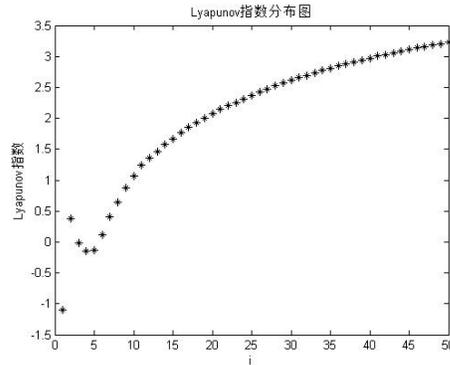

Figure 1: The Relationship Of Points And Lyapunov Exponent

From Fig.1, the positive Lyapunov exponent of Henon map can be generated as nodes of networks is increased. In following, we study the relationship between the parameter a and Henon map. For $y_n+1=b*x_n$, we can simple get picture y is similar with picture x, the difference is only a coefficient. So we can get the relationship between the parameter a and Henon map only by studying the relationship between the parameter a and the parameter x.

(a)
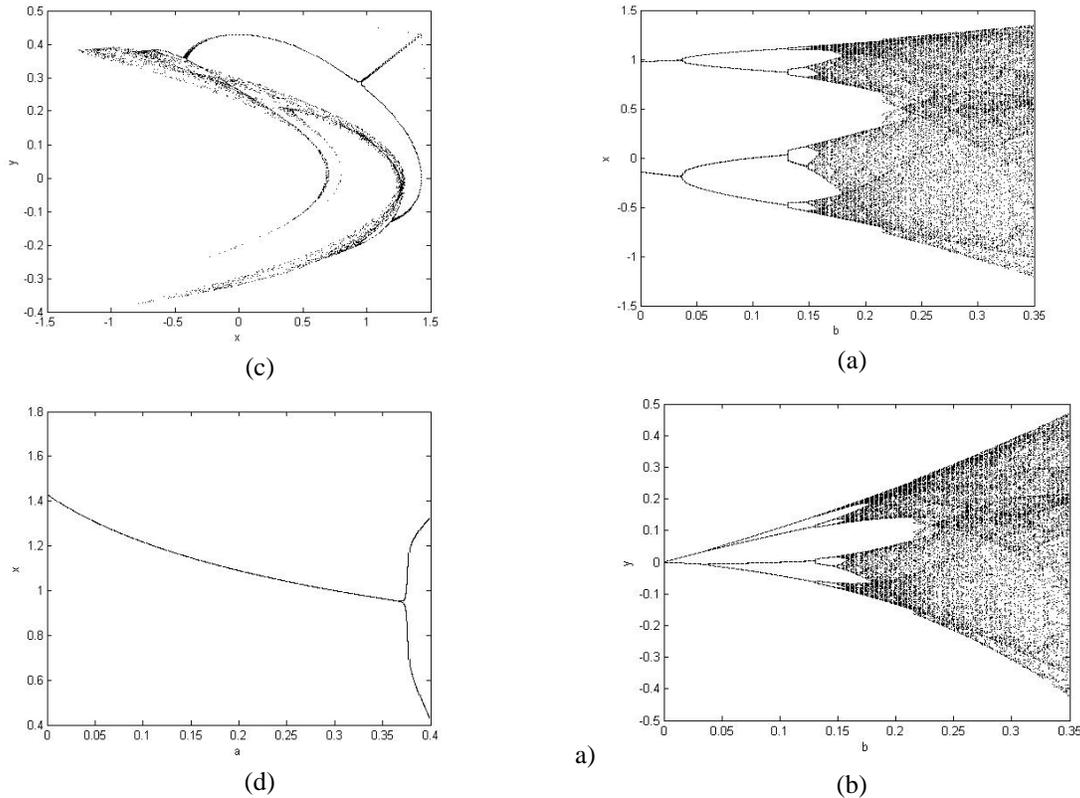

(b)

(c)

(a)

(d)
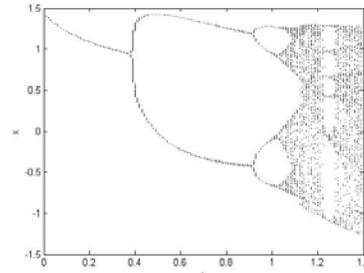
a)
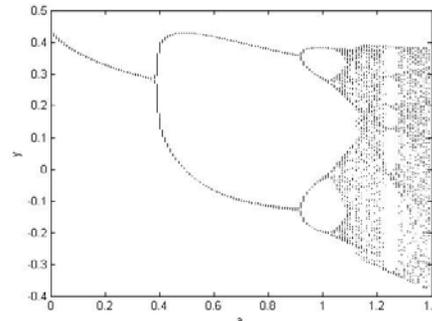
(b)

Figure 2: The Relationship Between The Parameter A
And Henon Map. A)Parameter A And Parameter X
B)Parameter A And Parameter Y C)Parameter X And
Parameter Y D) Parameter A And Parameter X.

From Figure 2, we can see when parameter *a* belongs to 0 and 0.38, it occurs to chaotic state, it influences both parameter x and parameter y. So the system is certain to become chaotic when the parameter a is large enough. So, with the certain nodes, it won't be chaotic when a belongs to 0 and 0.38, and it change nothing with nodes. While with the increase of *a*, it will exhibit periodic bifurcation and chaotic phenomena.

Second, we study the relationship between the parameter b and Henon map. The influences are as follows when *a* belongs to 0 and 0.38 or not. Figure 2 (a)and (b) stands for the relationship between the parameter b and Henon map when a can result in chaos .whereas a belongs to 0 and 0.38 in the Figure 2(c)and (d).
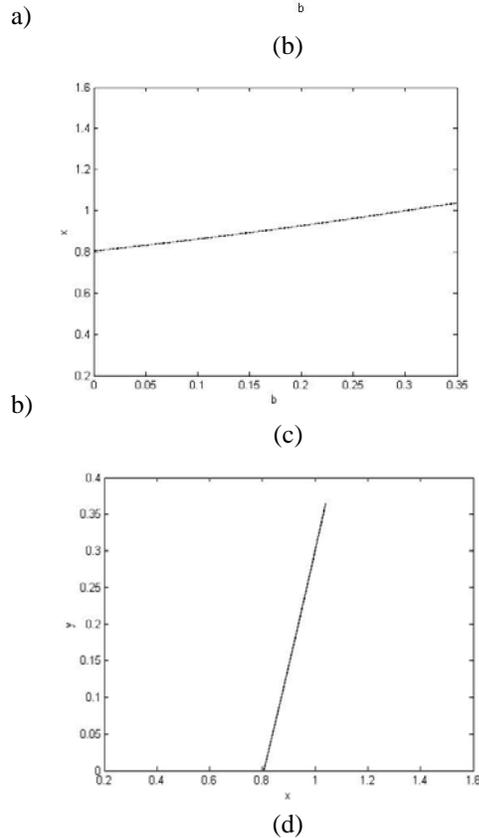

b)

(c)

(d)

*Figure 3: The Relationship Between The Parameter B
And Henon Map(A) Parameter B And X B) Parameter B
And Y C) Parameter B And X   D) Parameter B And Y .*

From Fig.3 a)and b), we can see the system become chaotic when *a* is very large; while, seeing from the picture c) and d), the system will not become chaotic when *a* is very small. We can conclude that the parameter b itself strength the influence of parameter *a* instead of resulting in bifurcation and chaos. So, we can say that instead of resulting in bifurcation and chaos, the parameter *b* itself strength the influence of parameter *a.*

3. THE ITERATIVE BEHAVIOR OF THE COUPLED NETWORK AND THE APPEARANCE OF CHAOS

3.1 The Coupled Network

It is clear that if A=(aij)$_{N×N}$, R$_{N×N}$ stands for the coupled configuration network ,if there exists a relationship between i and j when i≠j ,we define a$_{ij}$=a$_{ji}$=1,otherwise,a$_{ij}$=a$_{ji}$=0,then we can know the take the diagonal elements

$$a_{ij} = -\sum_{j=1,j\neq i}^{N} a_{ij} = -\sum_{j=1,j\neq i}^{N} a_{ji}, i = 1, 2...N \qquad (5)$$

(1)The global network

Every two nodes relate with each other in the global coupled network, moreover, there are n*(n-1)/2edges in total. According to the reference we can get its coupled matrix A:

$$A = \begin{pmatrix} -N-1 & 1 & 1 & \cdots & 1 \\ 1 & -N-1 & 1 & \cdots & 1 \\ \vdots & & \ddots & \ddots & \cdots & 1 \\ 1 & 1 & 1 & \cdots & 1 \\ 1 & 1 & 1 & \cdots & -N-1 \end{pmatrix}$$

(6)

(2) The star-network

There is only one central node in star-network with N nodes. Moreover, each node only associated with central node .The corresponding coupled matrix A is:

$$A = \begin{pmatrix} -N-1 & 1 & \cdots & 1 \\ 1 & -1 & \cdots & 0 \\ \vdots & & \ddots & \cdots & 0 \\ 1 & 0 & \cdots & -1 \end{pmatrix} \qquad (7)$$

(3)The nearest-neighbor network

In the nearest neighbor coupled network N nodes, each node only associate with the nearest node. Its coupled matrix A is:

$$A = \begin{pmatrix} -2 & 1 & 0 & \cdots & 1 \\ 1 & -2 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 & -2 & 1 \\ 1 & 0 & \cdots & 1 & -2 \end{pmatrix} \qquad (8)$$

3.2 The Iterative Behavior Of The Coupled Network And The Appearance Of Chaos

If we iterative data in the coupling network, in spite of itself iteration, nodes will also influence each other. We use a specific and a common number to describe the interaction influence between node and node, which we use symbol c to define it.

If the system is a discrete process, the iteration equation of the x node is x$_i$(k)=f(x$_i$(k)), and then the x+1 node's corresponding iterative equation is

$$x_i(k+1) = f(x_i(k)) - c\sum_{j=1}^{N} a_{ij} f(x_j(k)), i = 1, 2...N$$

(9)

If the system is a continuous process, the first x node's corresponding iteration equation is x$_i$(k)=f(x$_i$(k)), then x+1 node's corresponding iterative equation is

$$x_i(t) = f(x_i(t)) - c\sum_{j=1}^{N} a_{ij} x_j(t), i = 1, 2...N \qquad (10)$$

We also assume that the node with a parameter is not chaotic before connecting with others in the initial system state. But with the increase number of the nodes and with the different iterative frequency, some certain coupling network will come to chaos phenomena.

For the globally coupled network and the star-network, when the nodes increase to a some value, the corresponding global coupled network and the star-network structure can transform form non-chaotic state to the chaotic state with limited superposition .For the nearest-neighbor network, as long as the network is large enough, there wouldn't occur to chaotic state.

For Henon map itself can change non-chaotic status to chaotic status by iterating. According to this, we select the ineffective Chaos of Henon map (that is, the value of parameter a is very small) in the three networks, the pictures are as follows (all take the first point as example, (the first picture shows the global coupling network, the second picture shows the nearest neighbor coupling network, the third picture shows the star network).We can see the chaotic phenomenon is not clear
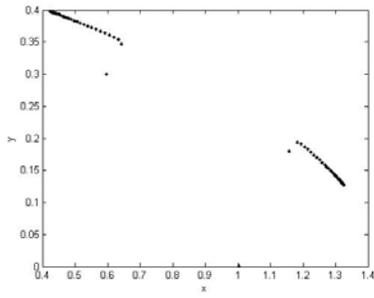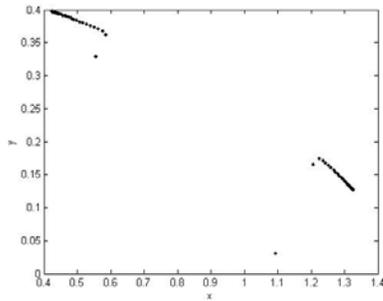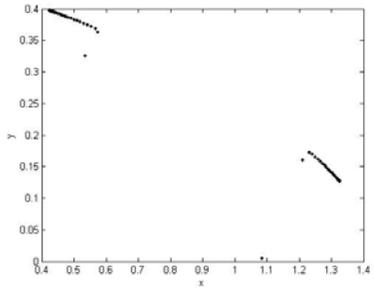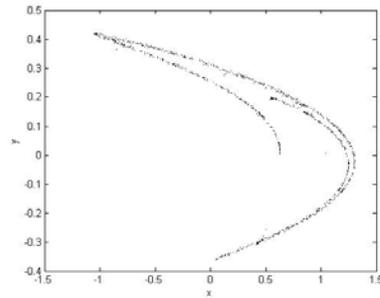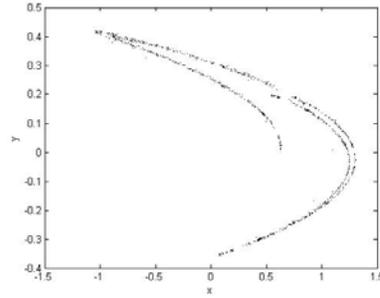
Fig.3 Three Typical Henon Maps In The Coupled Networks When Parameter A<0.38(With 100 Points) A)The Globally Coupled Network B)A Star-Network C)A Nearest-Neighbor Coupled Network

Fig.4 Three Typical Henon Maps In The Coupled Networks When A>0.38(With 100 Points) A) The Globally Coupled Network B)A Star-Network C) A Nearest-Neighbor Coupled Network

From the above figure, the chaos is not generated in three systems. We verify that Henon map can turn to be chaotic when the initial state is on the contrary, and parameter *a* is too smaller.
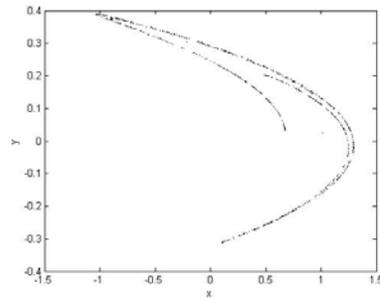
In general studies, we usually adopt parameter a=1.2,b=0.3 to the Henon map. At the moment, the initial state has become a chaotic state following iteration. In this regard, it is necessary to study the different coupled networks under different conditions
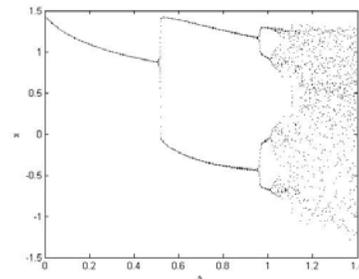
From Fig.4, the chaos are generated in the three types of systems when a=1.2. From the simulation result, we can see the chaos of the system is influenced by parameter *a,* and there is nothing about parameter b.
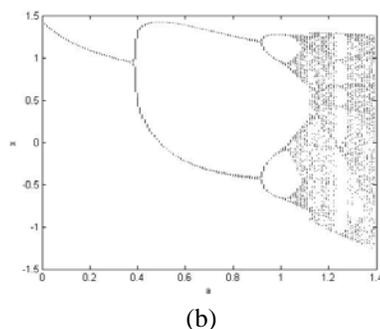

(a)

(b)

Figure 5: The Relationship Between The Parameter A And The Parameter X. A)Star Network (15 Iteration) B) A Nearest Neighbor Coupled Network(100iteration)

From Figure 5, although both of the system become more chaotic when parameter *a* is very large, the difference is that a nearest-neighbor coupled network will be more easily to adapt.

Thus, we can draw that when the parameter a is lightly larger, the node of Henon map will affect each other in the overlay process, corresponding global coupled network and star network structure can be superimposed after finite increase its chaotic, and the effect is distinct; when the number of nodes is certain, the global coupled network and star network is close to infinite ;while, as long as the network is large enough, the nature of the nearest neighbor coupling network system is not be changed.

4. CONCLUSION

With a certain number of nodes, from non-chaotic to a chaotic state occurs in the case of different coupling network of Henon map for the same initial point is discussed in this paper. And then in certain different coupled network states under different conditions is analyzed. The situation from non-chaotic to chaotic is verified and the network with more nodes can be more practical used. Furthermore, the relationship between different kinds of networks and different parameters has been studied. Drawn form the above, as long as the nearest neighbor coupled network is large enough, the chaos will be not occurred if the initial state is not a chaotic system, while if the initial state is a chaotic system, the chaotic state will not be changed. In the same case of other conditions we can better and more rational use the large nearest neighbor coupled network to make it to a higher level of application efficiency.

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