# APPLICATION OF NESTED COST OPTIMIZATION MODELS TO TSPD

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## ABSTRACT

Through the analysis of Travelling Salesman Problem with Pickup and Delivery (TSPD) in the related research, two kinds of logistics systems including whole logistics system and partial logistics system under one-to-many logistic network are considered. And mathematical model is set up, then, nested cost optimization model is set up in order to meet the constraint condition of nested order cycle time. According to an application pickup and delivery instance of iron and steel in one factory, by using the method of adaptive response surface in Hyperstudy software, the initial solutions are figured out, and further the optimum direction of iteration is found so as to obtain the optimum solution.

**Keywords:** *Nested, Logistics System, Pickup and Delivery, Cost models, Optimal Solution*

## 1. INTRODUCTION

System integrated and cost optimization are considered, which complement each other, are the integral goal of modern logistics distribution system. And cost control is the most important procedure for optimizing the system. In fact, the cost mainly comes from transport and inventory, which take the main parts. So to integrate Vehicle Routing Problem and inventory control is the key point of the researching on this paper. The research purpose is minimizing the sum of cost through optimizing and controlling the different types of costs respectively.

In the course of planning and operating logistics distribution system, Travelling Salesman Problem with Pickup and Delivery(TSPD), which is characterized by a set of customers, each of them supplying (pickup customer) or demanding (delivery customer) a given amount of product[1], usually occurs. And it is affected by many factor, this article only considers both the demand of pickup and delivery for logistics distribution system and the most loading of vehicles. Therefore, it is not allowed to neglect to the influence of the collection cost. TSPD is actually an NP-hard problem, so the related researches focus on algorithm. About simultaneous pickup and delivery, a heuristic solution approach based on particle swarm optimization is presented [2]. The problem and LIFO loading by considering the use of multiple vehicles and a limitation on the total distance is extended by Cheang B [3]. And a variable neighborhood search approach is showed by Mladenovic N [4]. Besides, scholars have researched some solutions of TSPD by genetic algorithm [5]-[6], ant colony algorithm [7]-[8] and so on. Nested viewpoint [9] and adaptive response surface algorithm is combined in order to get a better solution.

In this paper whole logistics system and partial logistics system under one-to-many logistic network are described. Then, nested cost optimization models that meets the constraints of nested order cycle time are set up. At last, a related analysis and calculation is carried out by taking the delivery and pickup of an iron and steel company as a case.

## 2. MODELING

This paper researches the one-to-many logistics delivery system, which is defined as one DC supplies goods for many retailers. The proposition sets for the distribution goods is single.

Firstly, the application of model to real-life problems is governed by the following design guidelines:

1) Whole logistics system refers to the delivery demand of all customers, which is not less than pickup demand, or is not more than pickup demand. But local logistics system is defined as the system

ISSN: **1992-8645**     www.jatit.org     E-ISSN: **1817-3195**

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that a part of customers delivery demand is not less than the pickup demand, others is not more than the pickup demand.

2) Nested order cycle time is the order cycle of one or more retailers that is nested another order cycle to reduce delivery costs. Furthermore, they are satisfied the rule of the second power. For example, the order cycle of the retailer One and Two is 2, means the nested cycle time. The retailer Three having nested order cycle time, means the nestable cycle time, is 4. The sketch map is shown in Figure 1.
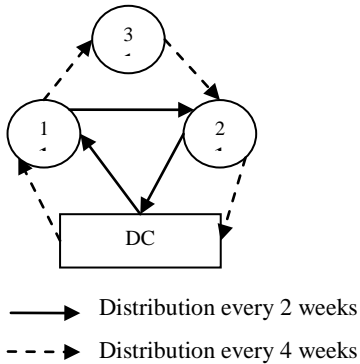


*Figure 1 Nested Order Cycle Time Distribution Sketch Map*

Secondly, the assumptions to build the model are used as follows:

①Transportation cost considers the start-up cost of vehicles but not vehicle traffic lines. And the order cost is not contained because of all order operations are carried out under the e-commerce platform.

②All retailers have the inventory and the supply of inventory meets the order cycle.

③The DC has pickup areas, and the capacity is Q. It is less than the total capacity of the pickup demand of all customers.

④Each distribution generates inventory cost.

⑤Each retailer makes sure the distribution number independently, and the number reaches the capacity of retailers.

⑥The pickup activities generate the cost, which is only related to the pickup number.

Finally, the symbols used in the model are described as follows:

Where i= {1,2, ..., N} is a retailer set ,with N being the number of all the retailers .and smaller one has shorter order cycle. The DC is i = 0. The delivery demand of the retailer i is $d_i$. The pickup demand of retailer i is $p_i$ ;  The unit inventory cost of the retailer i is $g_i$. . The unit pickup cost of the retailer i is $w_i$. The order cycle is $T_i$.

The objective function is to seek the optimal nested order cycle time which can get a minimum cost including of delivery cost, inventory cost and pickup cost. The paper assumes that the capacity of all vehicles in the DC is the same as R .And the DC supplies service for n customers having delivery and pickup demand. So the lest number of vehicles

is $$h = \max \left\{ \, ent[\left.\sum_{i=1}^{n} d_i \middle/ R \right.]+1 \, , \, ent[\left.\sum_{i=1}^{n} p_i \middle/ R \right.]+1 \, \right\}$$ .In this logistics system, the number of vehicles is H, they provide service for all the retailers. So

$$L(1) = \sum_{i \in P_h} d_i, \quad C_d(i_h) = \sum_{k=1}^{i_k} d_i, \quad C_p(i_h) = \sum_{k=1}^{i_k} p_i.$$

Where $P_h$ is the running route of vehicle h, L (1) is an initial capacity of the vehicle. $i_h$ is the current point when the vehicle h runs along $P_h$, $c_d(i_h)$ is the cumulative delivery demand along $P_h$ , $c_p(i_h)$ is the cumulative pickup demand along $P_h$. This paper indicates $d_i \geq p_i$ (i=2,3…n) in whole logistics system ,but $d_i \geq 0$， $p_i \geq 0$ in partial logistics system.

## 2.1 COST CALCULATION

A customer follows nested order cycle time，which is satisfied the rule of the second power, and $T_1 \leq T_2 \leq … \leq T_n$. Furthermore, smaller customers have shorter order cycle time.So, the distribution routes include{1}，{1,2}，…，{1,2,…,i}，…，{1,2,…,n}. This paper presents a method of expressing the transportation cost of {1,2,…,i} by M（{1,2,…,i}）,and $f_i$= M （{1,2,…,i}）.So the calculation formula of transportation cost in the whole logistics system is $\sum_{i=1}$ [M({1,2,…,i}) -

M({1,2,…,(i-1)})]$T_i^{-1} = \sum_{i=1}^{n}$ ($f_i$ - $f_{i-1}$)$T_i^{-1}$.

In partial logistics system, the most loading of customers is denoted $L(i_h) = L(1) - c_d(i_h) + c_p(i_h)$. It is maybe greater than the one of vehicles.Therefore the TSPD in existence should be improved. In the DC, the vehicles must be started regardless of the number of the order goods, and the start-up cost is regarded as the fixed one. The model of transportation cost from these customers is presented as follows:

$$f_i = \min \sum_{k=1}^{n} \sum_{l=1}^{n} (c_{kl} z_{kl} + c_0 h)$$

subject to     $L(ih) \leq R$,

$$\sum_{l=1}^{n} z_{kl} \leq 1 (k=1,2,\ldots,n),$$

$$\sum_{k=1}^{n} z_{kl} \leq 1 (l=1,2,\ldots,n),$$

$$z_{kl} \in \{0,1\} ((k,l=1,2,\ldots,n).$$

Where $c_o$ is the average startup cost. $c_{kl}$ is the distance between k and l. $z_{kl}$ is the number of passing through $c_{kl}$, and it is 1 if passing otherwise is 0. Then, $\sum_{i=1}^{n} (f_i - f_{i-1})T_i^{-1}$ denotes the total cost of transportation in the planning.

If each customer meets the condition of $L(i_h) \leq R$ ,this route is the best. $\sum_{i=1}^{n} d_i g_i T_i / 2$ expresses the total cost of inventory taking no account of the cost of DC. And $\sum_{i=1}^{n} w_i p_i T_i$ is the total cost of pickup.

## 2.2 INITIAL MODEL

Based on the above analysis, the problem can be formulated as:

$$\min[F(T)] = \sum_{i=1}^{n} (f_i - f_{i-1})T_i^{-1} + \sum_{i=1}^{n} d_i g_i T_i / 2 + \sum_{i=1}^{n} w_i p_i T_i$$

subject to $\quad 0 < T1 \leq T2 \leq \ldots \leq Tn (i=1,2,\ldots,n)$ (1)

$\quad T_i = 2^{m_i}$ (m is a integer， Unit period is 1 ) (2)

$$\sum_{i=1}^{n} d_i T_i \leq hR \quad (i=1,2,\ldots,n)$$
(3)

$$\sum_{i=1}^{n} p_i T_i \geqslant Q$$
(4)

$$0 \leq p \leq d_i \quad Or \quad 0 \leq p_i, \quad 0 \leq d_i.$$
(5)

Where the objective minimizes the total cost of transportation, inventory and pickup, nested order cycle time is ensured by(1)-(2), the capacity of vehicles is enforced by(3), the capacity of pickup area in DC is ensured by(4) and the demand of pickup meets constraint (5),but it is $0 \leq p \leq d_i$ in the whole logistics system , while $0 \leq p_i$, $0 \leq d_i$ in the partial logistics system.

## 2.3 NESTED COST OPTIMIZATION MODEL

For the finally optimal solution $T^*$, it does not necessarily satisfy the nested constraint. So the new model is set up in order to solve this problem and defined as nested cost optimization model, as follows:

$$\min[F(T^*)] = \sum_{i=1}^{n} (f_i - f_{i-1})T_i^{-1} + \sum_{i=1}^{n} d_i g_i T_i / 2 + \sum_{i=1}^{n} w_i p_i T_i$$

subject to $\quad T_i^o = 2^{k-1}, T_i^u = 2^k$ (k is a integer, $2^{k-1} \leq T_i^* \leq 2^k$)

$$\sum_{i=1}^{n} d_i T_i \leq hR \quad (i=1,2,\ldots,n)，$$

$$\sum_{i=1}^{n} p_i T_i \geq Q,$$

$$0 \leq p_i \leq d_i.$$

Where $2^{k-1} \leq T^* \leq 2^k$, and k is an integer. And $T_i^o = 2^{k-1}$, $T_i^u = 2^k$.

## 3. EXPERIMENTATION

The paper takes the delivery and pickup of an iron and steel company (in Chongqing) as the experimentation.

*Table 1 The Vehicle Operating Cost Of Each Customer (Unit:RMB)*

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 6 | 8 | 12 | 17.5 | 14.8 | 16.4 | 14.5 | 16 | 10.6 |
| 1 |   | 0 | 4.8 | 12 | 17 | 16 | 17.5 | 19.8 | 15 | 14.3 |
| 2 |   |   | 0 | 8.5 | 13 | 12.5 | 14.7 | 18.6 | 14 | 12.8 |
| 3 |   |   |   | 0 | 6.1 | 4.6 | 7.7 | 12.9 | 11 | 8.6 |
| 4 |   |   |   |   | 0 | 4.6 | 9 | 15 | 15 | 12.5 |
| 5 |   |   |   |   |   | 0 | 5 | 11 | 11 | 8.5 |
| 6 |   |   |   |   |   |   | 0 | 6.5 | 8 | 5.8 |
| 7 |   |   |   |   |   |   |   | 0 | 6.8 | 6.5 |
| 8 |   |   |   |   |   |   |   |   | 0 | 3 |
| 9 |   |   |   |   |   |   |   |   |   | 0 |

*Table 2 The Delivery And Pickup Demand In Whole Logistics System*

| Customer | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $d_i$ | 0 | 600 | 100 | 150 | 50 | 650 | 100 | 550 | 100 | 600 | 2900 |
| $p_i$ | 0 | 600 | 100 | 150 | 50 | 650 | 100 | 550 | 100 | 600 | 2900 |
| $g_i$ | 0 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 225 |
| $w_i$ | 0 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 72 |

Unit：$d_i$ and $p_i$ is kg each unit time, $g_i$ is RMB each kg each unit time, $w_i$ is RMB each time.

Table 3 The delivery and pickup demand in local logistics system

| Customer | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $d_i$ | 0 | 600 | 100 | 150 | 50 | 650 | 100 | 550 | 100 | 600 | 2900 |
| $p_i$ | 0 | 350 | 300 | 200 | 250 | 200 | 250 | 650 | 150 | 550 | 2900 |
| $g_i$ | 0 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 225 |
| $w_i$ | 0 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 72 |

Unit is just the same as Table 3.

The company has a DC to serve 9 customers with TSPD. The vehicle operating cost of each customer is shown as Table 1, and 0 is DC. The delivery and pickup demand in whole logistics system and local logistics system is given in Table 2 and Table 3 below.

Where the capacity of vehicle is 1000 kg, the least vehicle number is 3. The average start-up cost is 10 RMB. The pickup capacity of DC is 2000 kg, and the unit order cycle is 1 week.

## 3.1 WHOLE LOGISTICS SYSTEM OPTIMIZATION

According to Table1and Table 2, the whole logistics system model is given as follows:

$$\min[F(T)] = \sum_{i=1}^{n} (f_i - f_{i-1})T_i^{-1} + \sum_{i=1}^{n} d_i g_i T_i / 2 + \sum_{i=1}^{n} w_i p_i T_i$$

subject to $\quad 0 < T_1 \leq T_2 \leq \ldots \leq T_9$

$600T_1 + 100T_2 + 150T_3 + 50T_4 + 650T_5 + 100T_6 + 550T_7 + 100T_8 + 600T_9 \leq 3000$

$600T_1 + 100T_2 + 150T_3 + 50T_4 + 650T_5 + 100T_6 + 550T_7 + 100T_8 + 600T_9 \geq 2000$

$0 \leq p_i = d_i , (i = 1, 2, \ldots, n)$

The paper applies the method of adaptive response surface in Hyperstudy software to optimize the problem.
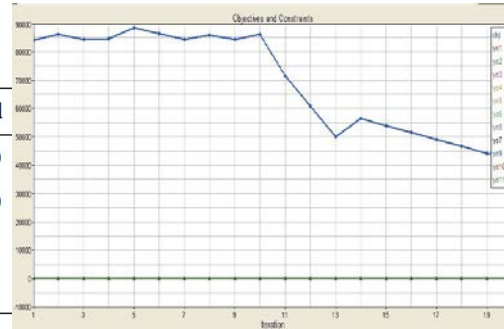


*Figure 2 The Iterative Process Of Whole Logistics System*

After 20 iterations, the optimum solution is obtained. The iterative process is shown in Figure 2. Due to limited space available, this paper only shows the result.

So, T = （$T_1, T_2, \ldots T_9$）= （0.02, 0.02, 0.02, 0.02, 0.53, 1.21, 1.21, 1.21, 1.21）,The target value is min [f (T)] = 42717.8 RMB. The solution does not meet the nested constraint condition. Therefore, nested cost optimization calculation should be continued. There are

$$2^{-3} \leq T_1 = T_2 = T_3 = T_4 \leq 2^{-2},$$

$$2^{-1} \leq T_5 \leq 1,$$

$$1 \leq T_6 = T_7 = T_8 = T_9 \leq 2.$$

So each order cycle may have two values,

$$T_1 = T_2 = T_3 = T_4 = \{2^{-3}, \ 2^{-2}\},$$

$$T_5 = \{2^{-1}, \ 1\},$$

$$T_6 = T_7 = T_8 = T_9 = \{1, \ 2\}.$$

The whole solutions are 29, with $T^*$ being the optimum solution. The meaning of the objective function is the minimum cost meeting the nested constraint. So $T^*$ is found, just as{0.125,0.125, 0.125, 0.125,1,1,1,1,1}, the target value is

$$\min[F(T^*)] = 46610.3 \text{ RMB}.$$

## 3.2 LOCAL LOGISTICS SYSTEM OPTIMIZATION

Similarly, the initial optimal solution in local logistics system is T=

（0.52,0.52,0.52,0.52,0.77,0.77,0.77,0.77,0.77）.

And the corresponding target value is

$$\min[F(T)] = 40693.9 \text{ RMB}.$$
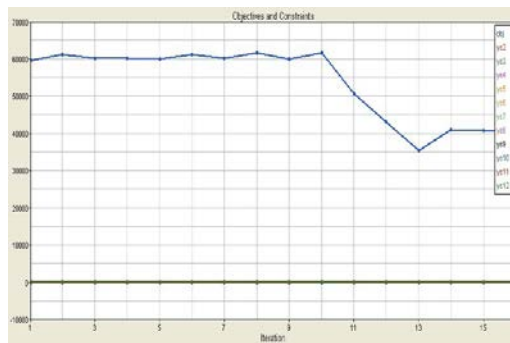
The iterations are as shown in Figure 3.

*Figure 3 The Iterative Process Of Partial Logistics System*

The optimal solution is get ,

$$T^* = \{0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 1, 1, 1\}.$$

And the target value is

$$\min[F(T^*)] = 43023.8 \text{ RMB}.$$

## 4. CONCLUSION

Based on the paper—cost Optimization Models of Nested Sequence Policy Under the One-to-Many Pickup and Delivery System—which is wrote by Xu Jiu-ping and Lei Zhen, this paper takes transportation, inventory and pickup as a whole after analyzing TSPD and introducing related work recently. Nested cost optimization model is set up respectively according to the different between whole logistics system and partial logistics system. The study explores the new algorithm combining with the experimentation about two kinds of logistics system and gains the solution. So as to reveal application of the nested cost optimization model to TSPD.

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