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PASSIVE LOCATION ALGORITHM USING KUSHNER EQUATION

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ABSTRACT

Kalman filtering algorithm is an effective linear filter, but in the nonlinear state its covariance is uncontrolled. Kushner equation is effective for the nonlinear filtering. According to the Kushner equation and the nonlinear of passive location, a new passive location algorithm based on the Kushner equation is presented. And it is used in passive location tracking. Experiments are made in MATLAB and its validity is proved.

Keywords: *Kushner Equation, Passive Location, Electronic Support Measures*

1. INTRODUCTION

Kalman filter is the analytical recursive solution for optimal filtering of linear-Gaussian systems. For nonlinear/non-Gaussian cases, such a perfect analytical solution is intractable[1]~[4]. Since most practical physical systems are nonlinear, nonlinear filtering is an important research theme in both theory and applications.

The nonlinear problem of the measurement equations or state equations exists in many areas, such as moving object tracking, modern signal processing, image processing and automatic control. So the state estimation of a nonlinear system has its significance in both theoretic and engineering applications[5]~[8]. However, the current nonlinear filtering schemes in practice are approximation algorithms which normally have the disadvantages of low precision and easy divergence so that these methods have limited performance to solve the nonlinear problems.

Passive location of emitter has been playing an important role in navigation, aviation, aerospace and electronic warfare with its outstanding features, such as longer range and better concealment. It is proved that the ability to obtain information is the key factor to determine the success of failure of the modern warfare. How to access the battlefield information stealthily, timely and accurately has been a vital problem. In the view of this, in the background of Network Centric Warfare, the research on passive localization by using observers is necessary and important. In order to quickly and accurately locate the emitters, some key technologies have been studied systematically and deeply in some literatures[9]~[11], including the localization precision analysis and sensor placement optimization, the estimation of observed parameters, the passive localization algorithm and the systematic error calibration.

The algorithm of linear minimum mean-square error filter is often applied in passive location. It has the specialty of tracking convergence fast but its location precision is low in the low precision of measuring azimuth. The extended kalman filter has the specialty of tracking location high precision but it is sensitivity for the first state estimation[12]~[16]. The Kushner equation is a sort of effective nonlinear filtering algorithm[17] and has the specialty of tracking high precision. And it needs not linearization process of state equation. A new passive location algorithm by using Kushner equation is presented.

The Kushner equation is briefly introduced in section 2. In section 3, the algorithm based on the Kushner equation for passive location is presented. In section 4, experiments are made in MATLAB and its validity is proved. Conclusions are drawn in section 5.

2. THE KUSHNER EQUATION BRIEF INTRODUCTION

It is supposed here that the state variable and the observation are the vector processes given by[18]

$$dx = f(x)dt + v(x)dW \tag{1}$$

$$dy = g(x,t)dt + \sigma dW \tag{2}$$

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Where W is Wiener process. Until mentioned otherwise, define $m = Ex$ and $m_i = E(x-m)^i$. Then

$$dm = E[f(x)]dt + (dy - Egdt)[E(gx) - Egm]/\sigma^2 \tag{3}$$

$$\begin{aligned} dm_2 = \; & [2E(x-m)f(x) + Ev^2]dt \\ & + \frac{(Egx - Egm)^2 dt}{\sigma^2} \\ & + \frac{(dy - Egdt)}{\sigma^2}[E(g-Eg)(x-m)^2] \end{aligned} \tag{4}$$

$$\begin{aligned} dm_i = \; & [-im_{i-1}Ef(x) + EL(x-m)^i]dt \\ & + \frac{1}{\sigma^2}[\frac{i(i-1)}{2}m_{i-2}(Egx - Egm)^2 \\ & - i(Egx - Egm)E(g-Eg)(x-m)^{i-1}]dt \\ & + \frac{(dy - Egdt)}{\sigma^2}[-im_{i-1}(Egx - Egm) \\ & + E(g-Eg)(x-m)^i] \end{aligned} \tag{5}$$

The operator

$$L = \sum_i f_i \frac{\partial}{\partial x_i} + \frac{1}{2}\sum_{i,j} v_{ij} \frac{\partial^2}{\partial x_i \partial x_j} \tag{6}$$

3. THE ALGORITHM BASED ON KUSHNER EQUATION FOR PASSIVE LOCATION

In Fig. 1, the position relation between observers and emitter is indicated.

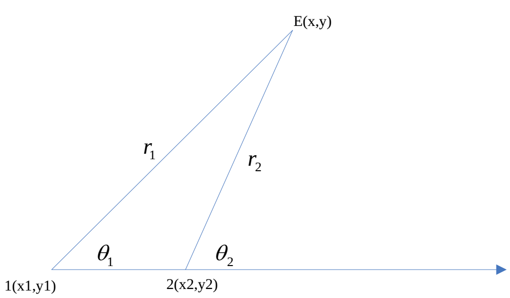

Figure 1. *Position Relation Between Single Observer And Emitter*

According to the geometry

$$a_1 = tg\theta_1 \tag{7}$$

$$a_2 = tg\theta_2 \tag{8}$$

$$x = \frac{y_1 - y_2 - a_1 x_1 + a_2 x_2}{a_2 - a_1} \tag{9}$$

$$y = \frac{a_2 y_1 - a_1 y_2 - a_1 a_2 x_1 + a_1 a_2 x_2}{a_2 - a_1} \tag{10}$$

The nonlinear problem of the measurement equations and state equations exists. However, the Kalman filtering schemes in practice are approximation algorithms which normally have the disadvantages of low precision and easy divergence so that the methods have limited performance to solve the nonlinear problem. Hence Kushner equation is used to solve the nonlinear filter. According to the formula (3), a numerical approximation to Kushner equation in the observers' passive location tracking is given in the following algorithm.

Let $g_1 = \frac{(x-x_1)v_y - (y-y_1)v_x}{(x-x_1)\sqrt{(x-x_1)^2 + (y-y_1)^2}}$,

$g_2 = \frac{(x-x_2)v_y - (y-y_2)v_x}{(x-x_2)\sqrt{(x-x_2)^2 + (y-y_2)^2}}$,

$p_1 = Ex$, $p_2 = Ey$, $p_{11} = E(x-p_1)^2$, $p_{12} = E[(x-p_1)(y-p_2)]$ and $p_{22} = E(y-p_2)^2$, then

$$dp_1 = v_x dt + \frac{\sqrt{(d\theta_1 - Eg_1 dt)(Eg_1 x - Eg_1 p_1)(d\theta_2 - Eg_2 dt)(Eg_2 x - Eg_2 p_1)}}{\sigma_1 \sigma_2} \tag{11}$$

$$dp_2 = v_y dt + \frac{\sqrt{(d\theta_1 - Eg_1 dt)(Eg_1 y - Eg_1 p_2)(d\theta_2 - Eg_2 dt)(Eg_2 y - Eg_2 p_2)}}{\sigma_1 \sigma_2} \tag{12}$$

Based on these, optimal estimators of the state parameters are presented.

Next we solve the equations of covariances. According to the formula from (4) to (6)

$$dp_{11} = \frac{-(Eg_1 x - Eg_1 p_1)(Eg_2 x - Eg_2 p_1)}{\sigma_1 \sigma_2} dt \tag{13}$$

$$dp_{12} = \frac{-\sqrt[4]{(Eg_1 x - Eg_1 p_1)^2 (Eg_1 y - Eg_1 p_2)^2 (Eg_2 x - Eg_2 p_1)^2 (Eg_2 y - Eg_2 p_2)^2}}{\sigma_1 \sigma_2} dt \tag{14}$$

$$dp_{22} = \frac{-(Eg_1 y - Eg_1 p_2)(Eg_2 y - Eg_2 p_2)}{\sigma_1 \sigma_2} dt \tag{15}$$

According to Taylor expansion

$$g_1(x,y) \approx g_1(p_1,p_2) + (x-p_1)\frac{\partial g_1(p_1,p_2)}{\partial x} + (y-p_2)\frac{\partial g_1(p_1,p_2)}{\partial y}$$

(16)

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$$g_2(x,y) \approx g_2(p_1,p_2) + (x-p_1)\frac{\partial g_2(p_1,p_2)}{\partial x} + (y-p_2)\frac{\partial g_2(p_1,p_2)}{\partial y} \tag{17}$$

If

$$\Delta_1 = (p_1 - x_1)^2 + (p_2 - y_1)^2 \tag{18}$$

$$A = \frac{\partial g_1(p_1,p_2)}{\partial x} = \frac{v_y}{(p_1-x_1)\sqrt{\Delta_1}} - \frac{[v_y(p_1-x_1)-v_x(p_2-y_1)][\Delta_1+2(p_1-x_1)^2]}{(p_1-x_1)^2\sqrt{\Delta_1^3}} \tag{19}$$

$$B = \frac{\partial g_1(p_1,p_2)}{\partial y} = \frac{-v_x}{(p_1-x_1)\sqrt{\Delta_1}} - \frac{[v_y(p_1-x_1)-v_x(p_2-y_1)]\cdot 2(p_1-x_1)(p_2-y_1)}{(p_1-x_1)^2\sqrt{\Delta_1^3}} \tag{20}$$

$$\Delta_2 = (p_1 - x_2)^2 + (p_2 - y_2)^2 \tag{21}$$

$$C = \frac{\partial g_2(p_1,p_2)}{\partial x} = \frac{v_y}{(p_1-x_2)\sqrt{\Delta_2}} - \frac{[v_y(p_1-x_2)-v_x(p_2-y_2)][\Delta_2+2(p_1-x_2)^2]}{(p_1-x_2)^2\sqrt{\Delta_2^3}} \tag{22}$$

$$D = \frac{\partial g_2(p_1,p_2)}{\partial y} = \frac{-v_x}{(p_1-x_2)\sqrt{\Delta_2}} - \frac{[v_y(p_1-x_2)-v_x(p_2-y_2)]\cdot 2(p_1-x_2)(p_2-y_2)}{(p_1-x_2)^2\sqrt{\Delta_2^3}} \tag{23}$$

Using Kushner equation, so

$$dp_1 = v_x dt + \frac{\sqrt{(d\theta_1 - g_1(p_1,p_2)dt)(Ap_{11}+Bp_{12})(d\theta_2 - g_2(p_1,p_2)dt)(Cp_{11}+Dp_{12})}}{\sigma_1\sigma_2} \tag{24}$$

$$dp_2 = v_y dt + \frac{\sqrt{(d\theta_1 - g_1(p_1,p_2)dt)(Ap_{12}+Bp_{22})(d\theta_2 - g_2(p_1,p_2)dt)(Cp_{12}+Dp_{22})}}{\sigma_1\sigma_2} \tag{25}$$

$$dp_{11} = \frac{-(Ap_{11}+Bp_{12})(Cp_{11}+Dp_{12})}{\sigma_1\sigma_2}dt \tag{26}$$

$$dp_{12} = \frac{-\sqrt[4]{(Ap_{11}+Bp_{12})^2(Ap_{12}+Bp_{22})^2(Cp_{11}+Dp_{12})^2(Cp_{12}+Dp_{22})^2}}{\sigma_1\sigma_2}dt \tag{27}$$

$$dp_{22} = \frac{-(Ap_{12}+Bp_{22})(Cp_{12}+Dp_{22})}{\sigma_1\sigma_2}dt \tag{28}$$

Finally, equations from (24) to (28) form a complete set of ordinary differential equations which can be solved to get the optimal parameter estimator.

4. NUMERICAL RESULTS

According to the algorithm based on Kushner equation, we make experiments comparing with LMS and EKF in the conditions: two observers' position (10, 10), the emitter moving from (100, 100) to (20, 60) in mean-velocity, tracking sampling dots $L = 100$, azimuth-measurement error 10. The tracking trajectory is given in Fig. 2.

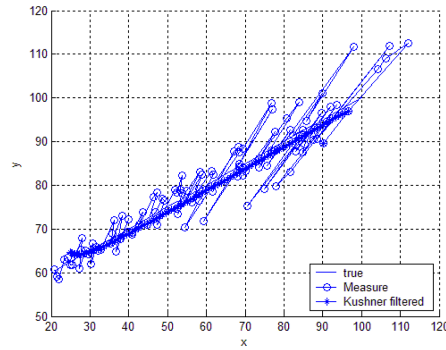

Figure 2. *The Target Tracking: Trajectory For The True, Measure And Kushner Equation Filtering*

The variance of tracking $p_{11} = E(x-p_1)^2$, $p_{12} = E[(x-p_1)(y-p_2)]$ and $p_{22} = E(y-p_2)^2$ is depicted in Fig. 3.

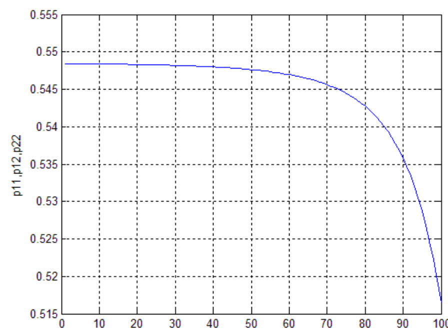

Figure 3. *The Variance Of Tracking*

So we can make the conclusion that the Kushner equation is a sort of effective nonlinear filtering algorithm and has the specialty of tracking high precision.

5. CONCLUSION AND DISCUSSION

We have discussed the algorithm based on Kushner equation for passive location tracking. Kalman filtering algorithm is an effective linear filter but in the nonlinear state its covariance is uncontrolled. Kushner equation is effective for the nonlinear filtering. According to the Kushner equation and the nonlinear of passive location, a new passive location algorithm is presented that is used in passive location tracking. Experiments are made in MATLAB and its validity is proved. In future, we would study high precision location algorithm based on multi-parameters, such as azimuth, frequency, and time of arrival.

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