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THE FORMAL TRIPLE I INFERENCE METHOD FOR LOGIC SYSTEM W_*UL

[1]MINXIA LUO, [2]NI SANG, [3]KAI ZHANG
[1]Department of Mathematics, China Jiliang University
Hangzhou, China
E-mail: minxialuo@163.com

ABSTRACT

In this paper, the formal triple I inference method based on logic system W_*UL is investigated. Firstly, a new complete formal system W_*UL, which is the schematic extension of uninorm logic system UL, is given. A complete many-sorted first-order formal system W_*UL_{ms} for fuzzy predicate logic is structured. Secondly, triple I solutions (including FMP-solutions, FMT-solutions) based on logic system W_*UL are given. Lastly, triple I algorithms are formalized in logic system $W_*UL\forall_{ms}$, and the strict logic proof of triple I algorithms are given. Moreover, the reductivities of the triple I algorithms are proved.

Keywords: *Fuzzy Logic, Fuzzy Reasoning, Triple I Algorithm, Many-sorted First-order Formal System, Logic System* W_*UL

1. INTRODUCTION

The research into logic systems is mainly to more closely simulate the ability of human reasoning in daily activities, and fuzziness in human thinking is a very important factor. As for fuzzy reasoning, the most basic models are given as follows(see [1,2]):

Given the input "x is A^*" and fuzzy rule "if x is A then y is B", try to deduce a reasonable output "y is B^*", **fuzzy modus ponens(FMP)**;

Given the input "y is B^*" and fuzzy rule "if x is A then y is B", try to deduce a reasonable output "x is A^*", **fuzzy modus tollens (FMT)**.

In the above models, A and A^* belong to fuzzy sets F(X) in the non-empty set X, B and B^* belong to fuzzy sets F(Y) in the non-empty set Y.

In regard to the models described above, the most in uential approach is the compositional rule of inference (CRI method for short) presented by Zadeh([3]) based on fuzzy set theory, in which propositions and inference rules involved are expressed as fuzzy sets and fuzzy relations and then calculate the output by synthesis from the new input and the known fuzzy relation. In reasoning processes, the CRI method has some arbitrariness and lacks solid logical basis. Wang([4,5]) proposed a triple I method for fuzzy reasoning based on R_0-implication operator by combining fuzzy logic and fuzzy reasoning, establishing the triple I principles for the models FMP and FMT. Subsequently based on regular implications and normal implications the unified triple I algorithms for FMP and FMT have been established by Wang and Fu([6]) which are equivalent to the unified forms designed by Pei([7]) for all residuated implications induced by left continuous t-norms to solve FMP and FMT problems. In addition, Pei ([8,9,10]) conducted a detailed research into the triple I algorithms based on the monoidal t-norms basic logical system MTL setting a sound logic foundation.

In 1996, Yager and Rybalov([11]) firstly proposed uninorm operator, which merged the common characteristics of t-norm and t-conorm. Uninorm operator has been widely applied in expert system, neural network, fuzzy system model, measure theory, mathematics morphology and many other domains. G. Metcalfe and F. Montagna ([12]) proposed uninorm fuzzy logic UL, which is Multiplicative additive intuitionistic linear logic $MAILL$ extended with the prelinearity axiom. Axiomatic extensions of UL include the well-known fuzzy logics such as monoidal t-norm logic MTL and basic fuzzy logic BL ([13]).

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In this paper, the weaker uninorm logic, W_*UL for shrot, which is the schematic extension of uninorm logic system UL is proposed. Moreover, the formal triple I inference method based on logic system W_*UL is investigated. This paper is organized as follows: in section 2, some basic concepts are introduced. In section 3, a formal system W_*UL and a many-sorted first-order formal system $W_*UL\forall_{ms}$ are structured. In section 4, triple I algorithms based on logic system W_*UL are formalized, and the strict logic proof for this triple I algorithms is given.

2. PRELIMINARIES

The following definitions are prepared for introducing important concepts and are used in proofs of main theorems in this paper.

Definition 2.1(see[11]) A uninorm is a binary functi $*:[0,1]^2 \to [0,1]$ such that for some $e_* \in [0,1]$, the following conditions hold for all $x, y, z \in [0,1]$:

(1) $x * y = y * x$ (Commutativity)

(2) $(x * y) * z = x * (y * z)$ (Associativity)

(3) $x \le y$ imples $x * y \le y * z$ (Monotonicity)

(4) $e_* * x = x$ (Identity)

If $e_* = 1$ or $e_* = 0$, then $*$ is a t-norm or t-conorm, respectively.

$*$ is residuated iff there exists a function $\to_*: [0,1]^2 \to [0,1]$ satisfying $z \le x \to_* y$ iff $x * z \le y$ for all $x, y, z \in [0,1]$.

In the formal system, $\otimes, \to, \wedge, \vee$ are binary, and $\bot, \bullet$, f, t are constants. We introduce propositional connectives as follows:

$\neg A =_{def} A \to f$,

$A \leftrightarrow B =_{def} (A \to B) \wedge (B \to A)$

Definition 2.2(see[12]) The formal system Uninorm Logic UL consists of the following axioms and inference rules:

(i) Axioms are:

(L1) $A \to A$

(L2) $(A \to B) \to ((B \to C) \to (A \to C))$

(L3) $(A \to (B \to C)) \to (B \to (A \to C))$

(L4) $((A \otimes B) \to C) \leftrightarrow (A \to (B \to C))$

(L5) $(A \wedge B) \to A$

(L6) $(A \wedge B) \to B$

(L7) $((A \to B) \wedge (A \to C)) \to (A \to (B \wedge C))$

(L8) $A \to (A \vee B)$

(L9) $B \to (A \vee B)$

(L10) $((A \to C) \wedge (B \to C)) \to ((A \vee B) \to C)$

(L11) $A \leftrightarrow (t \to A)$

(L12) $\bot \to A$

(L13) $A \to \bullet$

(L14) $((A \to B) \wedge t) \vee ((B \to A) \wedge t)$

(ii)The inference rules are:

Modus ponens (MP): $\{A, A \to B\} \vdash B$;

Adjunction rule (ADJ): $\{A, B\} \vdash A \wedge B$.

Definition 2.3(see[12]) A pointed bounded commutative residuated lattice is an algebra: $\mathrm{L} = (L, \wedge, \vee, \otimes, \to, t, f, \bot, \bullet)$ with universe L, binary operations $\wedge, \vee, \otimes, \to$, and constants $t, f, \bot, \bullet$, such that:

(1) $(L, \wedge, \vee, \bot, \bullet)$ is a bounded lattice;

(2) $(L, \otimes, t)$ is a commutative monoid;

(3) $z \le x \to y$ iff $x \otimes z \le y$ for all $x, y, z \in L$.

We also define the operations: $-x =_{def} x \to t$ $u =_{def} -\bullet$

Definition 2.4(see[12]) A UL-algebra is a pointed bounded commutative residuated lattice satisfying the prelinearity condition:

$t \le ((x \to y) \wedge t) \vee ((y \to x) \wedge t), x, y \in L$.

Definition 2.5(see[12]) A UL-algebra is standard iff its lattice reduct is $[0,1]$.

Theorem 2.1(see[13]) If T is theory, A is a formula, then $\mathrm{T} \vdash_{UL} A$ iff $\mathrm{T} \models_{GEN(UL)} A$. $GEN(UL)$ for UL-algebra.

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3. THE SCHEMATIC EXTENSION OF THE SYSTEM *UL* AND THE MANY-SORTED FIRST-ORDER FORMAL SYSTEM $W_*UL\forall_{ms}$

In this section, a weaker uninorm logic W_*UL which is an extention of the uninorm fuzzy logic system UL is structured.

Definition 3.1 The weaker uninorm logic, W_*UL for shrot, consists of axioms of UL plus the axiom $(W_*) A \to t$, and inference rules:

Modus ponens (MP): $\{A, A \to B\} \vdash B$;

Adjunction rule (ADJ) : $\{A, B\} \vdash A \wedge B$

Using the results given in Chapter 3 of [13], we can prove the completeness of the system W_*UL.

Theorem 3.1 If T is a theory, A is a formula, then $T \vdash_{W_*UL} A$ if $T \models_{LIN(W_*UL)} A$, $LIN(W_*UL)$ for linearly ordered W_*UL -algebra.

Monoidal t-norm basic logic MTL is the system W_*UL extended with the axiom $f \to A$.

Furthermore, we construct a complete many-sorted first-order formal system $W_*UL\forall_{ms}$. In the new system $W_*UL\forall_{ms}$, the variables and constants have different sorts, and the predicates have different types.

Introducing sort signs into the general first-order language L, we get a many-sorted first-order language L_{ms}, in which variables and constants both have decided sorts, and predicates have decided types. The definitions of terms and formulas of language L_{ms} are similar to the ones of language L. A first-order system can be structured in the language L_{ms}, which we call a many-sorted first-order formal system $W_*UL\forall_{ms}$.

Definition 3.2 The many-sorted first-order formal system $W_*UL\forall_{ms}$ consists of axioms of W_*UL plus the following axioms and inference rules:

(i) Axioms are:

($\forall$ 1) $(\forall x)A(x) \to A(v)$ (v and x are the same sort, t substitutable for x in A)

($\forall$ 2) $(\forall x)(A \to B) \to (A \to (\forall x)B)$

(x not free in A)

($\forall$ 3) $(\forall x)(A \vee B) \to (A \vee (\forall x)B)$

(x not free in A)

($\exists$ 1) $A(v) \to (\exists x)A(x)$

(v substitutable for x in A)

($\exists$ 2) $(\forall x)(A \to B) \to ((\exists x)A \to B)$

(x not free in A)

(ii) The inference rules are: Modus ponens (MP): $\{A, A \to B\} \vdash B$;

Adjunction rule (ADJ): $\{A, B\} \vdash A \wedge B$;

The generalization rule (GEN): $A \vdash (\forall x)A$.

Notice, term v and variable x have the same sort in system $W_*UL\forall_{ms}$ in the axiom ($\forall 1$).

The following parts list some important properties of the many-sorted first-order formal system $W_*UL\forall_{ms}$, which will be used in proofs of main theorems in this paper.

Proposition 3.2 Hypothetical syllogism (HS) is the inference rule of the system $W_*UL\forall_{ms}$ $\{A \to B, B \to C\} \vdash A \to C$.

Theorem 3.3 $W_*UL\forall_{ms}$ proves the following properties:

(B1) $A \to (B \to A)$

(B2) $A \to (B \to (A \otimes B))$

(B3) $(A \to B) \to ((A \otimes C) \to (B \otimes C))$

(B4) $(B \otimes A) \to (A \otimes B)$

(B5) $(\forall x)(A \to B) \to ((\forall x)A \to (\forall x)B)$

Proof: (B1) 1° $B \to t$ (W_*)

2° $(B \to t) \to ((t \to A) \to (B \to A))$ (L2)

3° $(t \to A) \to (B \to A)$ ($1^\circ, 2^\circ, MP$)

4° $(A \to (t \to A)) \to (((t \to A) \to (B \to A)) \to (A \to (B \to A)))$ (L2)

5° $A \to (t \to A)$ (L11)

6° $((t \to A) \to (B \to A)) \to (A \to (B \to A))$ ($4^\circ, 5^\circ, MP$)

7° $A \to (B \to A)$ ($3^\circ, 6^\circ, MP$)

(B2) 1° $(A \otimes B) \to (A \otimes B)$ (L1)

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$2^{\circ}\ A\to(B\to(A\otimes B))$ ($1^{\circ}, L4, MP$)

(B3) $1^{\circ}\ (A\to B)\to(A\to B)$ (L1)

$2^{\circ}\ A\to((A\to B)\to B)$ ($1^{\circ}, L3$)

$3^{\circ}\ A\to(((A\to B)\to B)\to((A\otimes(A\to B))\to B))$ ($2^{\circ}, L4$)

$4^{\circ}\ (A\otimes(A\to B))\to B$ ($2^{\circ}, 3^{\circ}, MP$)

$5^{\circ}\ B\to(C\to(B\otimes C))$ (B2)

$6^{\circ}\ (A\otimes(A\to B))\to(C\to(B\otimes C))$ ($4^{\circ}, 5^{\circ}, HS$)

$7^{\circ}\ A\to((A\to B)\to(C\to(B\otimes C)))$ ($6^{\circ}, L4$)

$8^{\circ}\ A\to(C\to((A\to B)\to(B\otimes C)))$ ($7^{\circ}, L3$)

$9^{\circ}\ (A\otimes C)\to((A\to B)\to(B\otimes C))$ ($8^{\circ}, L4$)

$10^{\circ}\ (A\to B)\to((A\otimes C)\to(B\otimes C))$ ($6^{\circ}, L3, MP$)

(B4) $1^{\circ}\ A\to(B\to(A\otimes B))$ (B2)

$2^{\circ}\ (A\to(B\to(A\otimes B)))\to(B\to(A\to(A\otimes B)))$

$3^{\circ}\ (B\to(A\to(A\otimes B)))$ ($1^{\circ}, 2^{\circ}, MP$)

$4^{\circ}\ (B\to(A\to(A\otimes B)))\to((B\otimes A)\to(A\otimes B))$

$5^{\circ}\ (B\otimes A)\to(A\otimes B)$ ($3^{\circ}, 4^{\circ}, MP$)

(B5) $1^{\circ}\ (\forall x)(A\to B)\to(A\to B)$ ($\forall 1$)

$2^{\circ}\ (\forall x)(A\to B)\to((\forall x)A\to B)$ ($1^{\circ}, GEN$)

$3^{\circ}\ (\forall x)(A\to B)\to((\forall x)A\to(\forall x)B)$ ($2^{\circ}, \forall 2$)■

Using Hájek's methods and the results given in [12,13], we can prove the completeness of the system $W_*UL\forall_{ms}$.

Theorem 3.4 (Completeness) If T is theory, A is a formula, then $T\vdash_{W_*UL\forall_{ms}} A$ iff $T\models_{LIN(W_*UL)} A$, $LIN(W_*UL)$ for linearly ordered W_*UL -algebra.

4. THE FORMAL TRIPLE I INFERENCE ALGORITHMS FOR FMP AND FMT

The basic fuzzy reasoning model of fuzzy reasoning machine are FMP(Fuzzy Modus Ponens) and FMT(Fuzzy Modus Tollens):

Rule $A\to B$

$$\frac{\text{Input } A^*}{\text{Output } B^*} \tag{4.1}$$

Rule $A\to B$

$$\frac{\text{Input } B^*}{\text{Output } A^*} \tag{4.2}$$

Wang gives the triple I principles of FMP and FMT problems (see[5]):

The Triple I Principle of FMP Problem: Suppose that X and Y are nonempty sets, $A, A^*\in F(X)$, $B\in F(Y)$, $F(X)$ and $F(Y)$ for the set consisted of the whole fuzzy sets over X and Y, respectively. Then B^* in the (4.1) is the minimum fuzzy set of $F(X)$, such that

$$(A(x)\to B(y))\to(A^*(x)\to B^*(y)), x\in X, y\in Y \tag{4.3}$$

has the maximum.

The Triple I Principle of FMT Problem: Suppose that X and Y are nonempty sets, $A\in F(X)$, $B, B^*\in F(Y)$. Then A^* in the (4.2) is the maximum fuzzy set of $F(X)$, such that (4.3) has the maximum.

We provide the triple I algorithms of FMP and FMT problems based on logic system W_*UL.

Theorem 4.1 The triple I algorithm of FMP problem is given by the following formula:

$$B^*(y)=\sup_{x\in X}\{R(A(x),B(y))\otimes A^*(x)\}, y\in Y$$

where R is a residuated implication operator of uninorm, $\otimes$ is a uninorm operator.

Theorem 4.2 The triple I algorithm of FMT problem is given by the following formula:

$$A^*(x)=\inf_{y\in Y}\{R(R(A(x),B(y)),B^*(y))\}, x\in X$$

Where R is a residuated implication operator of uninorm.

Making use of the structure and properties of system $W_*UL\forall_{ms}$, the FMP and FMT problems can be formalized. Based on system $W_*UL\forall_{ms}$, we discuss the reasonableness of the formal triple I algorithms of FMP and FMT problems.

For FMP and FMT problems, the language of the system $W_*UL\forall_{ms}$ contains only two sorts s_1, s_2.

The fuzzy sets A, B on the universes X, Y can be viewed as two unary predicates with types $(s_1),(s_2)$ respectively. The fuzzy sentences " x is A " and " y is B " can be viewed as two atomic formulas $A(x)$ and $B(y)$, where x, y are object variables with sorts s_1, s_2, respectively. So the fuzzy conditional sentence that if " x is A then y is B " can be represented as $A(x)\to B(y)$. Moreover, FMP and FMT problems can be formalized as follows:

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Rule $A(x) \to B(y)$

$$\frac{\text{Input } A^*}{\text{Output } B^*} \qquad (4.4)$$

Rule $A(x) \to B(y)$

$$\frac{\text{Input } B^*}{\text{Output } A^*} \qquad (4.5)$$

where x is a variable with sort s_1, y is a variable with sort s_2, A and A^* are unary predicates with type (s_1), and B, B^* are unary predicates with type (s_2).

In order to solve FMP and FMT problems, we indeed need to find a unary predicate B^* or A^* with type (s_2) or (s_1) so that some suitable conditions hold.

Therefore we introduce the following concepts.

Definition 4.1 Suppose that A, A^* and B are unary predicates, the type of A and A^* is (s_1), the type of B is (s_2). If in the system $W_*UL\forall_{ms}$, there exists some unary predicate B^* with type (s_2) such that:

(TI1) $\vdash (\forall x)(\forall y)((A(x) \to B(y)) \to (A^*(x) \to B^*(y)))$;

(TI2) If C is the unary predicate with type (s_2),

$\vdash (\forall x)(\forall y)((A(x) \to B(y)) \to (A^*(x) \to C(y)))$, we have $\vdash (\forall y)(B^*(y) \to C(y))$, then we call B^* the formal triple I solution of (4.4).

Definition 4.2 Suppose that A, B and B^* are unary predicates, the type of A is (s_1), the type of B, B^* is (s_2). If the system $W_*UL\forall_{ms}$, there exists some unary predicate A^* with type (s_1) such that (TI1) hold and (TI3) If D is the unary predicate with type (s_1), and

$\vdash (\forall x)(\forall y)((A(x) \to B(y)) \to (D(x) \to B^*(y)))$, we have $\vdash (\forall x)(D(x) \to A^*(x))$, then we call A^* the formal triple I solution of (4.5).

Definition 4.3(see[9]) Suppose that A, A^* and B are unary predicates, the type of A and A^* are (s_1), the type of B is (s_2). A is normal (ie. $\vdash (\exists x)A(x))$. If $A^* = A$, we have $B^* \approx B$, ie. B^* and B are provably equivalent (briefly, equivalent. $\vdash B^* \leftrightarrow B$, we say that triple I algorithm of FMP problem (4.4) is reductive.

Definition 4.4(see[9]) Suppose that A, B and B^* are unary predicates, the type of A is (s_1), the type of B and B^* is (s_2). B is co-normal (ie. $\vdash (\exists y)\neg B(y))$. If $B^* = B$, we have $A^* \approx A$, we say that triple I algorithm of FMT problem (4.5) is reductive.

Based on the system $W_*UL\forall_{ms}$, we can obtain the following theorems.

Theorem 4.3 In the system $W_*UL\forall_{ms}$, the formal triple I solution B^* of FMP problem is given by following formulas:

$$B^* = (\exists x)((A(x) \to B) \otimes A^*(x)) \qquad (4.6)$$

for variable y with sort (s_2),

$$B^*(y) = (\exists x)((A(x) \to B(y)) \otimes A^*(x)) \qquad (4.7)$$

Proof: We will prove (4.7) by (TI1) and (TI2) in Definition 4.1.

The following sequence is a proof of (TI1):

$1^\circ ((A(x) \to B(y)) \otimes A^*(x))$

$\quad \to (\exists x)((A(x) \to B(y)) \otimes A^*(x))$ $(\exists 1)$

$2^\circ (((A(x) \to B(y)) \otimes A^*(x))$

$\quad \to (\exists x)((A(x) \to B(y)) \otimes A^*(x)))$

$\quad \to ((A(x) \to B(y)) \to (A^*(x) \to$

$\quad (\exists x)((A(x) \to B(y)) \otimes A^*(x))))$ (L4)

$3^\circ (A(x) \to B(y)) \to (A^*(x) \to$

$\quad (\exists x)((A(x) \to B(y)) \otimes A^*(x)))$ $(1^\circ, 2^\circ, MP)$

$4^\circ (\forall x)(\forall y)(A(x) \to B(y)) \to (A^*(x)$

$\quad \to (\exists x)((A(x) \to B(y)) \otimes A^*(x)))$ $(3^\circ, GEN)$

The following sequence is a proof of (TI2):

$1^\circ (\forall x)(\forall y)(A(x) \to B(y)) \to (A^*(x) \to C(y)))$

(Hypot)

$2^\circ (A(x) \to B(y)) \to (A^*(x) \to C(y))$ $(\forall 1)$

$3^\circ ((A(x) \to B(y)) \to (A^*(x) \to C(y)))$

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$\to (((A(x)\to B(y))\otimes A^*(x))\to C(y))$ (L4)

$4^\circ ((A(x)\to B(y))\otimes A^*(x))\to C(y)$ $(2^\circ,3^\circ,MP)$

$5^\circ (\exists x)((A(x)\to B(y))\otimes A^*(x))\to C(y)$

$(4^\circ,\exists 1,MP)$

$6^\circ B^*(y)\to C(y)$ $(5^\circ,Abbr.)$

$7^\circ (\forall y)(B^*(y)\to C(y))$ $(6^\circ,GEN)$ ■

Theorem 4.4 In the system $W_*UL\forall_{ms}$, the formal triple I solution A^* of FMT problem is given by following formulas:

$$A^* = (\forall y)((A\to B(y))\to B^*(y)) \quad (4.8)$$

for variable x with sort s_1,

$$A^*(x) = (\forall y)((A(x)\to B(y))\to B^*(y)) \quad (4.9)$$

Proof: We will prove (4.9) by (TI1) and (TI3) in Definition 4.1 and 4.2.

$1^\circ ((A(x)\to B(y))\to B^*(y))\to$

$((A(x)\to B(y))\to B^*(y))$ (L1)

$2^\circ (\forall y)((A(x)\to B(y))\to B^*(y))\to$

$((A(x)\to B(y))\to B^*(y))$ $(\forall 1)$

$3^\circ (A(x)\to B(y))\to ((\forall y)((A(x)\to$

$B(y))\to B^*(y))\to B^*(y))$ $(2^\circ,L3)$

$4^\circ (A(x)\to B(y))\to (A^*(x)\to B^*(y))$ $(3^\circ,Abbr.)$

$5^\circ (\forall x)(\forall y)(A(x)\to B(y))\to (A^*(x)\to B^*(y))$

The following formula sequence is a proof of (TI3):

$1^\circ (\forall x)(\forall y)(A(x)\to B(y))\to (D(x)\to B^*(y))$

(Hypot)

$2^\circ (A(x)\to B(y))\to (D(x)\to B^*(y))$ $(1^\circ,\forall 1)$

$3^\circ D(x)\to ((A(x)\to B(y))\to B^*(y))$ $(2^\circ,L3,MP)$

$4^\circ (\forall y)(D(x)\to ((A(x)\to B(y))\to B^*(y)))$

$(3^\circ,GEN)$

$5^\circ D(x)\to (\forall y)((A(x)\to B(y))\to B^*(y))$

$(4^\circ,\forall 2,MP)$

$6^\circ D(x)\to A^*(x)$ $(5^\circ,Abbr.)$

$7^\circ (\forall x)D(x)\to A^*(x)$ $(6^\circ,GEN)$ ■

Theorem 4.5 The algorithm of Theorem 4.4 is reductive.

Proof: Suppose that A and A^* are normal, and $A=A^*$. We will prove $B\approx B^*$, with

$B^*(y)=(\exists x)((A(x)\to B(y))\otimes A(x))$.

First, we prove $\vdash (\forall y)(B^*(y)\to B(y))$.

$1^\circ (A(x)\to B(y))\to (A(x)\to B(y))$ (L1)

$2^\circ ((A(x)\to B(y))\otimes A(x))\to B(y)$ $(1^\circ,L4,MP)$

$3^\circ ((\forall x)((A(x)\to B(y))\otimes A(x)))\to B(y)$ $(2^\circ,GEN)$

$4^\circ ((\exists x)((A(x)\to B(y))\otimes A(x)))\to B(y)$

$(3^\circ,\exists 2,MP)$

$5^\circ B^*(y)\to B(y)$ $(4^\circ,Abbr.)$

$6^\circ (\forall y)(B^*(y)\to B(y))$ $(5^\circ,GEN)$

Moreover, we prove $\vdash (\forall y)(B(y)\to B^*(y))$.

$1^\circ (\forall x)A(x)$ (Hypot)

$2^\circ (\forall x)(\forall y)(A(x)\to B(y))$ (Hypot)

$3^\circ A(x)\to (B(y)\to (A(x)\otimes B(y)))$ (B2)

$4^\circ B(y)\to (A(x)\to B(y))$ (B1)

$5^\circ (A(x)\otimes B(y))\to (A(x)\otimes (A(x)\to B(y)))$
$(4^\circ,B3,MP,B4)$

$6^\circ (A(x)\otimes B(y))\to ((A(x)\otimes (A(x)\to B(y)))$

$\to ((A(x)\to B(y))\otimes A(x)))$ $(5^\circ,B4)$

$7^\circ (A(x)\otimes (A(x)\to B(y)))\to ((A(x)\otimes B(y))$

$\to ((A(x)\to B(x))\otimes A(x)))$ $(6^\circ,L3,MP)$

$8^\circ (A(x)\otimes (A(x)\to B(y)))$

$\to ((A(x)\otimes B(y))\to B^*$ $(7^\circ,\exists 1)$

$9^\circ (A(x)\otimes (A(x)\to B(y)))\to$

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$((A(x)\otimes B(y))\to B^*(y))$ $(8^\circ, Abbr.)$

10° $A(x)\to((A(x)\to B(y))\to((A(x)\otimes B(y))\to B^*(y)))$ $(9^\circ, L4)$

11° $(A(x)\otimes B(y))\to B^*(y)$ $(1^\circ, 2^\circ, B6, 10^\circ, MP)$

12° $A(x)\to(B(y)\to B^*(y))$ $(11^\circ, L4)$

13° $((\forall x)A(x))\to(B(y)\to B^*(y))$ $(12^\circ, GEN)$

14° $B(y)\to B^*(y)$ $(1^\circ, 13^\circ, MP)$

15° $(\forall y)(B(y)\to B^*(y))$ $(14^\circ, GEN)$ ■

Theorem 4.6 The algorithm of Theorem 4.4 is reductive.

Proof: Suppose B and B^* are co-normal, and $B^*=B$. We will prove $A^*\approx A$, with $A^*(x)=(\forall y)((A(x)\to B(y))\to B(y))$.

First, we prove $\vdash(\forall x)(A(x)\to A^*(x))$

1° $(A(x)\to B(y))\to(A(x)\to B(y))$ (L1)

2° $A(x)\to((A(x)\to B(y))\to B(y))$ $(1^\circ, L3, MP)$

3° $(\forall y)(A(x)\to((A(x)\to B(y))\to B(y)))$ $(2^\circ, GEN)$

4° $A(x)\to((\forall y)((A(x)\to B(y))\to B(y)))$ $(3^\circ, \forall 2, MP)$

5° $A(x)\to A^*(x)$ $(4^\circ, Abbr.)$

6° $(\forall x)(A(x)\to A^*(x))$ $(5^\circ, GEN)$

Moreover, we prove $\vdash(\forall x)(A^*(x)\to A(x))$

1° $(\neg(\forall y)B(y)\to A(x))\to(((A(x)\to(\forall y)B(y))\to(\forall y)B(y))\to(\neg(\forall y)B(y)\to A(x)))$ (B1)

2° $(\neg(\forall y)B(y)\to A(x))\to(\neg(\forall y)B(y)\to(((A(x)\to(\forall y)B(y))\to(\forall y)B(y))\to A(x)))$ $(1^\circ, L3)$

3° $A(x)\to(\neg(\forall y)B(y)\to A(x))$ (B1)

4° $(\forall x)A(x)$ (Hypot)

5° $\neg(\forall y)B(y)\to(((A(x)\to(\forall y)B(y))\to(\forall y)B(y))\to A(x))$ $(2^\circ, 3^\circ, 4^\circ, MP, HS)$

6° $(\exists y)\neg B(y)$ (Hypot)

7° $\neg(\forall y)B(y)$ (6°)

8° $((A(x)\to(\forall y)B(y))\to(\forall y)B(y))\to A(x)$ $(5^\circ, 7^\circ, MP)$

9° $((\forall y)((A(x)\to B(y))\to B(y)))\to((A(x)\to((\forall y)B(y)))\to((\forall y)B(y)))$ $(B5, \forall 2)$

10° $((\forall y)((A(x)\to B(y))\to B(y)))\to A(x)$ $(8^\circ, 9^\circ, HS)$

11° $A^*(x)\to A(x)$ $(10^\circ, Abbr.)$

12° $(\forall x)(A^*(x)\to A(x))$ $(11^\circ, GEN)$ ■

5. CONCLUSION

In this paper, fristly we provide a new complete formal logic system W_*UL, which is the schematic extension of logic system UL, and constructe a complete many-sorted first-order formal system $W_*UL\forall_{ms}$ of fuzzy predicate logic. Then, based on the logic system W_*UL, we propose the triple I algorithms for FMP and FMT problems. It is not only that, making use of the structure and properties of system $W_*UL\forall_{ms}$, triple I solutions of the FMP and FMT problems are formalized, but also the reasonableness and redutivity of triple I algorithms of FMP and FMT problems are explained. We have put fuzzy reasonings into the framwork of pure fuzzy logic and provided a new inference method for processing complicated problems in the real world.

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