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A PROBABILISTIC FUZZY SET FOR UNCERTAINTIES-BASED MODELING IN LOGISTICS MANIPULATOR SYSTEM

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ABSTRACT

The probabilistic fuzzy set (PFS) and the related probabilistic fuzzy logic system (PFLS) are designed for handling the uncertainties in both stochastic and nonstochastic nature. In this paper, a novel probabilistic fuzzy set is proposed by randomly varying widths of the Gaussian membership function. Then the related PFLS is constructed and applied to logistics manipulator system problem to study its stochastic modeling capability. It clearly discloses that the novel PFS perform better than the previous PFS. The work presented can extend the potential application of probabilistic fuzzy set.

Keywords: *Probabilistic Fuzzy Set, Secondary Probability Density Function, Rigid-Flexible Manipulator, Modeling Capability*

1. INTRODUCTION

There exist different uncertainties in many real-world applications. These uncertainties can be classified into stochastic and nonstochastic uncertainties [1]. Generally, stochastic uncertainties can be captured well by the probabilistic modeling [2]. On the other hand, the fuzzy technique has been witnessed to be a powerful modeling tool to nonstochastic uncertainties. Type-1 fuzzy set is proposed [3] and it uses crisp membership grade for modeling imprecise and vague information. However, when the uncertainties are very complex, it may not be suitable to use a crisp membership grade in [0, 1]. To capture the uncertainties in membership function (MF) more sufficiently, the type-2 fuzzy set is first defined by Zadeh [4]. It blurs the boundaries of the type-1 MF for directly modeling the more complex uncertainties and has membership grades that are themselves fuzzy [5]. Currently, type-1 and type-2 set have been successfully applied in many fields such as decision making [6], function approximation [7] and so on.

In most of real-world applications, both stochastic and deterministic uncertainties exist simultaneously. However, the traditional fuzzy theory and probabilistic models are only good at processing one aspect of uncertainties. So it would be valuable to integrate the probability theory with the fuzzy theory [8-9]. Consequently, some concepts and methods are proposed, such as probability measures of fuzzy events [10], fuzzy random set [11], fuzzy random variable [12-13], nonstationary fuzzy sets [14] and fuzzy model with probability-based rule weights [15], etc. Fundamentally, two kinds of integration principles underlie these methods. One assumption is to introduce the fuzzy uncertainties into the statistical framework. Another assumption is to introduce the stochastic uncertainties into the fuzzy system. Based on the second assumption, the probabilistic fuzzy set (PFS) is proposed and developed by introducing the probabilistic theory into the traditional fuzzy set described by center and width [16]. As such, the fuzzy grades in the traditional fuzzy set become the stochastic variables described by the secondary probability density function (PDF), which make it able to capture both stochastic and nonstochastic uncertainties. Recently, based on probabilistic fuzzy set, the probability fuzzy logic system is proposed and it has been applied for stochastic modeling and control [16], function approximation problem [17] and so on.

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However, research about probabilistic fuzzy sets still remains at the beginning phase. The previous probabilistic fuzzy set is constructed through randomizing the center of the Gaussian type fuzzy set. The randomization of the width of the fuzzy set has not yet been considered. In this paper, a novel probabilistic fuzzy set is proposed by randomizing the width of Gaussian fuzzy sets. Furthermore, based on the novel probabilistic fuzzy set, the related PFLS is applied to logistics manipulator system modeling problem. It discloses that the novel probabilistic fuzzy set performs better than the previous probabilistic fuzzy set. Thus, it makes the probabilistic fuzzy set more applicable in engineering applications.

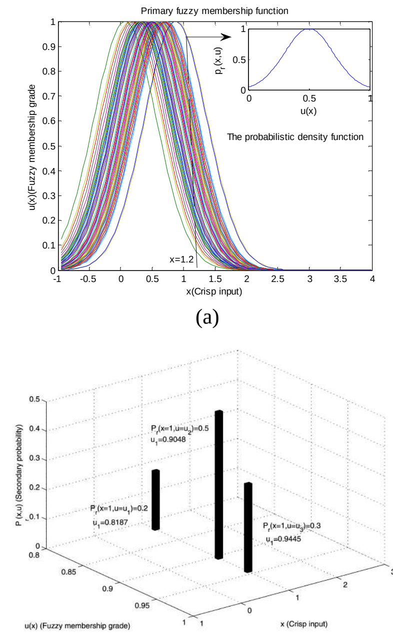
This paper is organized as following: the problem formulation is presented in section 2. In section 3, a new type of probability fuzzy set will be constructed. The modeling analysis of the novel probabilistic fuzzy set is conduced in section 4. Finally, the conclusion is given in section 5.

2. PROBLEM FORMULATION

2.1 Probabilistic Fuzzy Set

The concept of probabilistic fuzzy sets have been proposed to capture uncertainties with both stochastic and fuzzy features [16] by introducing probability into the traditional fuzzy set described by center and width. Based on considering the random variation from the center of the traditional Gaussian fuzzy set, the random variation was introduced into the membership functions. So in probabilistic fuzzy set, for an input x, there no longer is a single value or values for the membership function; instead, the membership function becomes a random variable that can be described by the secondary PDF as shown in Figure1 (a).

As such, a 3-dimention membership function including the fuzzy dimension and the probabilistic dimension is hinted in the probabilistic fuzzy set as shown in Figure1 (b), which makes it able to handle the information with both fuzzy and stochastic uncertainties existing in the process.



(b)

Figure 1: (A) Fuzzy MF In The Probabilistic Fuzzy Set For The Perturbed Center; (B) The Illustration Of 3-Dimension Structure In PFS.

2.2 Probabilistic Fuzzy Logic System

Similar to the ordinary fuzzy logic system, the PFLS still has operations of fuzzification, inference engine and defuzzification. Different to the ordinary fuzzy logic system, the PFLS uses the probabilistic fuzzy set that is described by a three-dimensional MF.

(1) Defuzzification: the original inputs will be transformed into the probabilistic fuzzy sets instead of traditional fuzzy sets based on probabilistic membership function.

(2) Inference engine: the inference in PFLS is based on the fuzzy rules as follows:

$$Rule\ j: If\ \ x_1\ is\ A_{1,j}\ and\ ...\ and\ x_i\ is\ A_{i,j}$$
$$and\ ...\ and\ x_n\ is\ A_{n,j},$$
$$Then\ \ y\ \ is\ \ B_j \quad (1)$$

where $A_{i,j}\ (i=1,2,...,n)\ (j=1,2,...,j)$ and B_j are probabilistic fuzzy sets. The probabilistic fuzzy relation can be written as:

$$R_{A_{1,i}\times\cdots\times A_{J,i}\rightarrow B_i}(x,\tau)=A_{1,i}\cap A_{2,i}\cap...\cap A_{J,i}\cap B_i \quad (2)$$

where $A_{1,i}\times...\times A_{J,i}$ is the Cartersian product of $A_{1,i},...,A_{J,i}$. And $A_{j,i}\equiv \underset{x_j\in X_j}{U}(U_{\tilde{A}_{j,i}},\wp_{A_{j,i}},P)$, $B_i\equiv U_{\tau\in Y}(U_{B_i},\wp_{B_i},P)$. The element number in

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$A_{1,i} \cap A_{2,i} \cap \ldots \cap A_{J,i} \cap B_i$ is $\prod_{j=1}^{J} S_{A_{j,i}} \cdot S_{B_i}$, where $S_{A_{j,i}}$ and S_{B_i} is the number of events in $U_{A_{j,i}}$ and U_{B_i} respectively.

Similar to the probabilistic fuzzy relation, the stochastic characteristic of probabilistic fuzzy set can be expressed as follow:

$$P(E_{\tilde{R}_j}) = P(E_{\tilde{A}_{1,j}}) \cdot P(E_{\tilde{A}_{2,j}}) \cdots\cdot P(E_{\tilde{A}_{n,j}}) \cdot P(E_{\tilde{B}_j}) \quad (3)$$

where $\mu_{\tilde{R}_j}$ is the fuzzy membership grade of the probabilistic fuzzy relation set $R_{A_{1,i}\times\cdots\times A_{J,i}\to B_i}(x,\tau)$, the $P(E_{\tilde{R}_j})$, $P(E_{\tilde{A}_{i,j}})$ and $P(E_{\tilde{B}_j})$ denote the corresponding probability of the fuzzy grade $\mu_{\tilde{R}_j}, \mu_{\tilde{A}_{i,j}}$ and $\mu_{\tilde{B}_j}$, with $E_{\tilde{A}_{i,j}}$ and $E_{\tilde{B}_j}$ as independent events.

(3) Defuzzification: mathematical expectation of the centroid output is computed as the final crisp output. The traditional defuzzification computes the centroid output y_C with the inference set $\mu_{\tilde{R}_j}$ as:

$$y_C = \frac{\sum_{j=1}^{\bar{J}} y_j \mu_{\tilde{R}}(x, y_j)}{\sum_{j=1}^{\bar{J}} \mu_{\tilde{R}}(x, y_j)} \quad (4)$$

$$\mu_{\tilde{R}} = Max(\mu_{\tilde{R}_1}, \cdots, \mu_{\tilde{R}_j}, \cdots, \mu_{\tilde{R}_{\bar{J}}}) \quad (5)$$

where y_j is the crisp consequent, y_C and $\mu_{\tilde{R}}$ are random variables. The crisp output y of the PFLS is the mathematical expectation of the traditional defuzzification:

$$y = Ex(y_C) \quad (6)$$

The established probabilistic fuzzy set only considers the random effects of the center of Gaussian MF, however random variations from the width are not considered.

3. CONSTRUCTION OF PROBABILISTIC FUZZY SET

In this section, considering randomizing the width of Gaussian fuzzy set, a new type of probabilistic fuzzy set will be proposed.

The primary MF as Gaussian type is described in (7) shown in Figure 2.

$$u_{Gau} = e^{(-\frac{(x-c)^2}{2\varsigma^2})} \quad (7)$$

where c is the center, ς is the width, and x is the input.

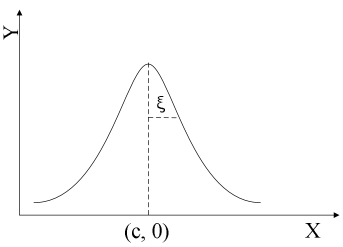

Figure 2: Gaussian Fuzzy MF

In engineering applications, based on the random sampling principle, in the process of repeatedly extracting samples that follows the normal distribution, if the samples number n is large enough [18], the distribution of variance of these samples will gradually approach to the normal distribution, which can be described as:

$$\theta \quad N(\alpha, \beta^2) \quad (8)$$

where θ is the variance, α denotes the mean of θ and β denotes the variance of θ.

In equation (7), the width ς of Gaussian is regarded as the variance θ of equation (8), it can be seen as a random variable following the normal distribution described as

$$\varsigma \sim N(\omega, \lambda^2) \quad (9)$$

Accordingly, shown as in Figure 3, the fuzzy grade u_{Gau} becomes a random variable $U_{Gau} = e^{(-(x-c)^2/2\varsigma^2)}$ with a certain distribution. Its probability distribution can be obtained as:

$$F_{U_{Gau}}(u_{Gau}) = \begin{cases} \int_0^{\frac{|x-c|}{\sqrt{-2\ln u_{Gau}}}} \frac{1}{\sqrt{2\pi}\lambda} e^{-\frac{(\varsigma-\omega)^2}{2\lambda^2}} d\varsigma & 0 < u_{Gau} < 1 \\ 0 & otherwise \end{cases} \quad (10)$$

And the secondary PDF is

$$\Pr ob_A(u_{Gau}) = \begin{cases} \frac{|x-c|(-2\ln u_{Gau})^{-\frac{3}{2}}}{\sqrt{2\pi}\lambda u_{Gau}} e^{-\frac{(\frac{|x-c|}{\sqrt{-2\ln u_{Gau}}}-\omega)^2}{2\lambda^2}} & 0 < u_{Gau} < 1 \\ 0 & otherwise \end{cases} \quad (11)$$

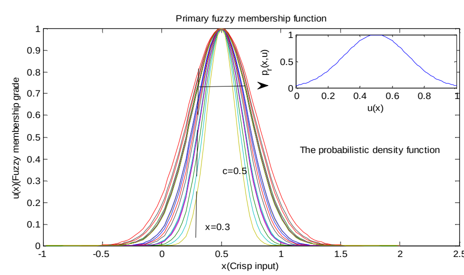

Figure 3: Fuzzy MF In The Probabilistic Fuzzy Set For The Perturbed Width

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Proof:

Suppose the width ς is a random variable following normal distribution which can be described as:

$$\varsigma \sim N(\omega, \lambda^2) \quad (12)$$

Then the density function is

$$\Phi(\varsigma) = \frac{1}{\sqrt{2\pi}\lambda} e^{-(\frac{1}{2})(\frac{\varsigma-\omega}{\lambda})^2} \quad (13)$$

The random variable fuzzy grade is $U_{Gau} = e^{(-(x-c)^2/2\varsigma^2)}$ $(u_{Gau} \in (0,1))$. Though U_{Gau} is non-monotonic, it is monotonically decreasing in $(0,+\infty)$, so the distribution function of fuzzy grade U_{Gau} can be obtained as following:

Obviously, when $u_{Gau} \le 0$, the distribution function is

$$F_{U_{Gau}}(u_{Gau}) = P(U_{Gau} < u_{Gau}) = 0 \quad (14)$$

When $0 < u_{Gau} < 1$, the distribution function can been obtained as:

$$\begin{aligned} F_{U_{Gau}}(u_{Gau}) &= P(U_{Gau} < u_{Gau}) \\ &= P(e^{-\frac{(x-c)^2}{2\varsigma^2}} < u_{Gau}) \quad (15) \\ &= P(\frac{(x-c)^2}{-2\ln u_{Gau}} > \varsigma^2) \end{aligned}$$

As variance ς must be positive, equation (15) can be written as:

$$P(\sqrt{\frac{(x-c)^2}{-2\ln u_{Gau}}} > \varsigma > 0) = \int_0^{\frac{|x-c|}{\sqrt{-2\ln u_{Gau}}}} \phi(\varsigma) d\varsigma \quad (16)$$

Thus, the probability distribution of U_{Gau} is equation (10).

Again, we consider the first derivative of u_{Gau}, the probability density function can be obtained from Variable Limit Integral Derivation Formula as:

$$\begin{aligned} F'_{U_{Gau}}(u_{Gau}) &= \frac{1}{\sqrt{2\pi}\lambda} e^{-\frac{(\frac{|x-c|}{\sqrt{-2\ln u_{Gau}}}-\omega)^2}{2\lambda^2}} (\frac{|x-c|}{\sqrt{-2\ln u_{Gau}}})' \\ &= \frac{|x-c|(-2\ln u_{Gau})^{-\frac{3}{2}}}{\sqrt{2\pi}\lambda u_{Gau}} e^{-\frac{(\frac{|x-c|}{\sqrt{-2\ln u_{Gau}}}-\omega)^2}{2\lambda^2}} \end{aligned} \quad (17)$$

And secondary PDF can be expressed as equation (11).

4. MODELING ANALYSIS

In this section, a logistics Rigid-Flexible manipulator problem is carried out to demonstrate some properties of the proposed PFS, and to investigate its distinctive modeling capability in engineering.

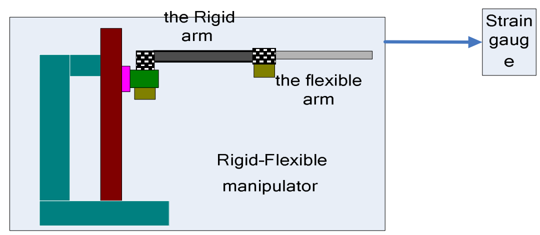

Figure 4: Rigid-Flexible Manipulator System

The Rigid-Flexible manipulator system is designed for verify the validity of the distributed parameter system as shown in Figure 4. When signal is applied to the revolute joint of the flexible arm, the strain will happen on the flexible link arms, then Strain gauge which is pasted in flexible link arms will appear distorted and produce strain voltage value. The strain voltage value can be used to Deduced the deformation of mechanical arm to verify the validity and accuracy of the distributed parameter system. In this system, it is very important for obtaining the strain voltage value

A nonlinear modeling is used to approximate the change process of strain voltage data collected from Strain gauge

$$y(k) = f(y(k-1), y(k-2), \ldots, y(k-n)) \quad (18)$$

where $y(k)$ is the input signal of first k moment, f is the desired Innuendo relationship.

A set of Strain gauge is pasted in flexible link arms, the signal that is applied to the revolute joint of the flexible arm is signal sin, the Sampling time is set to 50ms, and the strain voltage value of Strain gauge is the output of the system.

Based on the proposed PFS, the related Gau_{width} -based PFLS is constructed to model the nonlinear system (18). The modeling progress in details is:

Step 1) Collect input–output data $n = 250$.

Step 2) Obtain the clustering results parameters (clustering center C, width ς) by the fuzzy c -mean variance (FCMV) algorithm. The number of clustering center $\tilde{c}=5$.

Step 3) The Gaussian membership function of the antecedent part is obtained to construct fuzzy

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$if-then$ rules. Then parameters (the width mean ω, the width variance λ) for second PDF which is expressed in equation (11) can be determined by randomizing the width. The $l-th$ rule in PFLS is:

$$Rule\ l: if\ x\ is\ A_l, then\ y\ is\ b_l \qquad (19)$$

Step4) The simulation comparison of Gau_{center} -based PFLS and Gau_{width} -based PFLS is carried out. RMSE is used here as:

$$RMSE = \sqrt{\frac{1}{n}\sum_{k=1}^{n}(y(k)-y_e(k))^2} \qquad (20)$$

where $n = 250$ is the number of testing data, $y(k)$ is the desired output and $y_e(k)$ is the estimated output.

Table I : The Parameter Of Gau_{width} -Based Pfls

Rule number	$(c_k, \omega_k, \lambda_k)$	(b_k)
1	(0.8091,0.2156,0.0802)	(0.1916)
2	(0.4924,0.0147,0.0492)	(0.5466)
3	(0.0864,0.2551,0.0358)	(0.8753)
4	(0.7686,0.1701,0.0397)	(0.8106)
5	(0.6730,0.1306,0.0131)	(0.2233)

The learning results of parameters obtained for Gau_{width} -based PFLS are given in Table I. Accordingly, the comparison of approximation error $y-y_e$ is shown in Figure 5.

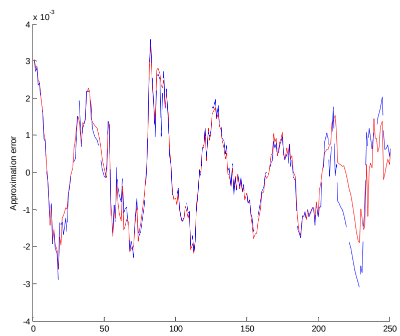

Figure 5: The Comparison Of Approximation Error $y-y_e$ For Gau_{width} -Based PFLS (Red Solid Line) And Gau_{center} -Based PFLS (Dotted Line).

The mean-squared error comparison of Gau_{width} -based PFLS and Gau_{center} -based PFLS is 6.3145e-004 and 6.1727e-004 respectively. The simulation comparison is based on statistical results. The average parameters are obtained from 100 Monte Carlo simulations. It is clearly that strong relations are found between modeling capability of PFS and the random disturb. The reason is that the Gau_{width} -based PFLS has the better potential ability to handle uncertainties than Gau_{center} -based PFLS under certain stochastic circumstance.

5. CONCLUSION

In this paper, a new probabilistic fuzzy set is proposed by randomly varying the width of Gaussian membership function. The related probabilistic fuzzy logic system is applied to a Rigid-Flexible manipulator problem to study its stochastic modeling capability. It is proved that the novel PFS perform better than the previous PFS under certain stochastic circumstance. This will broaden the potential application of probabilistic fuzzy set. In the future, more designation may be conducted for PFS, it is believed that the PFS will be very promising for many engineering application.

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