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APPLICATION OF UNCERTAIN NONLINEAR SYSTEMS PARTIAL STATE VARIABLES CONTROL TO A CLASS OF PENDULUM SYSTEMS

YANJUN LIANG, GUOYING LIU

School of Computer and Information Engineering, AnYang Normal University, Anyang 455002, Henan, China

ABSTRACT

Because of inaccuracy of modeling and parameters measurement, the controlled systems are subjected by the inaccuracy inevitably and the control of uncertain systems is studied by many researchers. Uncertain nonlinear systems partial state variables control technique is developed, and applied to single pendulum systems. In order to make a certain state variable asymptotically stable and adaptive parameters identify the uncertain parameters, using partial state stability theory and adaptive control method, two controllers and adaptive regulators are designed separately for two state variables and uncertain parameters. Our controllers can guarantee the uniformly ultimate boundedness of the solution of the closed-loop system, and make the tracking error arbitrarily small for uncertain parameters. Numerical simulation results illustrate the effectiveness of the proposed controllers and adaptive regulators.

Keywords: *Nonlinear Systems, Partial State Variables Control, Uncertain Systems*

1. INTRODUCTION

In the actual control systems, there are many nonlinearity facts. One often using linear control theory ignoring the nonlinearity of the control systems, it is useful for the weak nonlinearity, but for the strong nonlinearity, one usually can not get expected control effect. Because of inaccuracy of modeling and parameters measurement, the controlled systems are generally subjected by the inaccuracy inevitably. Therefore uncertain nonlinear systems control is taken much attention recently.

There is not a general control method for all the uncertain nonlinear systems control. One tries to study kinds of uncertain nonlinear systems by many means. In [1], by incorporating adaptive backstepping design technique into a neural network based adaptive control design framework, adaptive backstepping design is developed for a class of nonlinear systems in strict-feedback form with arbitrary uncertainty. In [2], the problem of finite-time stabilization for nonlinear systems is studied; they prove that global finite-time stabilizability of uncertain nonlinear systems that are dominated by a lower-triangular system can be achieved by Hölder continuous state feedback. A robust adaptive fuzzy control design approach is developed for a class of multivariable nonlinear systems with modeling uncertainties and external disturbances [3]. In [4], an adaptive control algorithm is proposed for output regulation of uncertain nonlinear systems in output feedback form under disturbances generated from nonlinear exosystems. In [5], they investigate the robust reliable H∞ control for a class of uncertain nonlinear system. A unified framework for adaptive iterative learning control design for uncertain nonlinear systems is proposed, and according to the value of a certain parameter gamma, the parametric adaptation law can be a pure time-domain adaptation, a pure iteration-domain adaptation or a combination of both [6]. A hybrid control system, integrating principal and compensation controllers, is developed for multiple-input-multiple-output (MIMO) uncertain nonlinear systems [7]. In [8], an adaptive fuzzy control approach is proposed for a class of MIMO nonlinear systems with completely unknown nonaffine functions, then in [9], external disturbances appear in each equation of each subsystem and the disturbance coefficients are assumed to be unknown functions rather than constant one, and the universal approximation theorem of the fuzzy logic systems is utilized to develop an adaptive control scheme for a class of nonlinear MIMO systems by the backstepping technique. For uncertain systems preceded by unknown dead-zone nonlinearity, in [10], a new scheme to design adaptive controllers is presented. In [11], they prove that the proposed nonlinear disturbance observer recovers not only the steady-state performance but also the transient

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performance of the nominal closed-loop system under plant uncertainties and input disturbances. If the uncertainties are bounded, while this bound is not known, in [12], a PI-adaptive fuzzy control architecture for a class of uncertain nonlinear systems is proposed that aims to provide added robustness in the presence of large and fast but bounded uncertainties and disturbances. Based on the integration of sliding mode control and adaptive fuzzy control, a novel adaptive fuzzy sliding mode control methodology is proposed in [13].

In the engineering, because some state variables of the systems are not available, partial state variables or a single state variable control is simple, effective and easy to implement. In this paper, uncertain nonlinear systems partial state variables control technique is developed for a class of pendulum systems. In order to make a certain state variable asymptotically stable and adaptive parameters identify the uncertain parameters, using partial state stability theory and adaptive control method, two controllers and adaptive regulators are designed separately for two state variables and uncertain parameters. Numerical simulation results illustrate the effectiveness of the proposed controllers and adaptive regulators.

2. UNCERTAIN PENDULUM CONTROL SYSTEMS

Pendulum is also named mathematical pendulum, and it is an important model in physics for its various dynamical behaviors. According to Newton laws of motion, if considering environmental damping and external driving force, simple pendulum movement is a two order nonlinear differential equation. Because of inaccuracy of modeling and parameters measurement, in the process of model establishment for pendulum dynamical systems, uncertainty is inevitable. Then we consider a class of pendulum systems as follows

$$\begin{aligned} \dot{x}_1 &= \varepsilon x_2 + \eta \tan x_1 \\ \dot{x}_2 &= -\theta \sin x_1 \end{aligned} \qquad (1)$$

in which x_1 and x_2 are state variables, ε, η and θ are uncertain parameters produced by the process of system modeling.

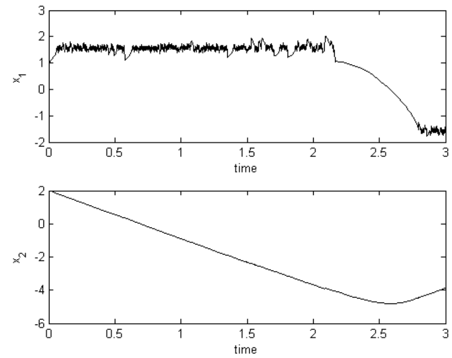

Figure 1: The Curves Of x_1 And x_2

We give the curves of the state variables x_1 and x_2 of systems (1) using Matlab software in figure 1, in which $\varepsilon = 1$, $\eta = 2$ and $\theta = 3$. Obviously, systems (1) are not asymptotically stable. The main purpose of this paper is designing controller u_1 and u_2 for control systems (2) to ensure a certain state variable asymptotically stable.

$$\begin{aligned} \dot{x}_1 &= \varepsilon x_2 + \eta \tan x_1 + u_1 \\ \dot{x}_2 &= -\theta \sin x_1 + u_2 \end{aligned} \qquad (2)$$

3. PARTIAL STATE VARIABLE CONTROLLER DESIGN

In order to design partial state variable controller for systems (2), we introduce two lemmas. Consider differential equations

$$\frac{d\boldsymbol{x}}{dt} = f(t, \boldsymbol{x}) \qquad (3)$$

in which

$f(t,\boldsymbol{x}) \in C[I \times R^n, R^n]$, $f(t,\boldsymbol{0}) \equiv \boldsymbol{0}$, $\boldsymbol{x} = col(\boldsymbol{y},\boldsymbol{z}) =$
$col(x_1, x_2, ..., x_m, x_{m+1}, x_{m+2}, ..., x_n) \in R^n$,
$\boldsymbol{y} = col(x_1, x_2, ..., x_m) \in R^m$,
$\boldsymbol{z} = col(x_{m+1}, x_{m+2}, ..., x_n) \in R^p$,
$(m + p = n)$,

$$\|\boldsymbol{x}\| := (\sum_{i=1}^{n} x_i^2)^{1/2}, \|\boldsymbol{y}\| := (\sum_{i=1}^{m} x_i^2)^{1/2},$$

$$\|\boldsymbol{z}\| := (\sum_{i=m+1}^{n} x_i^2)^{1/2}.$$

Lemma 1[14] A necessary and sufficient condition of function $V(t,\boldsymbol{x}) \in C(I \times R^n, R)$, and $V(t,\boldsymbol{0}) \equiv 0$ positive definite for y is $\exists \varphi(r) \in K$ in $\Omega := \{\boldsymbol{x} \mid \|\boldsymbol{x}\| \le H\}$ satisfying

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$$V(t,\mathbf{x}) \geq \varphi(\|\mathbf{y}\|). \qquad (4)$$

Lemma 2[14] If function $V(t,\mathbf{x}) \in C(I \times R^n, R)$, and $V(t,\mathbf{0}) \equiv 0$ satisfies(4), and its derivative makes $\frac{\mathrm{d}V}{\mathrm{d}t}\Big|_{(3)} \leq -c(\|\mathbf{y}\|)(c \in K)$, trivial solution of differential equations (3) is stable for y.

3.1 Design Controller u_2 Making x_2 Asymptotically Stable

In theorem 1, we provide controller u_2 and adaptive control law $\lambda(t)$.

Theorem 1 Controller u_2 and adaptive control law $\lambda(t)$ in (5) make state variable x_2 of the pendulum control systems (2) asymptotically stable.

$$\begin{aligned} u_2 &= g(t)\sin x_1 - x_2 \\ \dot{\lambda} &= -x_2 \sin x_1 \end{aligned} \qquad (5)$$

in which $\lambda(t) = g(t) - \theta$.

Proof Take a Lyapunov function

$$V(t,x_2) = \frac{1}{2}(x_2^2 + \lambda(t)^2) \qquad (6)$$

for the pendulum control systems(2). According lemma 1, it is positive definite for x_2, and its derivative along the systems (2) is as follows

$$\begin{aligned} \frac{\mathrm{d}V}{\mathrm{d}t}\Big|_{(2)} &= x_2\dot{x}_2 + \lambda(t)\dot{\lambda} \\ &= x_2(-\theta\sin x_1 + u_2) + \lambda(t)\dot{\lambda} \\ &= x_2(-\theta\sin x_1 + g(t)\sin x_1 - x_2) \\ &\quad - (g(t) - \theta)x_2 \sin x_1 \\ &= -x_2^2 \leq 0 \end{aligned} \qquad (7)$$

By the lemma 2, the control systems (2) are asymptotically stable for state variable x_2.

Then we employ Matlab program to simulate the control effect of the controller u_2 and adaptive control law $\lambda(t)$. The results are shown in figure 2-4, in which the uncertain parameters are chosen as $\varepsilon = 1$, $\eta = 2$ and $\theta = 3$.

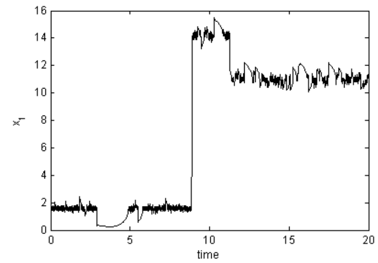

Figure 2: The Curve Of x_1

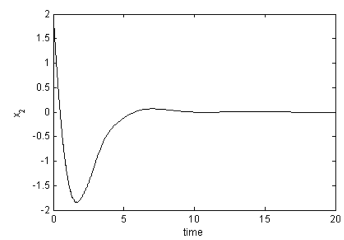

Figure 3: The Curve Of x_2

We study asymptotically stability of partial variables in this paper. Now our main purpose is control state variable x_2, and makes it asymptotically stable. From figure 2 and figure 3, we can see x_2 is asymptotically stable, and not for x_2. It illustrates that adaptive control parameter $g(t)$ recognizes uncertain parameter θ of the pendulum control systems rapidly in figure 4.

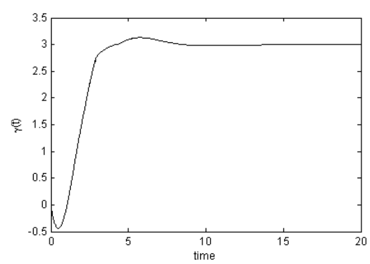

Figure 4: The Curve Of $g(t)$

3.2 Design Controller u_1 Making x_1 Asymptotically Stable

In the flowing, we design controller u_1 and adaptive control laws to make x_1 asymptotically stable and to recognize any given uncertain parameters ε and η in theorem 2.

Theorem 2 The controller u_1 and adaptive control laws described in (8) make x_1 asymptotically stable.

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$$\begin{aligned} u_1 &= (-\alpha(t)^2 + \alpha(t)\xi(t) - \beta(t)^2 \\ &\quad + \beta(t)\varphi(t))x_1 - \alpha(t)x_2 - \beta(t)\tan x_1 \\ \dot{\xi} &= x_2 x_1 + (\alpha(t) - \xi(t))x_1^2 \\ \dot{\varphi} &= x_1 \tan x_1 + (\beta(t) - \varphi(t))x_1^2 \end{aligned} \tag{8}$$

in which $\xi(t) = \alpha(t) - \varepsilon,\ \varphi(t) = \beta(t) - \eta$.

Proof choose a Lyapunov function

$$V(t, x_1) = \frac{1}{2}(x_1^2 + \xi(t)^2 + \varphi(t)^2) \tag{9}$$

for the pendulum control systems (2). Apparently, it is positive definite for x_1, and its derivative along the systems (2) is as follows

$$\begin{aligned} \left.\frac{dV}{dt}\right|_{(2)} &= x_1\dot{x}_1 + \xi(t)\dot{\xi} + \varphi(t)\dot{\varphi} \\ &= x_1(\varepsilon x_2 + \eta \tan x_1 + u_1) \\ &\quad + \xi(t)(x_2 x_1 + (\alpha(t) - \xi(t))x_1^2) \\ &\quad + \varphi(t)(x_1 \tan x_1 + (\beta(t) - \varphi(t))x_1^2) \\ &= -(\varepsilon^2 + \eta^2)x_1^2 \\ &\le 0 \end{aligned} \tag{10}$$

According lemma 2, the control systems (2) are asymptotically stable for state variable x_1.

In order to further check the effectiveness of the controller and adaptive control laws described in(8), we use Matlab software to simulate the result of the control systems (2) with the effect of the controller and adaptive control laws. The results are shown in figure 5-8, in which the uncertain parameters are chosen as $\varepsilon = 7$, $\eta = 6$ and $\theta = 1$.

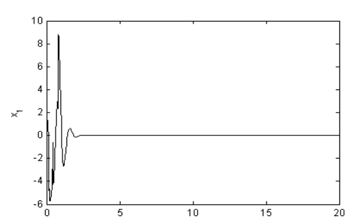

Figure 5: The Curve Of x_1

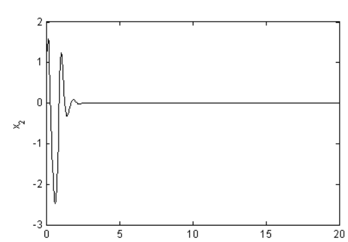

Figure 6: The Curve Of x_2

Figure 5 and figure 6 are the control curves of the state variables x_1 and x_2, from which we can see that he state variables x_1 has very good asymptotically stability with the effect of the controller and adaptive control laws. Figure 7 and figure 8 are the curves of the adaptive control parameters $\alpha(t)$ and $\beta(t)$, and they show that $\alpha(t)$ and $\beta(t)$ can respectively recognize the values of uncertain parameters ε and η, and have very good real-time trait and strong robustness.

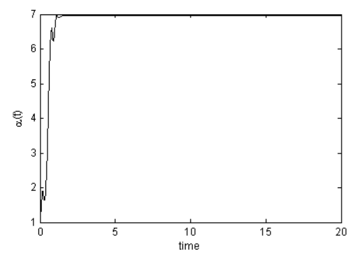

Figure 7: The Curve Of $\alpha(t)$

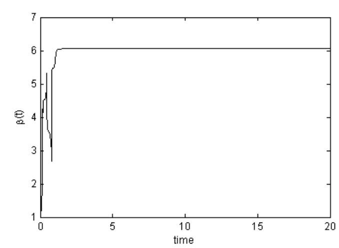

Figure 8: The Curve Of $\beta(t)$

4. CONCLUSION

In the practical application, the control systems often have some uncertainty, and the problems of partial variable control are paid attention to in many cases. In this paper, using Lyapunov stability theory, and employing adaptive control method, we studied the problem of designing partial state variable controllers and adaptive laws for a class of pendulum uncertain control systems. Two partial state variable controllers were designed respectively, and adaptive control laws were given for the uncertain parameters of a class of pendulum uncertain systems. The numerical simulation were carried out, and it shows that the controllers and adaptive control laws are simple and easy, directly perceived through the senses, and have strong robustness, good control performance, and good application.

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