

EMULATION ANALYSIS OF TARGET DAMAGE MODELS BASED ON POISSON DISTRIBUTION

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ABSTRACT

In order to study the damage efficiency of the fragment for the target, puts forward to adopt Poisson probability distribution function analysis fragment group distribution rule; According to the target features and Poisson probability, using the target center to establish damage vulnerable area of the mathematical model; Through the flinders damage each tank section full probability and mutual independence hypothesis, research and analysis the target damage coordinate law, estimated the average number of fragments hit the target; According to the related parameters of fragment dispersion area and target intersection, calculating the target damage probability. According to the design model, and gives out the fragment of target damage probability distribution of simulation results.

Keywords: *Poisson Distribution, Fragments Field, Target Vulnerability, Damage Model*

1. INTRODUCTION

When designing and researching weapons, the actual known data must be closely related to their performance, and whether the design meets the demands of the damage power to the target, which needs actual tests and mathematical calculations. Most of the shooting range test is going on the condition of fixed target and measurer, target damage model and simulation can be assessed given weapons' performance at a lower cost. At the interception distance, whether fragment field can damage the target relies on the following factors: the covering degree of fragmentation on targets; fragments mass, density; the speed that the fragments hit the target; the fragility of the target; the intersection conditions of broken piece field with the target. If the weapons' power can not satisfy the required performance, the designer can change the parameters involved in the above factors, such as modifying the intersection angle of the fragments with the target which can reduce the phenomenon of ricochet. You can also modify the initial velocity of the fragments; when the fragment velocity is high, fragments around bomb-axis scatter uniformly, which shows on a Poisson distribution state. Therefore, the study on the target damage model and simulation can serve the research on target damage and supply relational development foundation which uses the damage probability to represent the target damage performance and quantify the damage parameters.

2. THE MODEL OF TARGET DAMAGE

2.1 The Basic Model Establishment

The damage performance of fragments field on target can be described with the relation between the probability of damage goal and coordinates of burst point which called as the target damage law. To study this law, it must be clear that the features that fragmentation field shows: the number of broken is uniformly scattered which obey the Poisson distribution; the vulnerability of the target and damage power of the fragments (here mainly refers to the number of broken piece) varies with interception distance. And the model must reflect these characteristics, too. Therefore, this damage law not only functions on coordinate y , but also the number of fragments, and is recorded as $G(y, n)$, which is called the target damage law.

The plane of y_2y_3 is created at the cross section of the target center, and y_2 's positive axis is upward and y_3 is perpendicular to the y_2y_3 plane, as shown in figure 1.

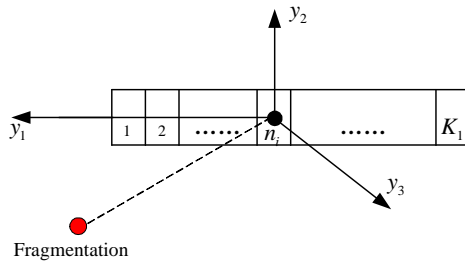


Figure 1: Damage Mathematical Model

Target vulnerability and fragments damage power are two equally important and closely related issues; they are two different side of the same phenomenon. Because of the vulnerability, so the flight target is divided into k_1 vulnerability cabins. According to scattered fragments flying range, the fragments field is divided into k_2 broken areas.

Assume that the number about the field of the j th fragments hit the i th wearing cabin obey to Poisson distribution, we can get:

$$p(k) = \frac{\lambda_{ij}^k}{k!} e^{-\lambda_{ij}} \quad (k = 0, 1, 2, \dots) \quad (1)$$

In equation (1), λ_{ij} is the mean number of the fragments in the j th fragment field which hit the i th vulnerable cabin

If we do not consider the damage accumulation, and each fragment smash up different tank section is an independent, when the number of the fragments in the j th fragment field which hit the i th vulnerable cabin is k , The probability of $R(k)$ that damage cabin meet the index damage rule, we could write:

$$R(k) = 1 - (1 - r_{ij})^k \quad (k = 0, 1, 2, \dots)$$

Where r_{ij} is the probability of the fragment which hit the i th vulnerable cabin that in the j th fragment field, which damage the i th vulnerable cabin.

Let $G_i(y, n)$ be the i th cabin damage probability, each tank section is not be damaged probability is $\prod_{i=1}^{k_1} [1 - G_i(y, n)]$, The probability of target damage is:

$$G(y, n) = 1 - \prod_{i=1}^{k_1} [1 - G_i(y, n)] \quad (2)$$

The fragments field is divided into k_2 broken areas, the speed and quantity of each broken piece inside the fragments is not the same. So we should consider each broken area. Assume that $G_i^j(y, n)$

is the damage probability that the fragments in the j th fragment field damage the i th vulnerable cabin, can be obtained:

$$G_i(y, n) = 1 - \prod_{j=1}^{k_2} [1 - G_i^j(y, n)] \quad (3)$$

By full Almost formula, we can get the probability that the fragments in the j th fragments field hit the i th vulnerable cabin of $G_i^j(y, n)$:

$$\begin{aligned} G_i^j(y, n) &= \sum_{k=0}^{\infty} p(k) R(k) \\ &= \sum_{k=0}^{\infty} \frac{\lambda_{ij}^k}{k!} e^{-\lambda_{ij}} [1 - (1 - r_{ij})^k] \\ &= \sum_{k=0}^{\infty} \frac{\lambda_{ij}^k}{k!} e^{-\lambda_{ij}} - \sum_{k=0}^{\infty} \frac{[\lambda_{ij} (1 - r_{ij})]^k}{k!} e^{-\lambda_{ij}} \\ &= e^{-\lambda_{ij}} e^{\lambda_{ij}} - e^{\lambda_{ij} (1 - r_{ij})} e^{-\lambda_{ij}} \\ &= 1 - e^{-\lambda_{ij} r_{ij}} \end{aligned} \quad (4)$$

Where k is taken the integer.

In summary the formula of (2)(3)(4), the damage law expression is:

$$\begin{aligned} G(y, n) &= 1 - \prod_{i=1}^{k_1} \prod_{j=1}^{k_2} [1 - G_i^j(y, n)] \\ &= 1 - e^{-\sum_{i=1}^{k_1} \sum_{j=1}^{k_2} \lambda_{ij} r_{ij}} \end{aligned} \quad (5)$$

2.2 The calculation of λ_{ij} .

As shown in figure 2, the space fragments field form an imaginary cone, space fragments field can be described by φ , which is the average scattering direction angle of the fragment. φ is:

$$\varphi = (\varphi_1 + \varphi_2) / 2.$$

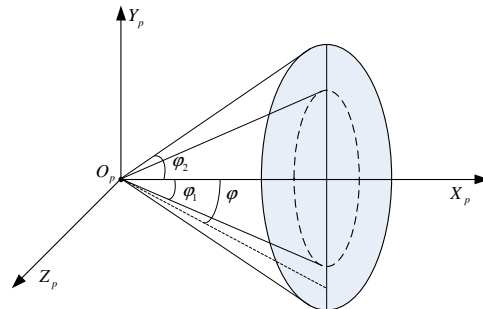


Figure 2: Space Fragments Field

The surface distribution density of the fragments in the space is:

$$\lambda = \frac{n_R}{2\pi D^2} \cdot \frac{\Delta f_\lambda(\varphi)}{\Delta\varphi \sin\varphi} \quad (6)$$

In Equation (6), R is fragments flying distance, n_R is the effective number of fragments when distance to D, while distribution is dynamic, $\Delta f_\lambda(\varphi)$ is the Fragment distribution probability with in the range of $\Delta\varphi$.

Along the flight direction of the fragment do target tangent plane, can be obtained the schematic diagram that broken area intersected the target, as shown in figure 3. Assume that , fragment after explosion, the region forms a cone, the size of the underside of the fragments in contact with the target radius, and the amount of off-target fragments, has more relationship with the size of the incident angle. The need to solve the fixed distance from the target that how far we need to detonate fuse, this distance can be see as the miss distance. Accurate distance fixed could effectively kill the target. And the size of the angle that the fragments incident determine the density and number of the fragments, it is also important for damaging targets. The width of the target threat district is related with the angle of incidence and the incident distance, if Incidence angle is too large, there will be a jump shot phenomenon, the angle of incidence is too small, the threatening will be significantly smaller. On the other hand, incident distance is not from the target nearer, the greater the threat to the target. We must find a reasonable best incident distance.

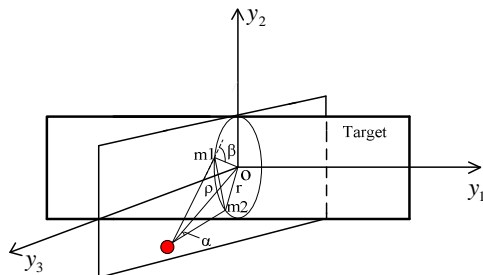


Figure 3: The Intersection Schematic Diagram Of Fragment Patches And The Target

Taking into account that fragmentation will produce the phenomenon of ricochet in the incident angle greater than β , from figure 3 , fragments dispersed regional and target intersection, we can get that the length of the radius of the bottom surface of the fragment distribution is half of $m1$ to $m2$ in the linear distance, the width of target cabin damage is the arc length between $m1$ and $m1$.So launch the R and the A as follows:

$$\frac{r}{\sin\alpha} = \frac{\rho}{\sin(\pi-\beta)} = \frac{\rho}{\sin\beta} \Rightarrow$$

$$R = 2r \sin(\beta - \alpha) = \frac{2\rho \sin\alpha \sin(\beta - \alpha)}{\sin\beta} \quad (7)$$

$$A = \frac{(\beta - \alpha)}{90} \pi r = \frac{\rho \pi \sin(\beta - \alpha)}{90 \sin\beta} \quad (8)$$

In equation (7) and equation (8):

$$0 \leq \alpha < \beta, \rho \text{ --Miss distance, } \rho = \sqrt{y_2^2 + y_3^2},$$

$$\alpha = \arcsin\left(\frac{r \sin\beta}{\rho}\right), r \text{ --The radius of target.}$$

Suppose that the hit length of the fragments in the jth fragment field on the y_1 axis is L_p . The equivalent length of the ith wearing cabin on the y_1 axis is $L_{y1}(i)$, and the number of fragments in the jth fragment field is N_j . By area ratio calculated λ_{ij} follows:

$$\lambda_{ij} = \frac{N_j L_{y1}(i) A}{2\pi r L_p} \quad (9)$$

3. SIMULATION ANALYSIS

Response to the damage mathematical model has been established, we have the necessary to verify the feasibility of the model, and master the influence of various parameters for the model. From the basic model of the target damage's law, we can see that $G(y, n)$ has the relationship with the parameters of λ_{ij} and r_{ij} . It can be obtained from the equation (9) that, the value of λ_{ij} depends the parameters of $N_j, L_{y1}(i), A, r, L_p$, among the parameters, the value of A is variable, and which is needed to carry on the quantitative analysis.

Equation (1) is the fragmentation field distribution function. As can be seen from the formula, λ_{ij} and k are two important parameters influence the distribution. When λ_{ij} takes 0 to 50,

k takes different integer, the simulation diagram of $P(k)$ is obtained. As can be seen from the figure (4), when the value of λ_{ij} and k takes 1, $P(k)$ reaches the maximum, however, with the increase in the number of fragments, The number of the fragments is more than 16, a takes different values, the value of $P(k)$ will tend to smooth, the number of the fragment under this steady state is required, and provide the necessary basis for future research.

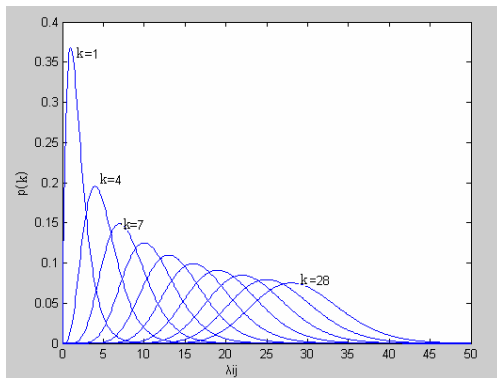


Figure 4: $P(k)$ Value Simulation

Equation (6) is the surface distribution density of the broken tablets in space. Figures 5 is obtained when the angle of φ takes from 0 to 30° and φ takes random value. It can be seen from the figure: φ takes 15°, with the angle of φ increases, the distribution density of the fragment change smoothly in the surface of the space, the surface density of the fragmentation distribution in the space as uniform as possible, so that it can provide the theoretical analysis for the design and testing.

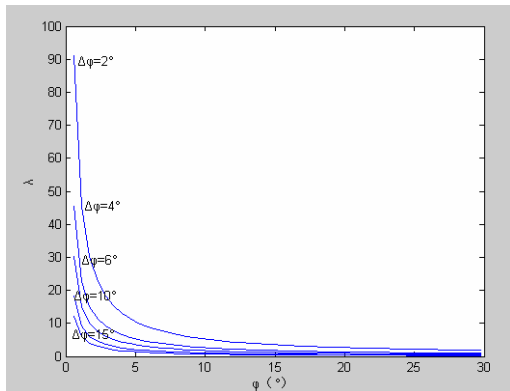


Figure 5: The Distribution Density of Fragment In Space

Equation (7) is the radius of the bottom surface of the fragments distribution zone. R is associated with the incident angle and the incident distance. A

value is fixed, and can't affect the value of A . Taking the value of ρ is 0-5 m, the value of α is 0- 80°, the simulation figure of bottom radius model of the fragment distribution area is obtained. As can be seen from the figure 6, when ρ is 2.5 m, with the incident angle is increased, the bottom surface area of the fragments field is greater, the greater the density and the number of fragments, the greater the extent of the target damaged.

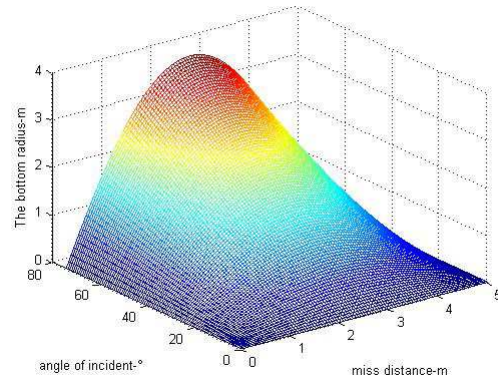


Figure 6: The Bottom Radius Model Of The Fragment Distribution Area

Equation (8) is the damage width of the broken area for the target cabin, can be obtained from the equation (8), β is a fixed value, when the incident angle α is constant, A value increases with the distance ρ increases, However, if ρ unchanged, A value decreases with the increase of the α value. Also taking value of ρ from 0m-5 m, the value of β up to 80°, the simulation diagram of figure 7 can be obtained. From the graph we can see that the threat width of A get to maximum for the target when the incident angle α is about 70°, ρ is 2.5 m

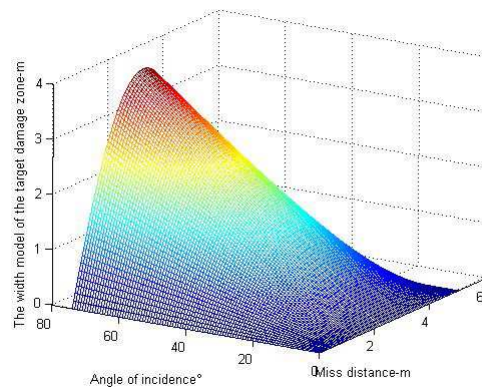


Figure 7: The Width Model Of The Target Damage Zone

Fig 8 is the simulation figure of perforation probability formula, to provide a reference for r_{ij} value.

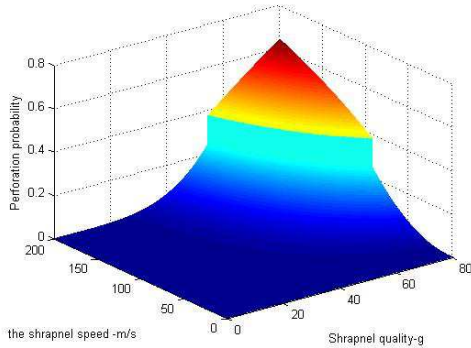


Figure 8: The Perforation Probability

As can be seen from the simulation graph, the higher the quality of the shrapnel, the higher the relative velocity of the shrapnel, the greater the probability of the cabin perforation. This shows that the fragmentation target damage and the kinetic energy of the fragments have a great relationship, from the kinetic energy formula $E = 1/2mv^2$ can be seen, fragments quality and speed decide the size of the shrapnel kinetic energy, in some extent decide the degree of the target damage.

4. CONCLUSION

In this thesis, starting from the distribution function, it has researched the calculation method that fragments damage on the target, through theoretical exposition and analysis of the example, the proposed mathematical model considering the influence of random factors, such as the quality of the fragments, the distance from explosion point to the target, the geometric dimensions of the target, the load capacity of the target, and so on. The vulnerability of the target in the proximity effect, which can provide a probabilistic assessment results, to compensate for the shortcomings of the more difficult to get the experimental data, therefore, this model has a certain degree of objectivity and practicality. Although the atmospheric impulse, flying speed and other parameters of the target for the damage is not taken into consideration in the calculation, however, due to the coordinate method and probability calculation method, simplified mathematical model of the damage. It has a good reference for the direction of such problems.

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