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CHAOS OF LOGISTIC MAPS IN COUPLED NETWORKS

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ABSTRACT

The emergence of chaos in complex dynamical networks is a commonly concerned issue. In this paper, we suppose that the networks consist of nodes which are in the non-chaotic and unattached state at first, then making them contact each other by choosing a suitable coupling strength. We found that the nodes and the degrees of each node in the network are greater, the value of coupling strength is smaller when the system transit from non-chaotic to chaotic. Our simulation result on Star-network with different nodes and adjacent network with 100 nodes of Logistic maps.

Keywords: *Chaos, Coupling Strength, Dynamical Networks, Logistic Map*

1. INTRODUCTION

Nearly two decades， the research of dynamic behavior is a hot topic[1]. Especially, one of dynamics that chaos is a very interesting nonlinear phenomenon, such as a lighted cigarette smoke in a stable airflow rising slowly and then curls into a fierce disturbance of smoke. Severe disturbance of smoke equal the system that is in chaotic state. The faucet, we continue to change the size of the taps, would drip from drops to current. Chaos, which can be applied to secure communication, the spread of disease and other fields, have been extensively studied in the past few decades[2-4]. In this paper, we first discuss the factors that cause chaos, then determine the range of the coupling strength.

The remainders of this paper is organized as follows: The relationship of Lyapunov exponents and chaos is discussed in Section 2. In Section 3, results of our simulation and study are presented. Finally, section 4 concludes the investigation.

2. LYAPUNOV EXPONENTS, CHAOS AND COUPLING STRENGTH

We know that in one-dimension discrete map, the value of the nodes will be roughly equal after iterations if Lyapunov exponent is less than zero, otherwise, it will be variational. So the emergence of chaos relate to whether Lyapunov exponent is positive or negative, then the calculation of Lyapunov exponent become very important. The state equations of the dynamical network are as follows:[5-7]

$$x_i(k+1) = f(x_i(k)) - c\sum_{j=1}^{N} a_{ij} f(x_j(k)),\ i = 1, 2 \ldots . N \quad (1)$$

If there is a connection between i and j $(i \neq j)$, then $a_{ij} = a_{ji} = 1$; otherwise, $a_{ij} = a_{ji} = 0$. Let $a_{ii} = -d_i, i = 1, 2 \ldots . N$. d_i represents the degrees of node i the numbers that node i connect with others.

$$\sum_{\substack{j=1 \\ j\neq i}}^{N} a_{ij} = \sum_{\substack{j=1 \\ j\neq i}}^{N} a_{ji} = d_i,\ i = 1, 2 \ldots . N \quad (2)$$

Matrix A can represent the structure of the dynamical network. Assume that A is a symmetric and irreducible matrix, which means that the network is fully connected in the sense of having no isolate clusters. Based on these assumptions on the coupling matrix A , it can be verified that zero is the largest eigenvalue, denoted as λ_1 ,and all the other eigenvalues $\lambda_2 \geq .... \geq \lambda_N$ are negative. The function $f(\cdot)$ in network (1) is a given nonlinear vector-valued map, describing the dynamics of a node, and $x_i(k) = (x_{i1}(k)), x_{i2}(k), \cdots x_{in}(k)) \in R^n$ are the state variables of node i . We let $n = 1$ in this discussion, and assume that the constant coupling strength c is positive, the nodes are in non-chaotic state. So Lyapunov exponent h_0 is less than zero.

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We change (1) to be the following form, and extend it to N dimension.

$$x(k+1)=F(x(k),c)\,,\ x(k)=\left[x_1(k)\cdots x_N(k)\right]^T\in R^N$$

The definition of N dimensional Lyapunov exponent is as follows:

$$\mu_i=\lim_{p\to\infty}\frac{1}{p}\ln\left|DF^p(x_0)\cdot u_i\right|\,,\ i=1,2\ldots.N \quad (3)$$

$DF^p(x_0)$ is the Jacobian matrix of the p-time iterated map starting from a random initial state x_0 ,and u_i is a set of orthonormal vectors in the tangent space of the map. In Ref.[2], the relationship between the Lyapunov-exponent spectra of diffusively coupled one-dimensional maps and the spectrum of the discrete SchrLodinger operator was discussed, which led to a conclusion that when the coupling strength is larger than a critical value, the Lyapunov-exponent spectrum extends to $-\infty$. We discuss the relationship between the Lyapunov exponent of an individual node, h_0 and the Lyapunov exponents of the coupled dynamical network. To calculate u_i , we differentiate network (1) and then evaluate the resulting derivatives at a random initial condition x_0 .[5]

$$\delta x_i(k+1)=f'(x_i(0))\delta x_i(k)-c\sum_{j=1}^{N}a_{ij}f'(x_j(0))\delta x_j(k)$$
(4)

Where $i=1,2\ldots.N$. yields

$$\mu_i(\lambda_i)=h_0+\ln\left|1-c\lambda_i\right|\,,\quad i=1,2\ldots.N \quad (5)$$

Due to the ordering of eigenvalues of the coupling matrix A , $0=\lambda_1>\lambda_2\geq....\geq\lambda_N$ and because the coupling strength c is positive, we can order the Lyapunov exponents u_i as follows:

$$\mu_N(\lambda_N)=h_0+\ln\left|1-c\lambda_N\right|\geq\mu_{N-1}(\lambda_{N-1})\geq\cdots\mu_2(\lambda_2)>\mu_1\lambda_1=h_0(<0)$$
(6)

If the coupled network (1) is chaotic, then there is at least one positive Lyapunov exponent, therefore $\mu_N>0$.[6-7]

From the above discussion, we can see that there is certain relation between the coupling strength and Lyapunov exponents. Lyapunov exponents are determined by the matrix which reflects the structure of the network, and coupling strength is positive. There will be following form when Lyapunov exponent is greater than zero.

$$\mu_N\geq\mu_{N-1}\geq\cdots\mu_{T+1}>0\,, \quad (7)$$

$$h_0=\mu_1<\mu_2\leq\cdots\leq\mu_T<0\,, \quad (8)$$

T $(1\leq T\leq N-1)$ and is positive integer.

From $(5)-(8)$, we can calculate the value range of the coupling strength. [8]

$$\frac{e^{-h_0}-1}{\left|\lambda_N\right|}<c<\frac{e^{-h_0}-1}{\left|\lambda_T\right|} \quad (9)$$

Now, there is at least one positive Lyapunov exponent, which means that some node is in chaotic state.

3. OUR WORK

First, we order dynamical network state equation as follows:

$$x_i(k+1)=p(x_i(k))(1-x_i(k))-c\sum_{j=1}^{N}a_{ij}\,p(x_j(k))(1-x_j(k))$$
(10)

Our simulation results on star network with different nodes and 100-node adjacent network. Constant p is fixed on $p=3.3$, $h_0=-0.6187$. We give one hundred initial value which approximately equal to 0.2. The structure of Star network is that there is only one node to connect with others, while other nodes don't connect with each other.

$$A=\begin{pmatrix}-N+1 & 1 & \cdots & 1\\ 1 & -1 & \cdots & 0\\ \vdots & \ddots & \cdots & 0\\ 1 & 0 & \cdots & -1\end{pmatrix}$$

The eigenvalues of the star network are $\lambda_1=0$, $\lambda_2=\lambda_3=......=\lambda_{N-1}=-1$, $\lambda_N=-N$. We can see that $\left|\lambda_N\right|$ change with N .

Figure 1 shows the relationship between '$\left|\lambda_N\right|$'and node number 'N'in star network. Figure two shows the relationship between node number'N'in star network and the coupling strength 'c' when the system is in chaotic state. We observe that with the increase of the number of the nodes, the coupling strength become small. Figure three is the network analysis selected with 30 nodes and 100 nodes in star network. Figure four shows the graphic analysis contained 100 nodes in the adjacent network, and each node has different

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degrees. Figure five is the graphic analysis selected the degrees of the nodes with 20, 42, 60, 84, in adjacent network with 100 nodes. From above diagram analysis, we can see that with the increase of the number of the nodes, the coupling strength gradually become small in star network, with the increase of the degrees of the nodes, the coupling strength gradually decrease in adjacent network.

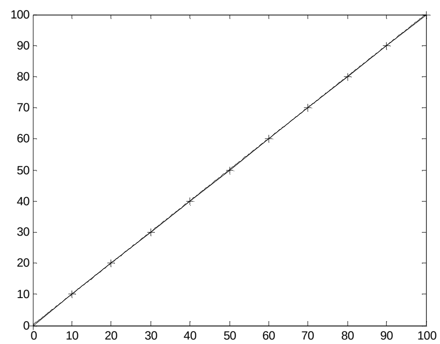


Figure 1: The Relationship Between '$|\lambda_N|$' And Node Number 'N' In Star Network.

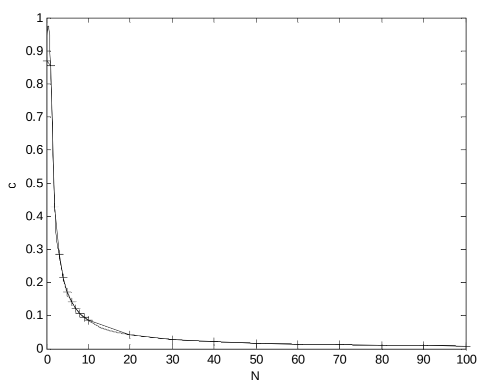
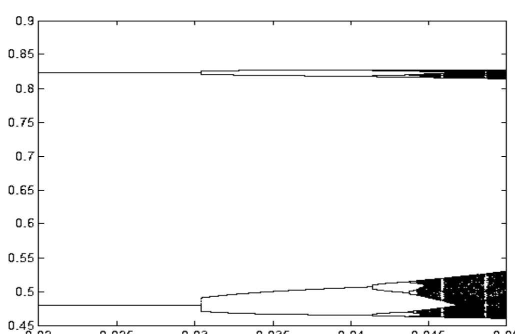
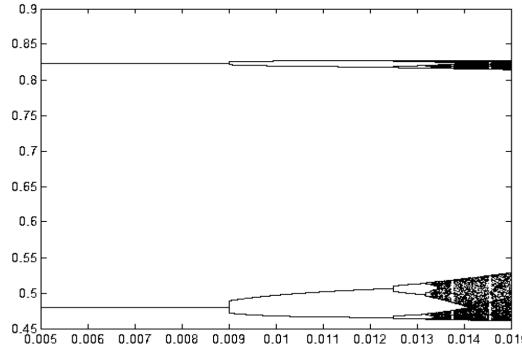


Figure 2: The System Is In Chaotic State, The Relationship Between The Node Number 'N' And The Coupling Strength 'c' In Star Network.

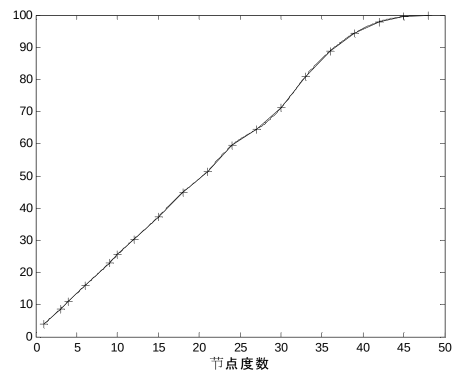


(a)



(b)

Figure 3: The System With The Change Of Coupling Transit To Chaos In Star Network. (A): 30-Node Star Network (B): 100-Node Star Network.



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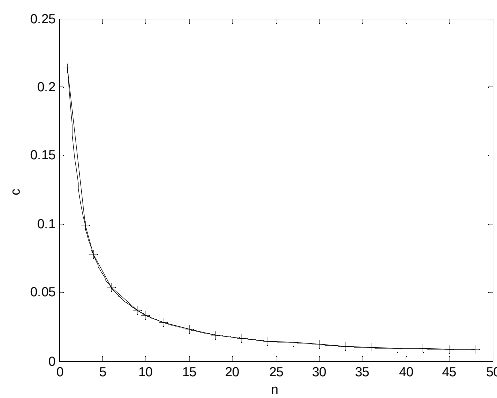

(b)

Figure 4: The Graphic Analysis Of Different Degrees Of The Node In 100-Node Adjacent Network. (A): The Relationship Between '$|\lambda_N|$' And The Degrees Of The Node 'n' (B): The Relationship Between The Degrees Of The Node 'n' And The Coupling Strength 'c'.

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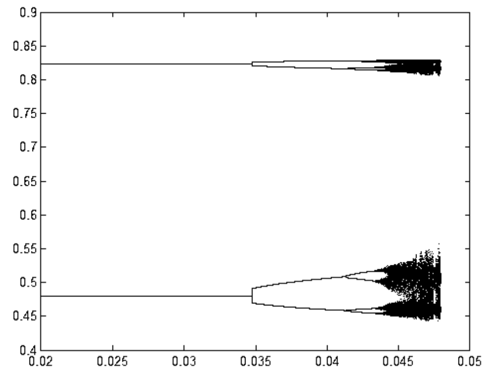

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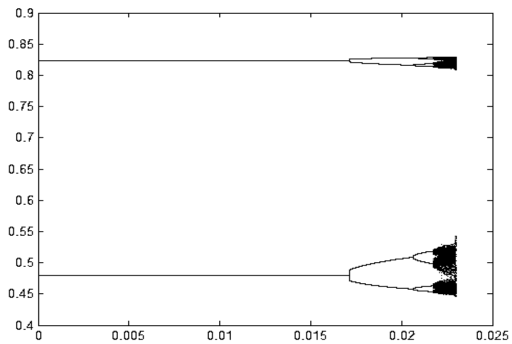

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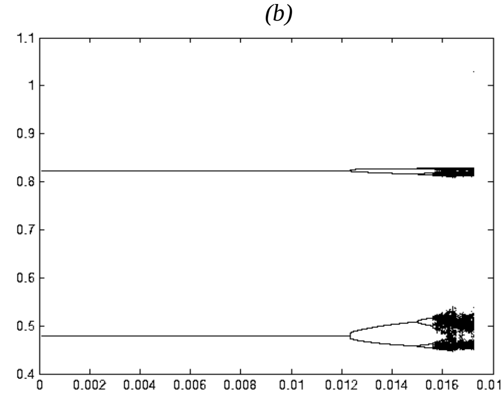
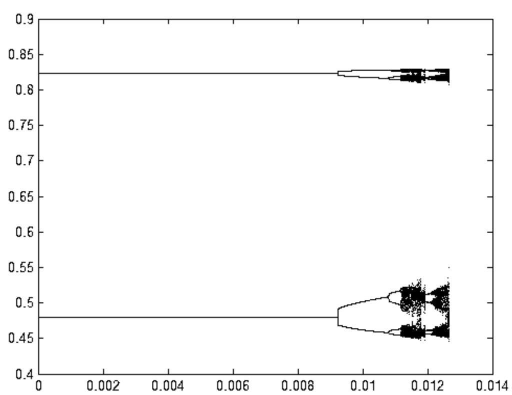


(c)



(d)

Figure 5: the system with different degrees transit to chaos with the change of coupling strength in 100-node adjacent network. (a): each node of the system with 20 degrees (b): each node of the system with 42 degrees (c): each node of the system with 60 degrees (d): each node of the system with 84 degrees.

4. CONCLUSION

From above analysis, we can observe that with the increase of the number of the nodes, the coupling strength the system transit to chaos gradually become smaller in star network. Of course it adapt to other dynamical coupling network that is similar to star network. In adjacent network, the more number of the degrees of the node, the smaller coupling strength that the system transit to chaos. It also adapt to other dynamical network that the degrees of each node increase, the coupling strength will become smaller.

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