



# GOAL PROGRAMMING BASED MASTER PLAN FOR CYCLICAL NURSE SCHEDULING

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## ABSTRACT

Constructing timetable for nurses in hospital is a challenging job for the head nurse or nurse manager as the task is done repeatedly for every specified period. It requires a lot of time and effort for generating a good and fair timetable. In order to overcome the problem, a cyclical nurse schedule using a 0-1 goal programming is proposed. The timetable will be the master plan that would help the head nurse or nurse manager to schedule the nurses in a yearly basis. The proposed model has also managed to satisfy the hospital's policies and the nurses' preferences. The model has provided an unbiased schedule and thus leads to an overall satisfaction and fairness to the nurses. The developed model has been solved using LINGO software and the result is found to be better compared to the present method used.

**Keywords:** *Mathematical Model; Goal Programming; Cyclical Schedule; Nurse Scheduling*

## 1. INTRODUCTION

The study on nurse scheduling is not new in the world of research related to human resource management in health care institutions [1]. Nurse scheduling is however a dynamic and challenging tasks to be addressed from time to time. At a local hospital in Malaysia, the nurse timetable is generated manually based on the experience and knowledge of head nurse and approval of administration. Since there is no written policies and procedures as a guidance in the preparation of the timetable, the schedule becomes inflexible and biased to some nurses at certain times.

In this study, a nurse scheduling model will be developed to solve unfairness in workload distribution among the nurses. A 0-1 goal programming (01GP) technique will be used to build the model. The 01GP is a variation from linear programming technique that considers more than one objective. The 01GP prefers to minimize slack variables for each objective functions according to their priorities rather than to minimize the objective functions itself [2][3]. There are a few research applied the 01GP technique to solve their problem such in acquisition allocation problem [4], recycling system [5], operation waiting list problem [6], facility location selection [7], and staff scheduling problem [8][9][10][11].

Musa and Saxena [8] have applied GP technique to one unit of a hospital that tackled the hospital's policies and nurses' requests. But the developed model is quite low complexity with a two weeks planning period and only one shiftwork. Ismail et. al. [9] also used GP technique to build a periodic nurse rostering model. However the model limited to a 14 days of planning period with three types of shift works.

Azaiez and Al-Sharif [10] have developed a nurse scheduling model with four weeks planning period and two shift works in Riyadh Al-Kharj Hospital. The model has considered the hospital's objectives and nurses' request. The result from the model has increased the hospital performance while reducing about 14% of their overtime cost. Nevertheless, the model has to be renewing for each four weeks.

Jenal et. al. [11] has solved a cyclical nurse scheduling problem using GP technique. The model has considered and achieved the hospital's objective and nurses' request for three weeks planning period and three shift works. The developed model has produced a schedule pattern for each of the 18 nurses in 21 days (three weeks). For the next 21 days, in order to preserve the fairness and continuity of the schedule, nurse 2 will take nurse 1's pattern, nurse 3 will take nurse 2's pattern and finally nurse 1 will take nurse 18's



pattern. The pattern for each nurse will change in a cascading manner until week 55 where the pattern will again return to the original schedule. Thus in this study, a master plan for cyclical nurse scheduling model will be developed to solve the nurse scheduling problem. The proposed model would be cyclic and have a shorter time to recycle again.

## 2. A MASTER PLAN FOR NURSE SCHEDULING MODEL

### 2.1 A Case Study

A case study considers a 12 days planning period and three shift works in one ward of Tuanku Mizan Army Hospital, Kuala Lumpur. The shift works divides into morning shift (7 am – 2 pm), evening shift (2 pm – 9 pm) and night shift (9 pm – 7 am). There are 12 nurses available in this ward. The model considers both hospital’s objectives and nurses’ request. The hospital’s objectives act as hard constraints that must be satisfied while the nurses’ requests act as soft constraints that can be violated.

Table 1. Notations

Notations	Definition
$n$	Number of days work per schedule ( $n=12$ ).
$m$	Number of nurses available in a ward ( $m=12$ ).
$i$	Index for days, $i=1,2,\dots, n$ .
$k$	Index for nurses, $k=1,2,\dots, m$ .
$P_i$	Number of nurses required for morning shift of day $i$ .
$T_i$	Number of nurses required for evening shift of day $i$
$M_i$	Number of nurses required for night shift of day $i$ .

Table 2. Decision Variables

Decision Variables	Definition
$A_{i,k}$	= 1 if nurse $k$ is assigned morning shift for day $i$ = 0 otherwise
$C_{i,k}$	= 1 if nurse $k$ is assigned evening shift for day $i$ = 0 otherwise
$E_{i,k}$	= 1 if nurse $k$ is assigned night shift for day $i$ = 0 otherwise
$F_{i,k}$	= 1 if nurse $k$ is assigned a day off for day $i$ = 0 otherwise

### 2.2 Notations and Decision Variables

Table 1 shows the notations while Table 2 shows the decision variables used in this study.

### 2.3 Hard Constraints

The hospital’s objectives or hard constraints considers in this study are as follow:

- to ensure the minimum number of nurses are fulfilled for each shift each day;

$$\sum_{k=1}^m A_{i,k} \geq P_i; \quad i = 1, 2, \dots, n \tag{1}$$

$$\sum_{k=1}^m C_{i,k} \geq T_i; \quad i = 1, 2, \dots, n \tag{2}$$

$$\sum_{k=1}^m E_{i,k} \geq M_i; \quad i = 1, 2, \dots, n \tag{3}$$

- to ensure each nurse has one and only one shift work in a day;

$$A_{i,k} + C_{i,k} + E_{i,k} + F_{i,k} = 1; \tag{4}$$

$i = 1, 2, \dots, n \quad k = 1, 2, \dots, m$



- to ensure each nurse has a three consecutive days of night shifts then followed by two consecutive days of days off;

$$E_{i,k} + E_{i+1,k} + E_{i+2,k} + F_{i+3,k} + F_{i+4,k} = 5; \quad (5)$$

$$i = 1 \quad k = 1, 2, 3$$

$$E_{i,k} + E_{i+1,k} + E_{i+2,k} + F_{i+3,k} + F_{i+4,k} = 5; \quad (6)$$

$$i = 4 \quad k = 4, 5, 6$$

$$E_{i,k} + E_{i+1,k} + E_{i+2,k} + F_{i+3,k} + F_{i+4,k} = 5; \quad (7)$$

$$i = 7 \quad k = 7, 8, 9$$

$$E_{i,k} + E_{i+1,k} + E_{i+2,k} = 3; \quad (8)$$

$$i = 10 \quad k = 10, 11, 12$$

- to ensure each nurse has the number of workload between eight to ten days per schedule;

$$\sum_{i=1}^n (A_{i,k} + C_{i,k} + E_{i,k}) \geq 8; \quad (9)$$

$$k = 1, 2, \dots, m$$

$$\sum_{i=1}^n (A_{i,k} + C_{i,k} + E_{i,k}) \leq 10; \quad (10)$$

$$k = 1, 2, \dots, m$$

- to ensure each nurse has no more than six consecutive days on; and

$$F_{i,k} + F_{i+1,k} + F_{i+2,k} + F_{i+3,k} + F_{i+4,k} + F_{i+5,k} + F_{i+6,k} \geq 1; \quad (11)$$

$$i = 1, 2, \dots, n-6 \quad k = 1, 2, \dots, m$$

- to ensure each nurse works according to the number of required shift works per schedule.

$$\sum_{i=1}^n A_{i,k} \geq 3; \quad k = 1, 2, \dots, m \quad (12)$$

$$\sum_{i=1}^n C_{i,k} = 3; \quad k = 1, 2, \dots, m \quad (13)$$

$$\sum_{i=1}^n E_{i,k} = 3; \quad k = 1, 2, \dots, m \quad (14)$$

## 2.4 Soft Constraints

The nurses' requests or soft constraints consider in this study are as follow:

- to avoid an isolated days on;

$$F_{i,k} + A_{i+1,k} + C_{i+1,k} + E_{i+1,k} + F_{i+2,k} \leq 2; \quad (15)$$

$$i = 1, 2, \dots, n-2 \quad k = 1, 2, \dots, m$$

$$F_{i,k} + A_{i+1,k} + C_{i+1,k} + E_{i+1,k} + F_{i+2,k} + \eta 1_k - \rho 1_k = 2; \quad (16)$$

$$i = 1, 2, \dots, n-2 \quad k = 1, 2, \dots, m$$

- to ensure each nurse has the same number of workload per schedule;

$$\sum_{i=1}^n (A_{i,k} + C_{i,k} + E_{i,k}) = 9; \quad (17)$$

$$k = 1, 2, \dots, m$$

$$\sum_{i=1}^n (A_{i,k} + C_{i,k} + E_{i,k}) + \eta 2_k - \rho 2_k = 9; \quad (18)$$

$$k = 1, 2, \dots, m$$

- to avoid an evening shift followed by morning or night shift;

$$C_{i,k} + A_{i+1,k} + E_{i+1,k} \leq 1; \quad (19)$$

$$i = 1, 2, \dots, n-1 \quad k = 1, 2, \dots, m$$

$$C_{i,k} + A_{i+1,k} + E_{i+1,k} + \eta 3_k - \rho 3_k = 1; \quad (20)$$

$$i = 1, 2, \dots, n-1 \quad k = 1, 2, \dots, m$$

- to avoid a morning shift followed by evening or night shift; and

$$A_{i,k} + C_{i+1,k} + E_{i+1,k} \leq 1; \quad (21)$$

$$i = 1, 2, \dots, n-1 \quad k = 1, 2, \dots, m$$

$$A_{i,k} + C_{i+1,k} + E_{i+1,k} + \eta 4_k - \rho 4_k = 1; \quad (22)$$

$$i = 1, 2, \dots, n-1 \quad k = 1, 2, \dots, m$$

- to avoid an isolated days off.

$$A_{i,k} + C_{i,k} + E_{i,k} + F_{i+1,k} + A_{i+2,k} + C_{i+2,k} + E_{i+2,k} \leq 2; \quad (23)$$

$$i = 1, 2, \dots, n-2 \quad k = 1, 2, \dots, m$$

$$A_{i,k} + C_{i,k} + E_{i,k} + F_{i+1,k} + A_{i+2,k} + C_{i+2,k} + E_{i+2,k} + \eta 5_k - \rho 5_k = 2; \quad (24)$$

$$i = 1, 2, \dots, n-2 \quad k = 1, 2, \dots, m$$



For each soft constraint, it will be formulated as a goal by adding up the slack variables. Equations (16), (18), (20), (22) and (24) show the goal 1, goal 2, goal 3, goal 4 and goal 5 for each soft constraint respectively.

**2.5 Objective Functions**

Each goal will be minimized according to their priority. The first priority is to minimize goal 1, followed by goal 2, goal 3, goal 4 and goal 5 in succession. Goal 1 is aimed to minimize the positive slack variable. Goal 2 is aimed to minimize both positive and negative slack variables. Goal 3, goal 4 and goal 5 are aimed to minimize the positive slack variables respectively. Thus the objective function for cyclical nurse scheduling model according to their priority is as follow:

$$\text{Minimize } \left( \begin{array}{l} \sum_{i=1}^{n-1} \sum_{k=1}^m \rho 1_{i,k}, \\ \sum_{k=1}^m \eta 2_k + \rho 2_k, \\ \sum_{i=1}^{n-1} \sum_{k=1}^m \rho 3_{i,k}, \\ \sum_{i=1}^{n-1} \sum_{k=1}^m \rho 4_{i,k}, \\ \sum_{i=1}^{n-1} \sum_{k=1}^m \rho 5_{i,k} \end{array} \right)$$

**3. RESULTS AND DISCUSSIONS**

The cyclical nurse scheduling model solves using LINGO 8.0 software. The result shows that the developed model has fulfilled goal 1, goal 2 and goal 3 while goal 4 and goal 5 are violated.

For goal 1, the positive slack variable gives zero value which means the objective of avoiding the isolated days on is achieved. There is no nurse have the isolated days on in their schedule’s pattern.

For goal 2, both the positive and negative slack variables give zero value which means the objective of having the same number of workload per schedule for each nurse is achieved. All nurses have the 9 days of workload per schedule.

For goal 3, the positive slack variable gives zero value which means the objective of avoiding

evening shift followed by morning shift or night shift is achieved. There is no nurse have the schedule’s pattern of having evening shift followed by morning shift or night shift.

For goal 4, there is a value for the positive slack variable. The objective of avoiding morning shift followed by evening shift or night shift is not achieved.

For goal 5, there is a value for the positive slack variable. The objective of avoiding the isolated days off is not achieved.

Table 3 shows the duty roster done manually while Table 4 shows the duty roster developed by using the 01GP technique. The schedule produced manually has shown that there is an inconsistency in the total number of workloads for the nurses and also the number of nurses on duty each day. The workloads vary from 11 days to 6 days with one nurse not having any shifts. The distributions of shifts are also unbalanced. There is one nurse who has not been assigned the night shifts at all in the 14 days period. These inconsistencies were the reasons for the nurse satisfaction and quality of work.

The developed model using the 01GP produced a very consistent and fair schedule for nurses. In the 12 day period, the total number of workloads and the number of nurses on duty daily are the same, and also the equal distributions of shift works. The reason why 12 day schedule is chosen is because the same pattern for each nurse can be repeated for the next 12 days without violating both hard and soft constraints. This will produce a master plan that would be cyclical 30 times in a year for 360 days.

With the master plan, the head nurse can have a good planning on the nurses’ activities such as attending the courses or seminars. While for the nurses themselves, they can have a good planning of their own activities such as having long holidays with their family. The master plan also demonstrates the fairness of schedule. The number of shift work and workload per schedule distributes equally among the nurses and this leads to the nurses satisfaction.



Table 3. The Duty Roster Done Maually

D <sup>a</sup>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	P <sup>ca</sup>	T <sup>da</sup>	M <sup>ea</sup>	Σ <sup>fa</sup>
J <sup>b</sup>	S	S	M	T	W	T	F	S	S	M	T	W	T	F				
J1	Office-hour duty														0	0	0	0
J2				T	T	T	P	P		T	T	P	M	M	3	5	2	10
J3	P		T	P	P	P	P			M	M	M			5	1	3	9
J4		P	P	T	P	P	M	M	M				T	P	5	2	3	10
J5	M	M	M				T	T	P	P	T	T		T	2	5	3	10
J6	M	M	M				T	T	T	P	P		T	T	2	5	3	10
J7	T	P		M	M	M								P	2	1	3	6
J8			T	P	P	P	P			M	M	M			4	1	3	8
J9					T	T	T		T	T	P	P	M	M	2	5	2	9
J10	P	T	P	T			T	P	P	P	P	T	P		7	4	0	11
J11	T		T	M	M	M				seminar			P	P	2	2	3	7
J12		T	P	P	T	T	M	M	M				T	T	2	5	3	10
P <sup>cb</sup>	3	2	3	4	4	4	3	2	3	3	3	3	3	4				
T <sup>db</sup>	2	2	4	3	3	3	4	3	2	3	3	2	3	3				
M <sup>eb</sup>	3	3	3	3	3	3	3	3	3	3	3	3	3	3				
Σ <sup>fb</sup>	8	7	10	10	10	10	10	8	8	9	9	8	9	10				

a. Days; b. Nurses; c. Morning shifts; d. Evening shifts; e. Night shifts; f. Total

Table 4. The Duty Roster Developed By Using The 01GP Technique

D <sup>a</sup>	1	2	3	4	5	6	7	8	9	10	11	12	P <sup>ca</sup>	T <sup>da</sup>	M <sup>ea</sup>	Σ <sup>fa</sup>
J <sup>b</sup>	S	S	M	T	W	T	F	S	S	M	T	W				
J1	M	M	M			T	T	T		P	P	P	3	3	3	9
J2	M	M	M			T	T	T		P	P	P	3	3	3	9
J3	M	M	M			T	T	T		P	P	P	3	3	3	9
J4	P	P	P	M	M	M			T	T	T		3	3	3	9
J5	P	P	P	M	M	M			T	T	T		3	3	3	9
J6	P	P	P	M	M	M			T	T	T		3	3	3	9
J7	T	T		P	P	P	M	M	M			T	3	3	3	9
J8	T	T		P	P	P	M	M	M			T	3	3	3	9
J9	T	T		P	P	P	M	M	M			T	3	3	3	9
J10			T	T	T		P	P	P	M	M	M	3	3	3	9
J11			T	T	T		P	P	P	M	M	M	3	3	3	9
J12			T	T	T		P	P	P	M	M	M	3	3	3	9
P <sup>cb</sup>	3	3	3	3	3	3	3	3	3	3	3	3				
T <sup>db</sup>	3	3	3	3	3	3	3	3	3	3	3	3				
M <sup>eb</sup>	3	3	3	3	3	3	3	3	3	3	3	3				
Σ <sup>fb</sup>	9	9	9	9	9	9	9	9	9	9	9	9				

a. Days; b. Nurses; c. Morning shifts; d. Evening shifts; e. Night shifts; f. Total

4. CONCLUSION

Scheduling the nurse to their duty roster is an important and repetitive task to the head nurse. The development of the master plan for cyclical nurse scheduling model helps the head nurse solving the nurse scheduling problem and gives a better result of the duty roster compared to the manual duty roster. Although the process of getting the result takes few times to run, but the output is reasonable

and being accepted by both the hospital and the nurses. The 01GP technique has been proved to solve more than one objective effectively. The cyclical scheduling aides the head nurse to have less effort during the scheduling process. The head nurse also can prepare the long term planning of activity for each nurse in advance. For further research, the model can be improved by adding up different sizes of nurses to cater different types of ward.



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